REQUIRED

AP Calculus Summer Work 2023-2024

Welcome to AP Calculus!

This packet is intended to prepare you for the course by:

• Reviewing prerequisite algebra and pre-calculus skills.

• **For BC students only**: Covering Chapter 1 of the calculus textbook. (With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter before the school year.)

Directions: AP Calculus AB students, complete to #103
AP Calculus BC students, complete entire packet AND email Mr. Grappone for an additional KHAN Academy Activity

It is due at the end of the first week of school and counts as a quiz grade (2 pts per completed page). The packet is lengthy, so please start early. While many of the exercises cover basic algebra skills, you will encounter a few tough exercises. If you need assistance completing this packet, please use khan academy or peers to help.

We are looking forward to a great school year! Calculus truly is a fascinating course—you will love it!

– Mrs. Catherine Deitelbaum and Mr. Michael Grappone

cdeitelbaum@sheltonpublicschools.org
mgrappone@sheltonpublicschools.org

The following formulas and identities will help you complete this packet.

Additionally, students are expected to know ALL of these by memory for the course.

Linear forms: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

Quadratic forms: $y = ax^2 + bx + c$ $y = a(x - h)^2 + k$ $y = a(x - p)(x - q)$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin(2x) = 2\sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x$

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$ $= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Exponential Properties: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

Logarithms: $y = \log_a x$ is equivalent to $a^y = x$

Logarithmic Properties: $\log_b mn = \log_b m + \log_b n$ $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

For #1-10, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

2. Crossing the x-axis at $x = -3$ and the y-axis at $y = 6$

3. Containing the points $(-6, -1)$ and $(3,2)$

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

6. Write an equation in point-slope form passing through $(0,5)$ with a slope of $\frac{2}{3}$.

7. Write an equation of a line passing through $(2,8)$ that is parallel to $y = \frac{5}{6}x - 1$.

8. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

9. Write an equation of a line with an x-intercept of $(2,0)$ and a y-intercept of $(0,3)$.

10. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

For #11-18, solve each equation for x. Note that some equations with have a specific value, but most will have a solution for x in terms of other variables. For example: $x = \frac{a+b}{c}$ *would be a solution.*

11. $x^2 + 3x = 8x - 6$

12. $\frac{2x-5}{x+y} = 3 - y$

13. $3xy + 6x - xz = 12$

14. $A = ax + bx$

15. $cx = vx$

16. $r = t - x(z - y)$

17. $\frac{3+x}{5-x} = 6 + y$

18. $\frac{y+2}{4-x} = 4(2 - z)$

For #19-24, solve each quadratic by factoring.

19. $x^2 - 4x - 12 = 0$

20. $x^2 - 6x + 9 = 0$

21. $x^2 - 9x + 14 = 0$

22. $x^2 - 36 = 0$

23. $9x^2 - 1 = 0$

24. $4x^2 + 4x + 1 = 0$

For #25-29, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

25. $f\left(\frac{1}{2}\right) =$

26. $g(-2) =$

27. $f(1) + g(0) =$

28. $f(0) \cdot g(0) =$

29. $\frac{g(-6)}{f(-6)} =$

Recall function composition notation $(f \circ g)(x)$ is the same thing as $f(g(x))$.

For #30-39, use $f(x) = x^2 - 1$, $g(x) = 3x$, *and* $h(x) = 5 - x$ *to find each composite function.*

30. $(f \circ g)(x) =$

31. $(g \circ f)(x) =$

32. $(f \circ f)(4) =$

33. $(g \circ h)(-4) =$

34. $(f \circ (g \circ h))(1) =$

35. $(g \circ (g \circ g))(5) =$

36. $f(g(x-1)) =$

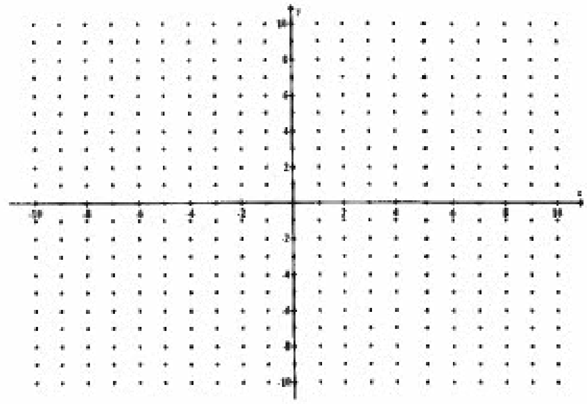
37. $g(f(x^3)) =$

38. $\frac{f(x+h)-f(x)}{h} =$

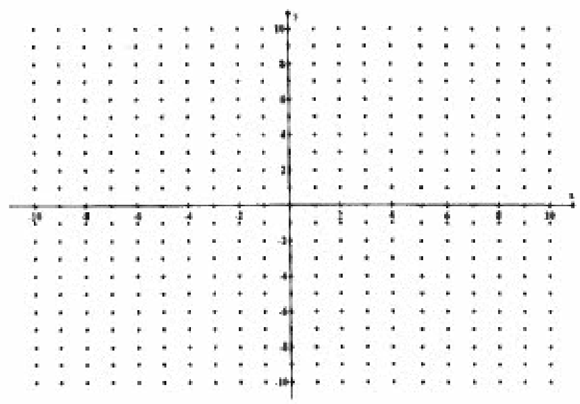
39. The expression in the previous problem is very significant and important in Calculus. Think back to Pre-Calculus... what is the name of that expression? ______________

For #40-42, graph each piecewise function.

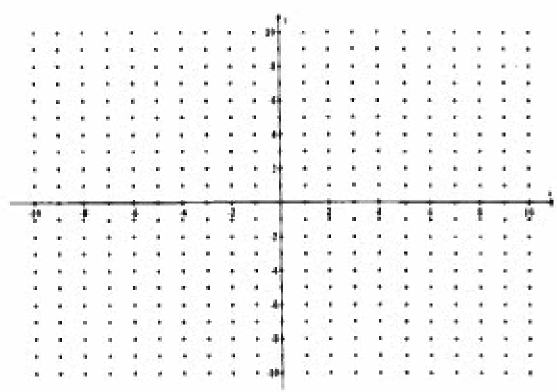
40. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



41. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



42. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2-|x-2| & ; \ x > 1 \end{cases}$



*An **exponential equation** is an equation in which the variable is in the exponent. To solve an exponential equation, you must use a logarithm to solve it.*

For #43-47, solve each exponential equation, rounding answers to the nearest thousandth. Note that some equations can be solved by writing each side as the same base instead of using a logarithm.

43. $5^x = \frac{1}{5}$

44. $6^x = 1296$

45. $6^{2x-7} = 216$

46. $5^{3x-1} = 49$

47. $10^{x+5} = 125$

For #48-51, simplify each expression without the use of a calculator.

48. $e^{\ln 4} =$

49. $e^{2\ln 3} =$

50. $\ln e^{9} =$

51. $5\ln e^{3} =$

For #52-57, solve each equation using natural logarithm. Round answers to the nearest thousandth.

52. $e^{x} = 34$

53. $3e^{x} = 120$

54. $e^{x} - 8 = 51$

55. $\ln x = 2.5$

56. $\ln(3x - 2) = 2.8$

57. $\ln(e^{x}) = 5$

For #58-66, find each exact value of the expression using the Unit Circle. ***NO CALCULATOR!***

58. $\sin 120° =$ ________

59. $\cos \frac{11\pi}{6} =$ ________

60. $\tan 225° =$ ________

61. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

62. $\sin 150° =$ ________

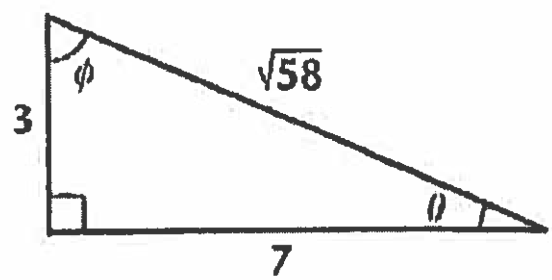
63. $\tan \frac{7\pi}{4} =$ ________

64. $\csc\left(\frac{\pi}{4}\right) =$ ________

65. $\sec(-210°) =$ ______

66. $\cot\left(\frac{5\pi}{4}\right) =$ ________

For #67-74, evaluate each trigonometric expression using the right triangle provided.



67. $\sin \theta =$ ________

68. $\cos \theta =$ ________

69. $\tan \phi =$ ________

70. $\csc \phi =$ ________

71. $\sec \theta =$ ________

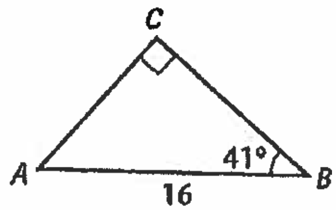
72. $\cot \theta =$ ________

73. $\sin \phi =$ ________

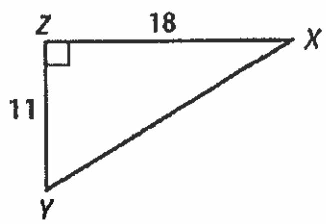
74. $\sec \phi =$ ________

For #75-76, solve each triangle, rounding all angles and sides to the nearest thousandth. Recall that "solve a triangle" means to find all missing sides and angles.

75.



76.



For #77-84, evaluate each inverse trigonometric function. ***NO CALCULATOR!***

77. $\sin^{-1}\left(\frac{1}{2}\right) =$ ______

78. $\sin^{-1}(-1) =$ ______

79. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ______

80. $\tan^{-1}(\sqrt{3}) =$ ______

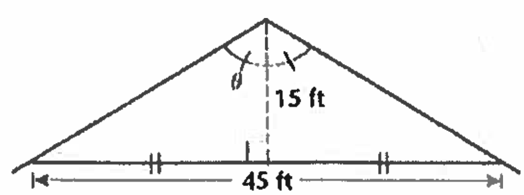
81. $\tan^{-1}(-1) =$ ______

82. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ ______

83. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ ______

84. $\sin^{-1}(\cos(0)) =$ ______

85. Find the angle at the peak of the roof, as shown in the picture. Round to the nearest thousandth.



86. Explain how the graph of $f(x)$ and $f^{-1}(x)$ compare.

Recall that to find an inverse of a function, simply switch the x and y and solve for y. We use the notation $f^{-1}(x)$ to define the inverse of $f(x)$.

For #87-89, find the inverse of each function.

87. $g(x) = \frac{5}{x-2}$

88. $f(x) = \frac{x^2}{3}$

89. $y = \sqrt{4-x} + 1$

90. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$?

For #91-94, convert the inequalities in to ***interval notation****. For example, $x > 3$ becomes $(3, \infty)$.*

91. $1 < x \leq 10$

92. $x < 0$ or $x \geq 4$

93. $x \geq -2$

94. $x \geq 4$ and $x > 10$

For #95-100, find the domain and range of each function. Write the answer in interval notation. Confirm your answer by graphing the function in your graphing calculator.

95. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

96. $f(x) = x^2 - 5$

D: ____________________ R: ____________________

97. $f(x) = \frac{1}{x+7}$

D: ____________________ R: ____________________

98. $f(x) = \frac{5}{x^2+1}$

D: ______________________ R: ______________________

99. $f(x) = \sqrt{x^2+5}$

D: ______________________ R: ______________________

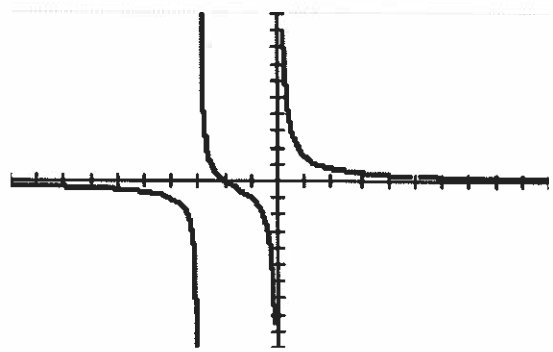
100. $g(x) = x^3 + 2x - 7$

D: ______________________ R: ______________________

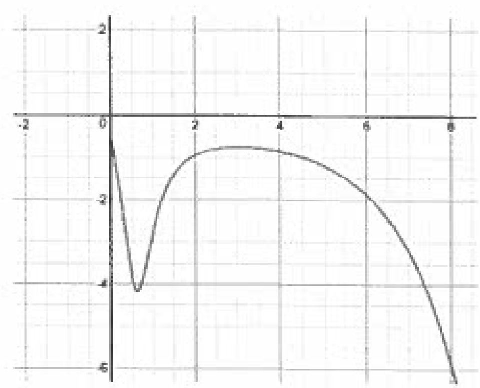
For #101-103, answer the question by referring to the function and its graph.

101. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$

Hint: There is a hole.



102. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

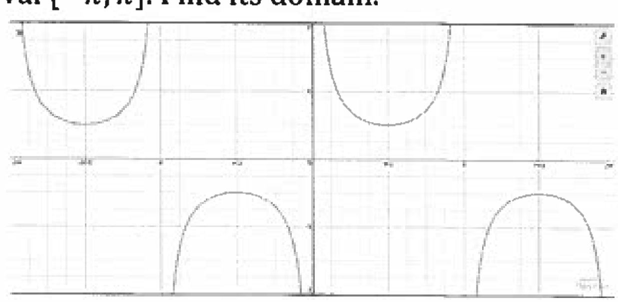


Find the maximum and minimum y-value of the function.

State the domain of $f(x)$.

State when the function is increasing and decreasing (write in interval notation).

103. Consider the function $f(x) = \csc x$ on the interval $[-\pi, \pi]$. Find its domain.



The difference quotient is defined to be $\frac{f(x+h)-f(x)}{h}$ *and is a core concept for the development of calculus. For #104-107, find the difference quotient of each function.*

104. $f(x) = 9x + 3$

105. $f(x) = 5 - 2x$

106. $f(x) = x^2 - 3x$

107. $f(x) = \frac{2}{x+1}$

108. Evaluate the following limits algebraically. If you cannot do it algebraically, view its graph.

$\lim_{x \to 1} e^{x^3 - x} =$

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} =$$

$$\lim_{x\to 5^+} \frac{x+5}{x-5} =$$

$$\lim_{x\to 0} \frac{\sin x}{3x} =$$

$$\lim_{h\to 0} \frac{(h-1)^3+1}{h} =$$

$$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} =$$

$$\lim_{x\to c} x =$$

$$\lim_{x\to a} c =$$

$$\lim_{h\to 0} \frac{h}{\sqrt{x+h}-\sqrt{x}} =$$

$$\lim_{x\to \infty} \frac{3x^3+5x^2-7x}{8x^3-13} =$$

$$\lim_{v\to 4^+} \frac{4-v}{|4-v|} =$$

$$\lim_{x\to \infty} \frac{\sqrt{x^2-9}}{2x-6} =$$