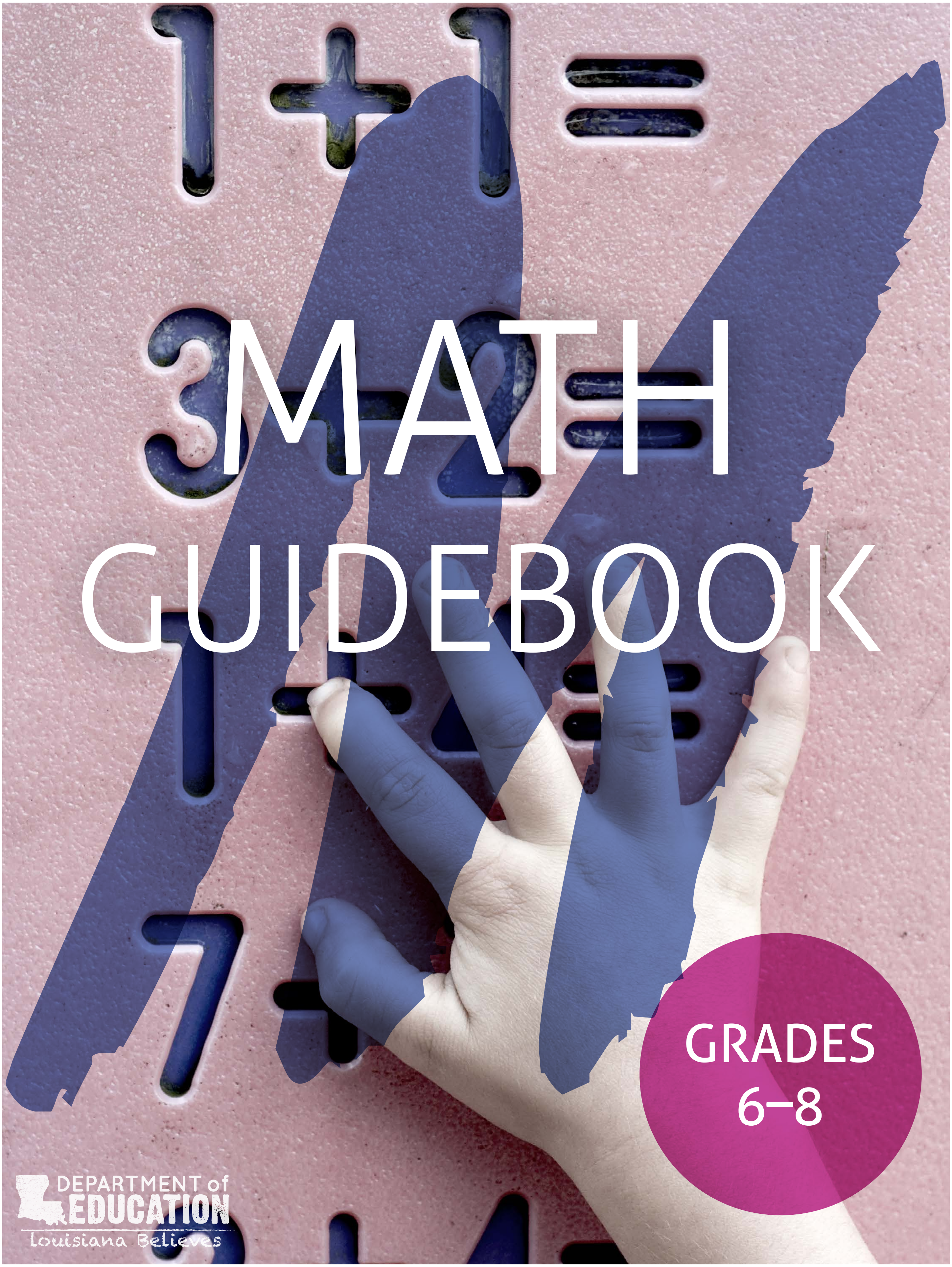


MATH GUIDEBOOK

GRADES 6–8

DEPARTMENT of EDUCATION
Louisiana Believes

CONTENTS

INTRODUCTION 5

- Louisiana Believes 5
- How to Use the Math Guidebook 6
- How to Read the Math Guidebook 6

MATHEMATICS OVERVIEW 8

- Standards Shifts 8
- Assessment and Instruction 11
    - Rigor 11
    - Remediation 11

TOOLS FOR TEACHING 13

6TH GRADE TOOLS 15

- 6th Grade Remediation Guide 15
- 6th Grade Tasks At a Glance 21
- 6th Grade Extended Constructed Response Tasks 23
- 6th Grade Instructional Tasks 47

7TH GRADE TOOLS 77

- 7th Grade Remediation Guide 77
- 7th Grade Tasks At a Glance 82
- 7th Grade Extended Constructed Response Tasks 84
- 7th Grade Instructional Tasks 108

8TH GRADE TOOLS 139

- 8th Grade Remediation Guide 139
- 8th Grade Tasks At a Glance 144
- 8th Grade Extended Constructed Response Tasks 146
- 8th Grade Instructional Tasks 174

APPENDIX (6-8 MATH STANDARDS) 223

This public document was published at a cost of $5,772.67. Five hundred (500) copies of this public document were published at this first printing at a cost of $5,772.67. The total cost of all printings of this document, including all reprints, was $5,772.67. This document was printed by the Office of Content; Louisiana Department of Education; P.O. Box 94064, Baton Rouge, LA 70804-9064. This material was printed in accordance with the standards for printing by State agencies established pursuant to R.S. 43.31.

Louisiana Believes...

Louisiana students...are **just as capable as students anywhere. They deserve high expectations** with support to reach them so that they are prepared to complete college and attain a professional career.

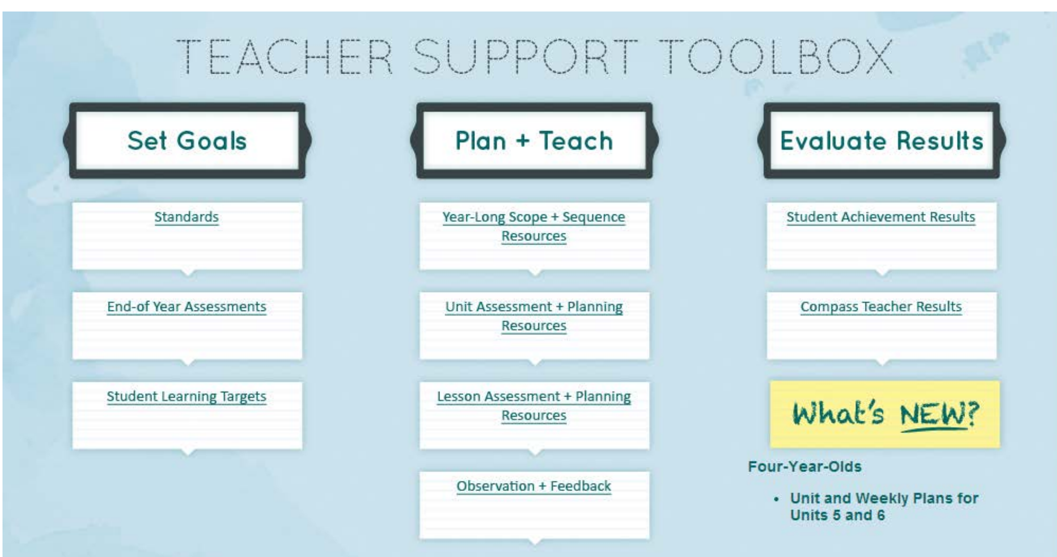
Louisiana teachers...will understand those expectations and work with their peers to make individual decisions to **meet their students' needs through planning and instruction.**

Louisiana principals and schools...will **create and lead meaningful structures of feedback and collaboration** to ensure teachers are able to learn and grow with support and guidance.

Louisiana districts...will **choose strong assessment and curricular** plans and **build systems that support school leaders** with goal setting, feedback, and collaboration.

Louisiana's Department of Education...will continue to shift away from prescribing local decisions and instead **provide resources, data, models, and direct teacher, principal, and district support.**

At the heart of these beliefs is good classroom teaching and learning. Effective instruction stems from the constant cycle of setting an ambitious goal, planning and teaching, and evaluating results. Our Teacher Support Toolbox in Louisiana is built to support these core actions of teachers. This instructional guidebook is a printed companion to our Teacher Support Toolbox. The guidebooks and the Teacher Support Toolbox, when used together, should support teachers and schools to make informed but independent decisions about how to provide rigorous but unique instruction in each classroom around the state.



http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox

How to Use the Math Guidebook

This guide is meant to support teachers in supplementing math instruction for students. Each group of students has a unique set of needs thus, the department is not mandating that teachers use the instructional tools shared in this guide. Instead, the models are provided as a starting point for teams of teachers to use in planning for the unique needs of their students.

This guide provides:

- An explanation of strong math instruction
- Grade-level and standard specific remediation guidance
- Instructional tasks aligned to the state standards for math

This guide **does not** provide:

- A complete curriculum
- A set of plans that should be taught exactly the same in every classroom
- Daily lesson plans that all math teachers must use in their classroom

How to Read This Guide

There are two sections of this guide, which function differently.

- ***Mathematics Overview*** (page 8): This section describes how teachers approach the shifts called for in Louisiana's new math standards.
- ***Tools for Teaching*** (page 13): This section provides grade-level instructional tasks and remediation guidance. These tasks are meant to serve as a supplement to a curriculum already in use in a classroom. Teachers should collaborate to adjust these tasks to meet the needs of their students.

In addition, this guide is a companion to additional resources within the Teacher Support Toolbox.[1] Thus, throughout the guide you will see the following icons that highlight key connections.

Online Teacher Toolbox Resources: Notes a recommendation to find more available resources in the Teacher Support Toolbox.

Multimedia Components: Notes a recommendation to find a resource or video hosted on an outside Internet site.

abc ***Statewide Assessment***: Illustrates how a component of this guide connects to the statewide assessment students will take.

CC ***Compass Connections***: Illustrates the connections between instructional content and the Compass rubric.

As always, we welcome questions and feedback on these materials. If you need any support, do not hesitate to reach us at classroomsupporttoolbox@la.gov.

[1] http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox

MATH OVERVIEW

MATHEMATICS OVERVIEW

STANDARDS SHIFTS

Louisiana's math standards (see "APPENDIX" on page 223) help students practice and master rigorous mathematical concepts. These new standards require students to answer complex math questions correctly and also to explain their thinking on how they arrived at the answer. Because these new standards ask students to go deeper in their exploration of math content, teachers will need to shift how they plan lessons and deliver instruction.

These major shifts include:

Shift 1: Focus strongly where the standards focus.

Definition of this shift: The standards call for a greater focus in mathematics. Rather than racing to cover topics in a mile-wide, inch-deep curriculum, the standards require teachers to significantly narrow the concepts covered in a single year and deepen student understanding of those concepts. The standards focus on the major work of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom. The major work of each grade includes:

- K-2: Addition and subtraction—concepts, skills, and problem solving; place value
- 3-5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
- 6: Ratios and proportional relationships; early expressions and equations
- 7: Ratios and proportional relationships; arithmetic of rational numbers
- 8: Linear algebra and linear functions

Shift 2: Coherence: Think across grades, and link to major topics within grades.

Definition of this shift: The standards progress from grade to grade in a coherent way. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new concept, but an extension of previous learning.

Illustration of this shift: The standards are written to connect between grade levels and within a grade level. The first illustration on page 9 shows a sample algebraic idea and how it is scaffolded from 6th grade to high school. The second illustration shows how standards connect to each other within one grade level.

One of several staircases to algebra.

A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression: add, subtract, multiply, and divide rational expressions.

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

8.EE.7b Solve linear equations (in one variable) with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coeffecients.

6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2+x) to produce the equivalent expression 6+3x; apply the distributive property to the expression 24x+18y to produce the equivalent expression 6(4x+3y); apply properties of operations to y+y+y to produce the equivalent expression 3y.*

Example: Equations and Expressions

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Standard 7.EE.3

Shift 3: Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Definition of this shift:

- ***Conceptual understanding***: The standards call for conceptual understanding of key concepts. Conceptual understanding helps students to explain how they got the correct answers and allows them to apply what they have learned to new types of problems.
- ***Procedural skill and fluency***: The standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions, such as single-digit multiplication, so that they have access to more complex concepts and procedures.
- ***Application***: The standards call for students to use math flexibly for applications in problem-solving contexts. In content areas outside of math, particularly science, students are given the opportunity to use math to make meaning of and access content.

Illustration from the Unit Plans: See the next section, Rigor, for an illustration of this shift.

To find your grade-level standards, go to the "APPENDIX" on page 223 of this document.

To find learning modules to help you better understand the standards, go to the standards page[2] in the Teacher Support Toolbox.

The new math standards are well researched.[3] Do not miss out on reviewing the research behind this approach to math instruction.

[2] http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox/standards

[3] http://www.achievethecore.org/dashboard/2/search/6/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/407/mathematics-research-and-articles

ASSESSMENT AND INSTRUCTION

Instructionally, the most challenging shift comes with the focus on rigor. Rigor in a math classroom can be extremely difficult to nail down. So often, educators are tempted to practice procedures with students rather than help them master the mathematical concepts. This shift from simply practicing and assessing procedures with students to practicing and assessing concepts is challenging, but critical. Even more, with the increased expectations brought on with the shift in rigor, remediation becomes critical. Teachers must work to identify which students need remediation and on which standards remediation would be most beneficial for these students. Thus, rigor and the needed remediation associated with increased rigor are two of the most important shifts teachers must be aware of as they change their instruction and assessment.

Rigor

While student fluency with math skills is critical, even more important is a student's ability to show mastery of a mathematical concept. State assessments will no longer demand that students simply perform based on memorized basic procedures. Rather, just as in real life, students are asked to solve complex problems based on their mathematical understanding.

So what does this really mean? Let's take a sample standard and consider what it would look like to teach and assess the procedure versus the concept.

Standard: **8.EE.C.8b**: Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

- The following is an item that would be used to teach and assess the procedure:
  - » Use the substitution method to solve the linear system:
    $2x + 5y = 12$
    $2x + 5y = 4$
- The following is an item that would be used to teach and assess the concept:
  - » Olivia said she can solve the system $2x + 5y = 12$ and $2x + 5y = 4$ in her head without doing any written work. Explain what you think she did, what you think her solution was, and how you know you are right.

In both examples, students need to know the steps and procedures to actually solve the equation. But in the second, memorizing the procedure alone is not enough. For students to apply mathematical understanding in the future, in a variety of settings, they must know why they are using the procedures and how to adapt them to fit new settings.

The tasks included in this guidebook deliberately help students explore, practice, and show mastery of the mathematical concepts demanded in the standards. This includes students' fluent use of basic math skills but pushes them beyond simple memorization to deep understanding of the content.

Remediation

As the rigor increases for students, so do the potential gaps in their understanding. Often, the instinct is to say that if a student is not at grade level, a teacher must completely go back to previous grade levels and remediate everything before moving on. In math, like other content areas, students do need quality remediation. But that remediation must be focused just on the content needed to quickly get students to practice at grade level. By practicing content at grade level, students more quickly improve their skill and understanding.

Let's look at an example.

Let's say 6th grade students are working on standard **6.EE.A.3:** Apply the properties of operations to generate equivalent expressions.

If the students are struggling in 6th grade with this standard, there are a few isolated standards from previous grade levels and from within 6th grade that will prepare students for this standard.

Standards from previous grades that prepare students include:

- **1.OA.B.3:** Apply properties of operations as strategies to add and subtract.
- **3.OA.B.5:** Apply properties of operations as strategies to multiply and divide.
- **5.OA.A.2:** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Standards from 6th grade that should be taught in advance of the above standard include:

- **6.NS.B.4:** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- **6.EE.A.2:** Write, read, and evaluate expressions in which letters stand for numbers.

Standards from 6th grade that should be taught at the same time as the above standard include:

- **6.EE.A.4:** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

The writers of the math standards created a tool[4] to help teachers more quickly determine the required previous content needed for each individual standard. This guide has taken that tool and created easy-to-access charts for teachers by grade level (in the "Tools for Teaching" section that follows). These charts will help teachers more quickly identify just the necessary remediation. This will allow students faster access to grade-level content, allowing them to grow and also practice basic skills in a more authentic setting.

Every task included in this guide includes the recommended remedial standards along with sample tasks to check on student readiness for the grade-level task. Below is an example of a chart included in every task included in this guide. The links provide sample practice problems to help teachers remediate with students in preparation for grade-level task.

Grade-Level Standard	The Following Standards Will Prepare Students:	Items to Check for Task Readiness:	Sample Remediation Items:
6.EE.A.3	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2 • 6.NS.B.4 • 6.EE.A.2	Use the distributive property to rewrite the expression 4(3y + 6x). 12y + 24x	http://www.illustrativemathematics.org/illustrations/139 http://www.illustrativemathematics.org/illustrations/255 http://learnzillion.com/lessonsets/567-apply-properties-of-operations-to-generate-equivalent-expressions http://learnzillion.com/lessonsets/480-apply-properties-of-operations-to-generate-equivalent-expressions http://learnzillion.com/lessonsets/237-apply-properties-of-operations-to-generate-equivalent-expressions

[4] http://www.edutron.com/0/Math/ccssmgraph.htm

TOOLS FOR TEACHING

6TH GRADE TOOLS

6TH GRADE TOOLS

6th Grade Remediation Guide

As noted in "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every sixth grade math standard[5].

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	• 4.OA.A.2 • 5.NF.B.5 • 5.OA.B.3		
6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	• 4.MD.A.1 • 5.NF.B.3 • 5.NF.B.7	• 6.RP.A.1	
6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	• 5.G.A.2	• 6.RP.A.1	• 6.EE.B.9
6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*		• 6.RP.A.2 • 6.RP.A.3a	• 6.EE.B.7 *(whole numbers)* • 6.EE.B.7 • 6.EE.B.9
6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.		• 6.RP.A.2	• 6.EE.B.7
6.RP.A.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.		• 6.RP.A.2	

[5] This content comes from the work of the math standards' authors found here: http://www.edutron.com/0/Math/ccssmgraph.htm

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (⅔) ÷ (¾) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (⅔) ÷ (¾) = 8/9 because ¾ of 8/9 is ⅔. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many ¾-cup servings are in ⅔ of a cup of yogurt? How wide is a rectangular strip of land with length ¾ mi and area ½ square mi?*	• 3.OA.B.6 • 5.NF.B.7		
6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm.	• 5.NBT.B.6		
6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	• 5.NBT.B.5 • 5.NBT.B.6 • 5.NBT.B.7	• 6.NS.B.2	
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	• 4.OA.B.4 • 5.OA.A.2		
6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	• None *Introduced in 6th Grade*		
6.NS.C.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.	• 3.NF.A.2	• 6.NS.C.5	
6.NS.C.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	• 5.G.A.1	• 6.NS.C.6a	• 6.NS.C.6c
6.NS.C.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	• 5.G.A.1	• 6.NS.C.6a	• 6.NS.C.6b

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.NS.C.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*		• 6.NS.C.6c	• 6.NS.C.7b
6.NS.C.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*		• 6.NS.C.6c	• 6.NS.C.7a
6.NS.C.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*		• 6.NS.C.6a	
6.NS.C.7d Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*		• 6.NS.C.7a • 6.NS.C.7b • 6.NS.C.7c	
6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	• 5.G.A.2	• 6.NS.C.6b	• 6.G.A.3
6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.	• 4.OA.B.4 • 5.NBT.A.2		

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = ½.*	• 5.OA.A.2 • 5.OA.B.3	• 6.EE.A.1	
6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2	• 6.NS.B.4 • 6.EE.A.2	• 6.EE.A.4
6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2	• 6.NS.B.4 • 6.EE.A.2	• 6.EE.A.3
6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.		• 6.EE.A.2	• 6.EE.B.7 *(whole numbers)* • 6.EE.B.8
6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		• 6.EE.A.2	• 6.EE.B.7 *(whole numbers)*

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.EE.B.7 (whole numbers) Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	• 5.NF.A.1 • 5.NF.B.4	• 6.NS.A.1	• 6.RP.A.3b • 6.EE.B.5 • 6.EE.B.6 • 6.EE.C.9
6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.		• 6.EE.B.7 (whole numbers)	• 6.RP.A.3b • 6.RP.A.3c • 6.EE.C.9
6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.		• 6.NS.C.6a • 6.NS.C.6c • 6.NS.C.7a • 6.NS.C.7b	• 6.EE.B.5
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	• 5.OA.B.3		• 6.RP.A.3a • 6.RP.B.3b • 6.EE.B.7 *(whole numbers)* • 6.EE.B.7
6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	• 4.MD.A.3 • 5.NF.B.4		
6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	• 5.MD.C.5a • 5.MD.C.5b		
6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	• 5.G.A.2		• 6.NS.C.8

6th Grade Standard	Previous Grade Standards	6th Gr. Stand. Taught in Advance	6th Gr. Stand. Taught Concurrently
6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.		• 6.G.A.1	
6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.	• 5.MD.B.2		
6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	• 5.MD.B.2		
6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.		• 6.SP.A.1 • 6.SP.A.2	
6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	• 5.MD.B.2		• 6.SP.B.5
6.SP.B.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.		• 6.SP.A.2 • 6.SP.A.3	• 6.SP.B.4

6th Grade Tasks At a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for sixth grade include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standard. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the sixth grade tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Gym Use Page 23	ECR	• 6.EE.A.2.a • 6.EE.A.2.b • 6.EE.B.6 • 6.EE.B.7	• 5.OA.A.2 • 5.OA.B.3 • 5.NF.A.1 • 5.NF.B.4 • 6.EE.A.2 • 6.NS.A.1
Student Council Popcorn Sales Page 27	ECR	• 6.EE.A.2b • 6.EE.A.3 • 6.EE.A.4	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2 • 5.OA.B.3 • 6.NS.B.4 • 6.EE.A.1 • 6.EE.A.2
Cutting Grass Page 32	ECR	• 6.NS.A.1	• 3.OA.B.6 • 5.NF.B.7
Low Temperatures Page 38	ECR	• 6.NS.C.5 • 6.NS.C.6 • 6.NS.C.7	• 3.NF.A.2 • 6.NS.C.6c • 6.NS.C.6a
Friends Meeting on Bicycles Page 42	ECR	• 6.RP.A.3b	• 6.RP.A.2 • 6.RP.A.3a
Chocolate Chip Cookies Page 47	IT	• 6.RP.A.3c • 6.RP.A.3d • 6.NS.A.1	• 3.OA.B.6 • 5.NF.B.7 • 6.RP.A.2
Word of Mouth Page 53	IT	• 6.EE.A.1	• 4.OA.B.4 • 5.NBT.A2
The Elevator Limit Page 57	IT	• 6.EE.B.5 • 6.EE.B.8	• 6.EE.A.2 • 6.NS.C.6a • 6.NS.C.6c • 6.NS.C.7a • 6.NS.C.7b

Title	Type	Task Standards	Task Remedial Standards
Crawfish Boil [Page 61]	IT	• 6.RP.A.1 • 6.RP.A.2 • 6.RP.A.3a • 6.RP.A.3b	• 4.OA.A.2 • 4.MD.A.1 • 5.G.A.2 • 5.NF.B.3 • 5.NF.B.5 • 5.NF.B.7 • 5.OA.B.3 • 6.RP.A.1 • 6.RP.A.2 • 6.RP.A.3a
Marathon Prep [Page 69]	IT	• 6.EE.B.7 • 6.EE.C.9	• 5.NF.B.4 • 5.OA.B.3

Gym Use (ECR)

Overview

Students are asked to write and use expressions representing basketball field goals and gym membership fees.

Standards

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $v = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with side lengths* $s = ½$.

Reason about and solve one-variable equations and inequalities.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equation of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.EE.A.2a	• 5.OA.A.2 • 5.OA.B.3	1. Write an expression to represent "the sum of a number and 9." a. *n + 9* 2. Write an expression to represent "Linzey makes some cookies, and John makes 6 more." a. c + 6	• http://www.illustrativemathematics.org/illustrations/556 • http://www.illustrativemathematics.org/illustrations/590 • http://learnzillion.com/lessonsets/69-read-write-and-evaluate-algebraic-expressions
6.EE.A.2c	• 5.OA.A.2 • 5.OA.B.3	1. What is the value of 3x if x = 5? a. 15 2. What is the value of x + 15 if x = 12? a. 27	• http://www.illustrativemathematics.org/illustrations/139 • http://learnzillion.com/lessonsets/69-read-write-and-evaluate-algebraic-expressions
6.EE.B.6	• 6.EE.A.2	1. If 3n is equal to 36, then what does the *n* represent? a. 12 2. If 5n is equal to 25, then what does 4n represent?	• http://www.illustrativemathematics.org/illustrations/421 • http://www.illustrativemathematics.org/illustrations/540 • http://learnzillion.com/lessonsets/556-

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		a. 20 3. http://www.illustrativemathematics.org/illustrations/425	understand-that-variables-represent-unknown-numbers-and-use-variables-to-solve-problems • http://learnzillion.com/lessonsets/490-use-variables-to-represent-numbers-and-write-expressions-to-solve-problems
6.EE.B.7	• 5.NF.A.1 • 5.NF.B.4 • 6.NS.A.1	1. Solve 5m = 60. a. m = 12 2. Solve 6 + w = 12. a. w = 6 3. http://www.illustrativemathematics.org/illustrations/1107	• http://learnzillion.com/lessonsets/577-solve-problems-by-writing-and-solving-equations-of-the-form-x-p-q-and-px-q • http://learnzillion.com/lessonsets/269-solve-problems-with-equations-xpq-and-pxq • http://learnzillion.com/lessonsets/72-write-problems-using-algebraic-expressions

Real-World Preparation: The following question will prepare students for some of the real-world components of this task:

What is a gym membership fee? A gym membership fee is an amount paid for a set time of use at a gym. For example, a yearly membership could be purchased. This membership would allow a member to use the gym for a year.

After the Task

Students may struggle with determining how many goals the two people make during the last 20 minutes. They may want to solve for the whole time. This could be a misunderstanding of the question or an inability to understand the parts of the expression and what they mean. They may copy totals from the table when answering questions about the gym membership.

Student Extended Constructed Response

1. Suzy is shooting basketballs at Bob's Gym. She makes 9 field goals. After 10 minutes, her friend Joshua joins her. They shoot basketballs together for another 20 minutes.

   a. Write an expression for the total number of field goals Suzy and Joshua make during the 30 minutes they spend at the gym. Be sure to define the variable used in the expression.

   b. Altogether, Suzy and Joshua made 27 goals. Use the expression you created to determine the number of field goals they made in the last 20 minutes. Explain your thinking.

2. The following chart details monthly membership costs at Bob's Gym.

Number of Months	Cost ($)
3	60
5	100
9	180

   a. Write an expression to represent the cost of (*m*) number of months of gym use.

   b. Suzy wants to purchase a 7-month membership. How much will the membership cost? Justify your answer.

   c. If she has $250, what is the maximum number of months for which she can purchase a membership? Justify your answer.

Extended Constructed Response Exemplar Response

1. Suzy is shooting basketballs at Bob's Gym. She makes 9 field goals. After 10 minutes, her friend Joshua joins her. They shoot basketballs together for another 20 minutes.

   a. Write an expression for the total number of field goals Suzy and Joshua make during the 30 minutes they spend at the gym. Be sure to define the variable used in the expression.
   b + 9; b represents the number of baskets that Suzy and Joshua make together during the last 20 minutes.

   b. Altogether, Suzy and Joshua made 27 goals. Use the expression you created to determine the number of goals they made in the last 20 minutes. Explain your thinking.
   Since they scored 27 field goals over 30 minutes, b + 9 = 27. I have to take away the 9 scored by Suzy, which leaves 18 goals scored during the last 20 minutes.

2. The following chart details monthly membership costs at Bob's Gym.

Number of Months	Cost ($)
3	60
5	100
9	180

   a. Write an expression to represent the cost of *m* (number of months) for gym use.
   The cost is 20m.

   b. Suzy wants to purchase a 7-month membership. How much will the membership cost? Justify your answer.
   20(7) = 140; A 7-month membership will cost her $140.

   c. With $250 to spend, what is the maximum number of months for which she can purchase a membership? Justify your answer.
   Suzy can purchase a membership for a maximum of 12 months because 20 (12) = 240. Since she only has $10 left, she cannot purchase the 13th month.

Student Council Popcorn Sales (ECR)

Overview

This task requires students to write and identify equivalent expressions to find the amount of popcorn sold. Students are asked specifically to use the distributive property to write some expressions. Students will also have to use variables to write expressions and use substitution to show that the expressions are equivalent.

Standards

6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms*.

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
6.EE.A.2b	• 5.OA.A.2 • 5.OA.B.3 • 6.EE.A.1	1. Write an equivalent expression for 5 + 4 + 5 + 4 + 8 + 10 2. 2(5+4) + 8 + 10 (other answers are possible) 3. Identify the factors in the problem 4(5 + 9) 4. The factors are 4 and (5+9) 5. http://www.illustrativemathematics.org/illustrations/540	• http://www.illustrativemathematics.org/illustrations/556 • http://www.illustrativemathematics.org/illustrations/590 • http://www.illustrativemathematics.org/illustrations/532 • http://learnzillion.com/lessonsets/198-write-read-and-evaluate-expressions-in-which-letters-stand-for-numbers
6.EE.A.3	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2 • 6.NS.B.4 • 6.EE.A.2	1. Use the distributive property to rewrite the expression 4(3y + 6x) a. 12y + 24x	• http://www.illustrativemathematics.org/illustrations/139 • http://www.illustrativemathematics.org/illustrations/139 • http://www.illustrativemathematics.org/illustrations/255 • http://learnzillion.com/lessonsets/567-apply-properties-of-operations-to-generate-equivalent-expressions • http://learnzillion.com/lessonsets/480-apply-properties-of-operations-to-generate-equivalent-expressions

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
			• http://learnzillion.com/lessonsets/237-apply-properties-of-operations-to-generate-equivalent-expressions
6.EE.A.4	• 1.OA.B.3 • 3.OA.B.5 • 5.OA.A.2 • 6.NS.B.4 • 6.EE.A.2	1. Rewrite 3x + 3x + 6y + 6y using the distributive property a. 2(3x + 6y) 2. http://www.illustrativemathematics.org/illustrations/542 3. http://www.illustrativemathematics.org/illustrations/461	• http://www.illustrativemathematics.org/illustrations/421 • http://learnzillion.com/lessonsets/238-identify-when-two-expressions-are-equivalent • http://learnzillion.com/lessonsets/71-identify-equivalent-equations-using-balancing-scales

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a student council? A student council is an organization of students within a school that helps organize fundraisers, dances, activities, and events at a school.

After the Task

This task shows students how to use expressions to help them solve and organize real-life problems. Have students find equivalent expressions using the other students in the table. Students can also substitute in dollar amounts for the expression used in part c and calculate the total amount of money Alysha and Cedric made. You could also assign values to the other student council members and have students find the total amount of money earned over the two-day period.

Student Extended Constructed Response

Student Council Popcorn Sale

The student council at Hillcrest Middle School is having a popcorn sale to raise money for an upcoming school dance. The table below represents the total number of boxes sold by student council members over a two-day period.

	Jonna	Cedric	Miles	Alysha
Thursday	9	8	5	6
Friday	10	8	7	6

a. Which expressions represent the number of boxes of popcorn that Jonna, Cedric, Miles, and Alysha sold both days? Choose all that apply.

  a.) 2(8) + 2(6) + 19 + 12
  b.) 9 + 8 + 5 + 6
  c.) 19 + 16 + 2(12)
  d.) 9 + 10 + 2(6) + 2(8) + 5 + 7
  e.) 8 + 8 + 6 + 6 + 9 + 7

b. The expression 2(8+6) can be used to find the amount of popcorn sold by Alisha and Cedric. Identify the factors used in the expression.

c. The student council realized not everyone would like the same type of popcorn, so Cedric sold butter popcorn, and Alysha sold caramel popcorn. Cedric sold his butter popcorn for *x* dollars, and Alysha sold her caramel popcorn for *y* dollars. Write two equivalent expressions that could be used to find the total amount of money collected by Cedric and Alysha on Thursday and Friday. One of the expressions must demonstrate the distributive property. Explain how you know the expressions you have written are equivalent.

Extended Constructed Response Exemplar Response

Student Council Popcorn Sale

The student council at Hillcrest Middle School is having a popcorn sale to raise money for an upcoming school dance. The table below represents the total number of boxes sold by student council members over a two-day period.

	Jonna	Cedric	Miles	Alysha
Thursday	9	8	5	6
Friday	10	8	7	6

a. Which expressions represent the number of boxes of popcorn that Jonna, Cedric, Miles, and Alysha sold both days? Choose all that apply.

   a.) 2(8) + 2(6) + 19 + 12
   b.) 9 + 8 + 5 + 6
   c.) 19 + 16 + 2(12)
   d.) 9 + 10 + 2(6) + 2(8) + 5 + 7
   e.) 8 + 8 + 6 + 6 + 9 + 7

   Of the choices above, a, c, and d are equivalent expressions that represent the total amount of popcorn sold over the two-day period for all four students.

b. The expression 2(8+6) can be used to find the amount of popcorn sold by Alysha and Cedric over the two-day period. Identify the factors used in the expression.

   The factors are 2 and (8 + 6)

c. The student council realized not everyone would like the same variety of popcorn, so Cedric sold butter popcorn, and Alysha sold caramel popcorn. Cedric sold his butter popcorn for *x* dollars, and Alysha sold her caramel popcorn for *y* dollars. Write two equivalent expressions that could be used to find the total amount of money collected by Cedric and Alysha on Thursday and Friday. One of the expressions must demonstrate the distributive property. Explain how you know the expressions you have written are equivalent.

   Sample Answers: 6x + 6x + 8y + 8y; 8y + 8y + 6x + 6x; 2(6x + 8y); 2(8y + 6x)

   **Note there are other equivalent forms that can be used, as long as one of the expressions involves the distributive property.*

Students can explain their reasoning through substitution, as long as they show equivalency. A sample explanation is given below:

I know that $6x + 6x + 8y + 8y$ and $2(6x + 8y)$ are equivalent because if I let $x = 2$ and $y = 4$, I can substitute the values for the variables. Because both expressions have the same value when I evaluate them, the expressions are equivalent.

$6(2) + 6(2) + 8(4) + 8(4)$	$2[6(2) + 8(4)]$
$12 + 12 + 32 + 32$	$2(12+32)$
88	$2(44)$
	88

Cutting Grass (ECR)

Overview

This instructional task requires students to use division of fractions by fractions to solve word problems.

Standards

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.NS.A.1	• 3.OA.B.6 • 5.NF.B.7	1. $\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$ 2. What is the area of a rectangle with length $\frac{7}{8}\ foot$ and width $\frac{2}{3}\ foot$? $\frac{7}{12}\ square\ feet$ 3. http://www.illustrativemathematics.org/illustrations/692 4. http://www.illustrativemathematics.org/illustrations/50 5. http://www.illustrativemathematics.org/illustrations/267	• http://www.illustrativemathematics.org/illustrations/12 • http://www.illustrativemathematics.org/illustrations/829 • http://www.illustrativemathematics.org/illustrations/1196 • http://learnzillion.com/lessonsets/701-interpret-and-compute-quotients-of-fractions • http://learnzillion.com/lessonsets/13-divide-fractions-by-fractions

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

1. What is a lawn mower? A lawn mower is a machine used to cut grass.
2. What is a gas can? A gas can is a container used to hold gas until you are ready to pour it into a machine.
3. If I am cutting grass, do I need to know the area or perimeter of the lot? This question is designed to ensure that students do not confuse area and perimeter. As you are explaining that students need to know the area of the lot, you can clarify any questions about what it means to cut grass.
4. What do you remember about volume? Volume is how much a container can hold—the capacity of a container.

After the Task

This task shows students how fractions appear in the real world. Remind students that most of the time real-world problems will involve fractions—not always simple numbers or integers. Have students write a word problem using division of fractions.

Student Extended Constructed Response

The lot next to Michael's house is empty. He wants to use it to play soccer with his friends, so he is going to cut the grass. Answer the following questions.

1. Michael's lawn mower requires $\frac{1}{3}$ of a gallon of gas to cut grass for one hour. He has $\frac{5}{6}$ of a gallon of gas in a gas can. How long can Michael cut the grass with this amount of gas? Show how you found your answer.

2. How wide is the rectangular lot if it has a length of $100\frac{1}{2}$ feet and an area of $5100\frac{3}{8}$ square feet? Show your work.

3. How long will it take Michael to cut the lot if he can cut $2550\frac{3}{16}$ square feet per hour? Show your work.

4. Does Michael have enough gas in his can to cut the lot? Justify your response.

5. Michael finds a second gas can in his garage. He uses $\frac{1}{4}$ of a gallon of the gas in the second gas can to cut the lot. If this is $\frac{2}{3}$ of the amount of gas that was originally in the second gas can, how much gas did Michael have in the second gas can when he started? Show your work.

Extended Constructed Response Exemplar Response

The lot next to Michael's house is empty. He wants to use it to play soccer with his friends, so he is going to cut the grass. Answer the following questions.

1. Michael's lawn mower requires $\frac{1}{3}$ of a gallon of gas to cut grass for one hour. He has $\frac{5}{6}$ of a gallon of gas in a gas can. How long can Michael cut the grass with this amount of gas? Show how you found your answer.

$$\frac{5}{6} \div \frac{1}{3} = \frac{5}{6} \times \frac{3}{1} = \frac{5}{2}\ hours\ or\ 2\frac{1}{2}\ hours$$

2. How wide is the rectangular lot if it has a length of $100\frac{1}{2}$ feet and an area of $5100\frac{3}{8}$ square feet? Show your work.

$$Area = length \times width$$

$$5100\frac{3}{8} = 100\frac{1}{2} \times width$$

$$width = 5100\frac{3}{8} \div 100\frac{1}{2}$$

$$width = 50\frac{3}{4}\ feet$$

3. How long will it take Michael to cut the lot if he can cut $2550\frac{3}{16}$ square feet per hour? Show your work.

$$5100\frac{3}{8} \div 2550\frac{3}{16}$$

$$\frac{40803}{8} \div \frac{40803}{16}$$

$$\frac{40803}{8} \times \frac{16}{40803}$$

$$\frac{40803 \times 16}{8 \times 40803}$$

It would take 2 hours.

4. Does Michael have enough gas in his can to cut the lot? Justify your response.

Yes, you do have enough gas to cut your lot.

You have enough gas to cut grass for $2\frac{1}{2}\ hours$. *It will take* $2\ hours$ *to cut your lot.*

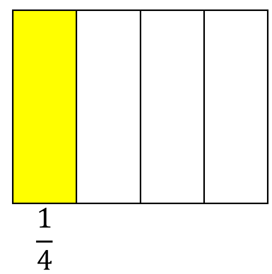
$$2\frac{1}{2}\ hours > 2\ hours$$

5. Michael finds a second gas can in his garage. He uses $\frac{1}{4}$ of a gallon of the gas in the second gas can to cut the lot. If this is $\frac{2}{3}$ of the amount of gas that was originally in the second gas can, how much gas did Michael have in the second gas can when he started? Show your work.

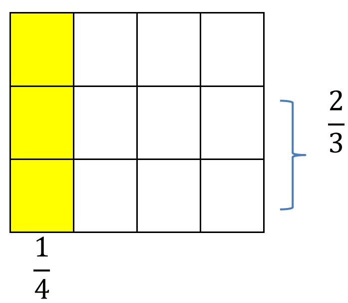
$$\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8} \text{ } gallon$$

Alternate solution method (students would likely only draw the diagram—the explanation is provided for understanding):

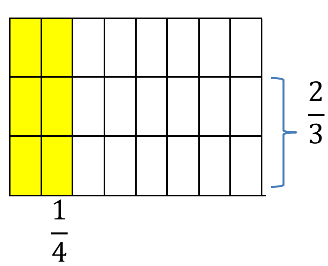
- *This model represents* $\frac{1}{4}$.



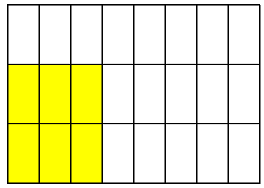
- *This model superimposes a square portioned into thirds horizontally onto the original model.*



- *Now we know the yellow* $\frac{1}{4}$ *area and the size of one of the factors that made that area. We have to subdivide the* $\frac{1}{4}$ *squares to make* $\frac{1}{8}$ *squares because we need to have pieces that will fit into the height that represents the 2/3 factor.*

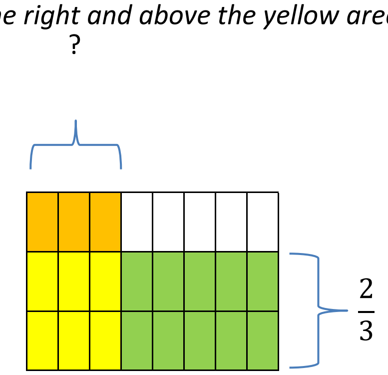


- *The product of 2/3 and another factor (the quotient) defines an area equivalent in size to 1/4. To find the quotient, move the top part of the yellow area so that it is the same height as the 2/3 factor.*



$$\frac{1}{4} \div \frac{2}{3}$$

- *Now shade the squares to the right and above the yellow area in order to see the factors one would multiply to find ¼.*



$$\frac{2}{3} \times ? = \frac{1}{4}$$

We know that $\frac{1}{4} \div \frac{2}{3}$ *can be thought of as* $\frac{2}{3} \times ? = \frac{1}{4}$*. There are 6 yellow parts out of 24 parts in all. The yellow area is* $\frac{1}{4}$ *of the rectangle. We know that one of the factors is* $\frac{2}{3}$*. From the orange and white areas of the rectangle, we can see that the other factor is* $\frac{3}{8}$*. Therefore* $\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$*.*

Low Temperatures (ECR)

Overview

Students will answer questions about positive and negative integers, including representing integers on a number line, comparing integers in the context of a real-world situation, and demonstrating an understanding of absolute value.

Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°\text{C} > -7°\text{C}$ *to express the fact that* $-3°\text{C}$ *is warmer than* $-7°\text{C}$*.*
b. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –3- dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
6.NS.C.5		1. What integer would you use to represent that New Orleans is 6 feet below sea level? a. *–6* 2. Which rational number would you use to represent a deposit of $15.36 to your bank account? a. *15.36* 3. http://www.illustrativemathematics.org/illustrations/278	• http://learnzillion.com/lessonsets/447-understand-the-relationship-between-positive-and-negative-numbers-interpret-zero-and-positive-or-negative-numbers-in-realworld-contexts • http://learnzillion.com/lessonsets/94-understand-how-positive-and-negative-numbers-describe-quantities
6.NS.C.6a	• 3.NF.A.2	1. What number is opposite of 25 on the	• http://www.illustrativemathematics.org/illust

		number line? a. *–25* 2. What is the value of –(–6)? *a. 6*	rations/169 • http://www.illustrativemathematics.org/illustrations/172 • http://learnzillion.com/lessonsets/210-position-numbers-and-their-opposites-on-a-number-line
6.NS.C.7b	• 6.NS.C.6c	1. New Orleans, Louisiana, has an elevation of –8 ft. Death Valley in California has an elevation of –282 ft Write an inequality to compare the elevations of these two places. Explain what your inequality means in terms of elevation. a. *–8 ft> –282 ft ; This means that an elevation of –8 ft is higher than an elevation of –282 ft.* 2. http://www.illustrativemathematics.org/illustrations/285	• http://learnzillion.com/lessonsets/138-interpret-statements-of-inequality-and-write-interpret-and-explain-statements-of-order-for-rational-numbers
6.NS.C.7c	• 6.NS.C.6a	1. The point *A* is placed on a number line such that its absolute value is 14. Which two integers could represent the location of point *A* on the number line? a. *14 or –14*	• http://learnzillion.com/lessonsets/191-understand-and-interpret-absolute-value-and-distinguishing-comparisons-of-absolute-value-from-statements-about-order

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is an almanac? An almanac is an annual calendar that contains important dates and statistical information such as astronomical data, tide tables, and weather information.

After the Task

When graphing the values on the number line for question 2, students may struggle since there are not enough marks for every number between –21 and 10. Students may need more practice with creating number lines that have different scales.

Students may also want to say that –7 is greater than –19 for question 3. Remind students to think about temperatures and to translate a greater temperature to it being warmer outside.

Students may want to explain their reasoning for question 4 as “I know the absolute value of 21 is 21.” Have students state what absolute value means in any situation (the distance a value is from zero); then have students use a number line to visualize what values would be 21 units from zero.

Student Extended Constructed Response

Low Temperatures for January 2, 2014, through January 10, 2014

	Jan. 2	Jan. 3	Jan. 4	Jan. 5	Jan. 6	Jan. 7	Jan. 8	Jan. 9	Jan. 10
New York, NY	27° C	11° C	10° C	26° C	37° C	6° C	10° C	21° C	30° C
Erie, PA	14° C	16° C	16° C	33° C	-2° C	-10° C	-10° C	4° C	26° C
Detroit, MI	12° C	0° C	0° C	14° C	-5° C	-12° C	-12° C	-3° C	21° C
Fargo, ND	-25° C	-25° C	-7° C	-19° C	-24° C	-24° C	-21° C	-9° C	-21° C

Source: http://www.almanac.com/weather/history

Use the table of temperatures above to complete the following.

1. Jerry said that one day the low temperature for his hometown, Detroit, Michigan, was 12 degrees below zero. On which date could this temperature have occurred?

2. Plot the temperatures for January 8 on the number line below. Name the two cities that had opposite temperatures on this day.

3. Alicia, who lives in Fargo, North Dakota, wrote the inequality $-7°C > -19°C$ to compare the temperatures on January 4 and January 5. Explain what this comparison means about the temperatures for these two dates.

4. On January 10, Rayshon noticed that the low temperature for the city he lives in had an absolute value of 21. In which city could Rayshon live? Explain your reasoning in terms of temperature.

Extended Constructed Response Exemplar Response

Low Temperatures for January 2, 2014, through January 10, 2014

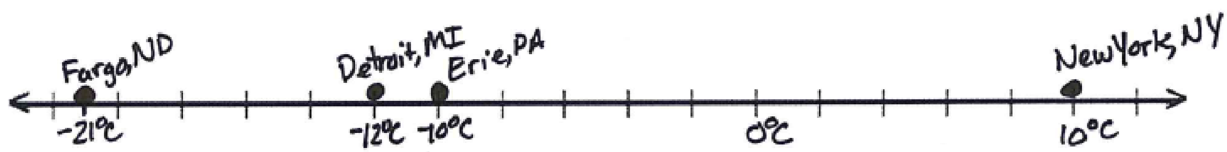
	Jan. 2	Jan. 3	Jan. 4	Jan 5	Jan. 6	Jan. 7	Jan. 8	Jan. 9	Jan. 10
New York, NY	27° C	11° C	10° C	26° C	37° C	6° C	10° C	21° C	30° C
Erie, PA	14° C	16° C	16° C	33° C	-2° C	-10° C	-10° C	4° C	26° C
Detroit, MI	12° C	0° C	0° C	14° C	-5° C	-12° C	-12° C	-3° C	21° C
Fargo, ND	-25° C	-25° C	-7° C	-19° C	-24° C	-24° C	-21° C	-9° C	-21° C

Source: http://www.almanac.com/weather/history

Use the table of temperatures above to complete the following.

1. Jerry said that one day the low temperature for his hometown, Detroit, Michigan, was 12 degrees below zero. On which date could this temperature have occurred?
   The temperature for Detroit, Michigan, could have been 12 degrees below zero on January 7 or January 8 (only one date is required).

2. Plot the temperatures for January 8 on the number line below. Name the two cities that had opposite temperatures on this day.



   The two cities with opposite temperatures are Erie, Pennsylvania, and New York, New York.

3. Alicia, who lives in Fargo, North Dakota, wrote the inequality $-7℃ > -19℃$ to compare the temperatures on January 4 and January 5. Explain what this comparison means about the temperatures for these two dates.

   The inequality $-7℃ > -19℃$ *means that the temperature on January 4 is warmer than the temperature on January 5.*

4. On January 10, Rayshon noticed that the low temperature for the city he lives in had an absolute value of 21. In which city could Rayshon live? Explain your reasoning in terms of temperature.

   Rayshon could live in either Detroit, Michigan, or Fargo, North Dakota (only one city needs to be given). If the absolute value of the temperature is 21, that means the temperature is 21 degrees from zero—it could be 21 degrees above zero (Detroit, Michigan) or 21 degrees below zero (Fargo, North Dakota).
   ***Note: students only need to give one city with a valid explanation.*

Friends Meeting on Bicycles (ECR)

Overview

This task requires students to use ratios and rates related to constant speed to determine the time that friends would meet when riding their bicycles.

Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.RP.A.3b	• 6.RP.A.2 • 6.RP.A.3a	1. Alex is training to run a half-marathon, which is 13.1 miles long. Alex runs at a pace of 4 miles per hour. If the half-marathon begins at 7:00 a.m., about what time will he cross the finish line, assuming he runs at a constant rate? a. *Alex will cross the finish line at approximately 10:16 am.* 2. Jane rode her bike to visit a friend that lives 12 miles away. It took her 1.5 hours to get to her friend's house. Assuming Jane rode her bike at a constant rate, how fast was Jane riding? a. *Jan was riding at a rate of 8 miles per hour.* 3. http://www.illustrativemathematics.org/illustrations/815 4. http://www.illustrativemathematics.org/illustrations/193	1. http://www.illustrativemathematics.org/illustrations/77 2. http://www.illustrativemathematics.org/illustrations/549 3. http://www.illustrativemathematics.org/illustrations/711 4. http://learnzillion.com/lessonsets/157-solve-unitrate-problems

After the Task

Suggestions are provided for additional assistance for students who may have struggled with different components of this task.

Questions 1 and 2: Students may have struggled with finding the distance between Taylor and Anya because of the different speeds. Have students represent the distance traveled in multiple ways: a table for each person, a number line diagram, or a tape diagram. Then discuss how they would determine the distance between the two friends at given intervals.

Questions 3 and 4: Students may experience difficulty when determining the time it takes to travel the remaining 9 miles as well as the total time traveled. Ask students to create a table to find the total number of miles the friends travel each hour (18 miles each hour). Then ask students to find how long it would take to travel the 63 miles total. The table created to assist with question 3 can also assist students with answering question 4.

Question 5: Students may struggle with finding the rate because the distance Anya travels is not stated in the question. Ask students how far Taylor traveled in the given time. Using Taylor's distance traveled, students should then be able to determine how far Anya would have had to travel in order to determine her speed.

Student Extended Constructed Response

Taylor and Anya live 63 miles apart. On some Saturdays, they ride their bikes toward each other's houses and meet somewhere in between. Taylor is a very consistent rider—she finds that her speed is always very close to 12.5 miles per hour. Anya rides more slowly than Taylor, but she is working out and is becoming a faster rider as the weeks go by.

1. On a Saturday in July, the two friends set out on their bikes at 8 a.m. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?

2. Make a table showing how far apart the two friends are after zero hours, one hour, two hours, and three hours.

3. At what time will the two friends meet? How do you know?

4. Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? How do you know if she is correct?

5. A couple of months later, on a Saturday in September, the two friends set out again on their bikes at 8 a.m. Taylor rides at 12.5 miles per hour. This time they meet at 11 a.m. How fast was Anya riding this time? Justify your answer.

Adapted from: http://www.illustrativemathematics.org/illustrations/137

Extended Constructed Response Exemplar Response

1. On a Saturday in July, the two friends set out on their bikes at 8 a.m. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?

   45 miles

2. Make a table showing how far apart the two friends are after zero hours, one hour, two hours, and three hours.

Hours	Miles Apart
0	63
1	45
2	27
3	9

Alternate Response:

Hours	Distance Anya Has Traveled	Distance Taylor Has Traveled	Miles Apart
0	0	0	63
1	5.5	12.5	45
2	11	25	27
3	16.5	37.5	9

3. At what time will the two friends meet? How do you know?
   At 3 hours, Taylor and Anya only have 9 more miles to travel before they meet. The number of miles between Taylor and Anya decreases by 18 miles per hour. Since 9 is half of 18, it will take a half hour to travel the 9 miles, so they will meet 3.5 hours later, at 11:30.

   Alternate answer: Since the distance between Taylor and Anya is decreasing at 18 miles per hour, 63/18 = 3.5 hours, so they will meet at 11:30.

4. Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? How do you know if she is correct?

   Taylor is correct and what she really means is that the distance between them is decreasing by 18 miles every hour, so the amount of time it will take them to meet is the same as if one person stays put and the other rides at 18 miles per hour. However, the place they meet will not be the same.

5. A couple of months later, on a Saturday in September, the two friends set out again on their bikes at 8 a.m. Taylor rides at 12.5 miles per hour. This time they meet at 11 a.m. How fast was Anya riding this time? Justify your answer.

Hours	Miles Apart
0	63
1	42
2	21
3	0

At 0 hours the friends are 63 miles apart, and at 3 hours they are 0 miles apart. The friends are getting closer at 21 miles per hour. Since Taylor is riding 12.5 miles per hour, Anya must be riding 8.5 miles per hour.

Alternate response: Students could also determine that Taylor rode 37.5 miles in the 3 hours (12.5 miles per hour x 3 hours), which would mean that Anya has to ride 25.5 miles in 3 hours (63 miles – 37.5 miles). 25.5 miles/3 hours = 8.5 miles per hour. This is Anya's rate.

Chocolate Chip Cookies (IT)

Overview

This instructional task requires students to use ratio and rate reasoning and division of fractions by fractions to solve word problems.

Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning with use of tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

c. Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percentage.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.RP.A.3c 6.RP.A.3d	• 6.RP.A.2	1. You have 30 baseball cards left after selling most of your collection. You have 15% of your collection left. How many baseball cards did you have originally? a. 200 2. How many ¼ cups are in 8 cups? a. $8 \div \frac{1}{4} = 32\ cups$ 3. http://www.illustrativemathematics.org/illustrations/54 4. http://www.illustrativemathematics.org/illustrations/899	• http://www.illustrativemathematics.org/illustrations/77 • http://www.illustrativemathematics.org/illustrations/549 • http://www.illustrativemathematics.org/illustrations/1611 • http://learnzillion.com/lessonsets/181-use-ratios-to-solve-percent-problems • http://learnzillion.com/lessonsets/87-use-ratios-to-convert-unit-measures
6.NS.A.1	• 3.OA.B.6 • 5.NF.B.7	1. $\frac{3}{4} \div \frac{2}{3}$ a. $\frac{9}{8}$ 2. http://www.illustrativemathematics.or	• http://www.illustrativemathematics.org/illustrations/12 • http://www.illustrativemathematics.org/illustrations/829

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		g/illustrations/692 3. http://www.illustrativemathematics.org/illustrations/50 4. http://www.illustrativemathematics.org/illustrations/267	• http://www.illustrativemathematics.org/illustrations/1196 • http://learnzillion.com/lessonsets/701-interpret-and-compute-quotients-of-fractions • http://learnzillion.com/lessonsets/13-divide-fractions-by-fractions

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a recipe? This question is designed to make sure that students understand the purpose of this task. A recipe tells you how to make something to eat. A recipe is a list of ingredients along with instructions on how to prepare the dish.

Why would you need to alter a recipe? A recipe is designed with a certain number of servings in mind. If a recipe will only make 5 servings but you are serving 10 people, you would need to double the recipe.

During the Task

Students may struggle with creating the shopping list. Encourage them to think about the size of the containers they are purchasing. They don't want to buy a container so big that they have lots of leftover food, but they shouldn't buy too little of an amount either.

For example, if they need 60 eggs, it makes more sense to purchase by 12-count or 18-count instead of by the half-dozen.

After the Task

This task shows students how math is useful in their own lives. Encourage students to find a recipe for their favorite food and alter it to feed their own families or the class.

Student Instructional Task

Your school is having a celebration, and you have to bring homemade chocolate chip cookies. You Googled and found a recipe that you'd like to try; this recipe yields 30 cookies.

Chocolate Chip Cookies

Ingredients

1/2 cup (1 stick) unsalted butter

3/4 cup packed dark brown sugar

3/4 cup sugar

2 large eggs

1 teaspoon pure vanilla extract

1 (12-ounce) bag semisweet chocolate chips, or chunks

2 1/4 cups all-purpose flour

3/4 teaspoon baking soda

1 teaspoon fine salt

Recipe source: http://www.foodnetwork.com/recipes/food-network-kitchens/chocolate-chip-cookies-recipe4.html?oc=linkback

1. To practice the recipe, you decide to follow the recipe as given.
    a. The eggs you used were 20% of the total number of eggs you had when you started. How many eggs did you have before you made this batch of cookies? Justify your answer.

    b. You are ready to add baking soda to the bowl, but you only have a $\frac{1}{8}$ teaspoon measuring spoon. How many $\frac{1}{8}$ teaspoons of baking soda will you need to add to your bowl? Show your work.

    c. How many $\frac{1}{4}$ cup measuring containers can you fill with sugar based on the ingredients listed? Show your work.

2. You are going to make a batch of cookies for every class at your school.

    a. Rewrite this recipe to show how much of each of these ingredients you will need.

    b. Research the ingredients in your recipe, and write a shopping list. For each ingredient, write how many of each item you will need to purchase. Also, determine the percentage of the item that will be left in the container after you make your batter.

    c. You are going to bake the cookies on three different afternoons, so you store your batter in three containers. Rewrite your recipe to show how much of each ingredient will be stored in each container.

Instructional Task Exemplar Response

Your school is having a celebration, and you have to bring homemade chocolate chip cookies. You Googled and found a recipe that you'd like to try on the Food Network website; this recipe yields 30 cookies.

Recipe source: http://www.foodnetwork.com/recipes/food-network-kitchens/chocolate-chip-cookies-recipe4.html?oc=linkback

1. To practice the recipe, you decide to follow the recipe as given.
   a. The eggs you used were 20% of the total numbers of eggs you had when you started. How many eggs did you have before you made this batch of cookies? Justify your answer.

$$. \frac{20}{100}x = 2$$

$$x = 2 \div \frac{20}{100}$$

$$x = 10\ eggs$$

or

20% = 2 eggs
40% = 4 eggs
60% = 6 eggs
80% = 8 eggs
100% = 10 eggs

Students may draw a tape diagram to show this reasoning.

20%	*20%*	*20%*	*20%*	*20%*
2 eggs	*2 eggs*	*2 eggs*	*2 eggs*	*2 eggs*

2. You are ready to add your baking soda to the bowl, but you only have a $\frac{1}{8}$ teaspoon measuring spoon. How many $\frac{1}{8}$ teaspoons of baking soda will you need to add to your bowl? Show your work.

$$\frac{3}{4} \div \frac{1}{8} = \frac{3 \times 8}{4 \times 1} = 6\ of\ the\ \frac{1}{8}\ teaspoons$$

   1. How many $\frac{1}{4}$ cup measuring containers can you fill with sugar based on the ingredients listed? Show your work.

$$\frac{3}{4} \div \frac{1}{4} = 3\ containers$$

3. You are going to make a batch of cookies for every class at your school.
   a. Rewrite this recipe to show how much of each of these ingredients you will need.

Students will have to research how many classes are at your school and how many students are in each class. An example response based on 30 classes is completed below.

Original Recipe:

Ingredients

1/2 cup (1 stick) unsalted butter

3/4 cup packed dark brown sugar

3/4 cup sugar

2 large eggs

1 teaspoon pure vanilla extract

1 (12-ounce) bag semisweet chocolate chips, or chunks

2 1/4 cups all-purpose flour

3/4 teaspoon baking soda

1 teaspoon fine salt

Recipe for 30 batches:

Ingredients

15 cups (30 sticks) unsalted butter

$\frac{45}{2}$ *or 22.5 or* $22\frac{1}{2}$ *cups packed dark brown sugar*

$\frac{45}{2}$ *or 22.5 or* $22\frac{1}{2}$ *cups sugar*

60 large eggs

30 teaspoons pure vanilla extract

30 (12-ounce) bags semisweet chocolate chips, or chunks

$\frac{135}{2}$ *or 67.5 or* $67\frac{1}{2}$ *cups all-purpose flour*

$\frac{45}{2}$ *or 22.5 or* $22\frac{1}{2}$ *teaspoons baking soda*

30 teaspoons fine salt

b. Research the ingredients in your recipe, and write a shopping list. For each ingredient, write how many of each item you will need to purchase. Also, determine the percentage of the item that will be left in the container after you make your batter.

This will vary depending on the recipe in part 2a and the size package bought by each student. For example, in the sample above, the recipe calls for 60 eggs. The student might do the following:

Buy eggs by the dozen:

$$\frac{60}{12} = 5 \text{ dozen}$$

The recipe will use 5 complete packages of a dozen eggs and 0% will remain.

Buy eggs by the 18 count:

$$\frac{60}{18} = 3\frac{1}{3}\ \text{packages}$$

You need to purchase 4 packages of 18-count eggs. You will have 66.67% of an 18-count package left.

c. You are going to bake the cookies on three different afternoons, so you store your batter in three containers. Rewrite your recipe to show how much of each ingredient will be stored in each container.
This will vary based on the answer in part a. The following is based on the example in part 2a:

Batter in each container:

Ingredients

5 *cups (10 sticks) unsalted butter*

$\frac{15}{2}$ *or 7.5 or* $7\frac{1}{2}$ *cups packed dark brown sugar*

$\frac{15}{2}$ *or 7.5 or* $7\frac{1}{2}$ *cups sugar*

20 large eggs

10 teaspoons pure vanilla extract

10 (12-ounce) bags semisweet chocolate chips, or chunks

$\frac{45}{2}$ *or 22.5 or* $22\frac{1}{2}$ *cups all-purpose flour*

$\frac{15}{2}$ *or 7.5 or* $7\frac{1}{2}$ *teaspoons baking soda*

10 teaspoons fine salt

Word of Mouth (IT)

Overview
Based on a given number of workers students will decide how to use word-of-mouth advertising to share the news of a new sandwich. They will express their findings using exponents.

Standards
Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Prior to the Task
Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.EE.1	• 4.0A.B.4 • 5.NBT.A2	1. How can you write 4 x 3 x 3 x 3 using exponents? a. 4×3^3 2. Write the expanded form of 2×6^4. a. 2 x 6 x 6 x 6 x 6 3. What is $2 \times 3^2 + 2 \times 3^3$? a. 90 4. http://www.illustrativemathematics.org/illustrations/532 5. http://www.illustrativemathematics.org/illustrations/891	• http://www.illustrativemathematics.org/illustrations/938 • http://www.illustrativemathematics.org/illustrations/1524 • http://www.illustrativemathematics.org/illustrations/1620 • http://learnzillion.com/lessonsets/196-write-and-evaluate-expressions-involving-whole-number-exponents

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is word-of-mouth advertising? Word-of-mouth advertising is when people share information by telling someone directly instead of putting the information on the radio or on television.

During the Task
1. Students need to understand the task means that each person will only tell two people. After they have shared with two people, they will not share the information again.
2. Students may have a hard determining the final answer. They may want to simply use the final expression in the table.
3. Students may struggle with finding the total when a person from week 5 becomes sick. They may want to reduce the 5 x 2 to 4 x 2. Teachers may need to assist with understanding why taking this person out from the whole group would change the entire answer. The person is simply unable to reach two people on the sixth day, and those two people are unable to reach people on the last day.

After the Task

Relate this task to the rising cost of advertisement. Discuss the impact on profit if advertisement costs are high.

Student Instructional Task

Dave owns a sandwich shop. He has 4 employees. Next week he will be introducing a new sandwich. In order to save money, he has decided to ask his employees, Rhonda, Lisa, Benny, and Jena, to spread the word about the new sandwich rather than spending money on advertisements. Dave and his employees will each tell two of their friends about the new sandwich. Dave assumes that each friend will tell two new people. Then each of these people will tell two new people and so on.

1. How many people will they reach by the end of seven days?
   In your answer include:

   - A table showing the number of people reached at the end of each day.
   - The expression for the number of people reached at the end of each day written in expanded form and exponential form (i.e., 2 x 4 x 4, 2 x 4^2).
   - The expression in exponential form you will use to determine the total number of people reached at the end of seven days.
   - The total number of people reached at the end of seven days, which will help Dave plan for the first day of sales.
2. How would your answer be affected if one person becomes sick on day 5 and is unable to share the news with two people? Explain your reasoning using an expression with exponents.
3. Develop an advertising plan that would reach more people per day. Show/explain how your plan would be more effective. Using expressions with exponents, share the total number of people reached for each of the seven days and the final number of people reached.

Instructional Task Exemplar Response

Dave owns a sandwich shop. He has 4 employees. Next week he will be introducing a new sandwich. In order to save money, he has decided to ask his employees, Rhonda, Lisa, Benny, and Jena, to spread the word about the new sandwich rather than spending money on advertisements. Dave and his employees will each tell two of their friends about the new sandwich. Dave assumes that each friend will tell two new people. Then each of these people will tell two new people and so on.

1. How many people will they reach by the end of seven days?
   In your answer include:

   - ❖ A table showing the number of people reached at the end of each day.
   - ❖ The expression written in expanded form and exponential form for each day (i.e., 2 x 4 x 4, 2 x 4^2).
   - ❖ The expression in exponential form you will use to determine the total number reached.
   - ❖ The total number reached, which will help Dave plan for the first day of sales.

   Students' table should look similar to this.

Day	*Number of people reached at the end of the day in expanded form*	*Number of people reached at the end of the day in exponential form*
1	*5 x 2*	5×2^1
2	*5 x 2 x 2*	5×2^2
3	*5 x 2 x 2 x 2*	5×2^3
4	*5 x 2 x 2 x 2 x 2*	5×2^4
5	*5 x 2 x 2 x 2 x 2 x 2*	5×2^5
6	*5 x 2 x 2 x 2 x 2 x 2 x 2*	5×2^6
7	*5 x 2 x 2 x 2 x 2 x 2 x 2 x 2*	5×2^7

   Total number reached: $5 \times 2^1 + 5 \times 2^2 + 5 \times 2^3 + 5 \times 2^4 + 5 \times 2^5 + 5 \times 2^6 + 5 \times 2^7 = 1{,}270$

2. How would your answer be affected if one person becomes sick on day 5 and is unable to share the news with two people? Explain your reasoning using an expression with exponents.
   The total number of people reached will be decreased by 1 person x 2 people not shared with on the sixth day. On the seventh day, 2 people will not be sharing with 2 people. This will give the expression: $1{,}270 – (2^1 + 2^2) = 1{,}264$ *people.*

3. Develop an advertising plan that would reach more people per day. Illustrate how your plan would be more effective. Using expressions with exponents, share your totals for each of the seven days and the final number reached.

Student responses will have different total answers. Key items to look for in their responses include:

- *An alternate advertising plan (i.e., sharing on Facebook, Twitter, etc.,), which includes more than 2 people shared with each day*
- *A description, whether written or drawn, that explains how this plan is better at reaching people*
- *Expressions to show how many people are reached each day*
- *Expression showing the total number*
- *Total number of people reached*

Sample Response:

Our plan uses email to help share the news. Instead of sharing the news with only two people we suggest that the employees and Dave send an email to four people on the first day and ask each of those people to send the email to four people. The email would ask everyone who receives it to send it to at least four people. Here's what the email might say:

> *Hi everyone! I wanted to let you know about a new sandwich that we will start serving at Dave's sandwich shop next week. We're excited about the sandwich, and we're asking you to help us spread the word. Please forward this email to at least four people who are not already listed on this email.*

We think this plan is better because people can still send an email when they are sick and it's easier to forward the email when you get it than it may be to find two new people who may not know about the sandwich already. Also, because we plan to share with four new people each day, more people will be reached.

Day	*Number of people reached at the end of the day in expanded form*	*Number of people reached at the end of the day in exponential form*
1	*5 x 4*	5×4^1
2	*5 x 4 x 4*	5×4^2
3	*5 x 4 x 4 x 4*	5×4^3
4	*5 x 4 x 4 x 4 x 4*	5×4^4
5	*5 x 4 x 4 x 4 x 4 x 4*	5×4^5
6	*5 x 4 x 4 x 4 x 4 x 4 x 4*	5×4^6
7	*5 x 4 x 4 x 4 x 4 x 4 x 4 x 4*	5×4^7

The total number reached would be Total number reached: $5 \times 4^1 + 5 \times 4^2 + 5 \times 4^3 + 5 \times 4^4 + 5 \times 4^5 + 5 \times 4^6 + 5 \times 4^7 =$ *109,220 people!*

The Elevator Limit (IT)

Overview

In this instructional task students are given two inequalities, one in words and one as a formula, and a set of possible solutions. Students must decide which of the possible solutions actually solves the inequalities.

Standards

Reason about and solve one-variable equations and inequalities.

6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.EE.B.5	• 6.EE.A.2	1. If 3x > 6, which value of x would make the inequality true: A. 1 B. 2 C. 3 a. The answer is C. 2. http://www.illustrativemathematics.org/illustrations/673	• http://www.illustrativemathematics.org/illustrations/540 • http://learnzillion.com/lessonsets/734-understand-solving-an-equation-or-inequality-as-the-process-of-finding-the-values-that-make-it-true
6.EE.B.8	• 6.NS.C.6a • 6.NS.C.6c • 6.NS.C.7a • 6.NS.C.7b	1. Write an inequality for the following situation: *There can be no more than 65 people on the bus.* a. $x \leq 65$; $x =$ the number of people on the bus 2. http://www.illustrativemathematics.org/illustrations/642	• http://www.illustrativemathematics.org/illustrations/283 • http://www.illustrativemathematics.org/illustrations/284 • http://www.illustrativemathematics.org/illustrations/285 • http://learnzillion.com/lessonsets/578-understand-write-and-represent-inequalities-of-the-form-x-c-or-x-c-and-recognize-that-they-have-infinitely-many-solutions • http://learnzillion.com/lessonsets/310-writing-using-and-understanding-inequalities

Real-World Preparation:

Teachers may need to have a discussion about field trips, and how student/teacher/chaperone ratios affect groupings of students.

During the Task

- Discuss with students what the inequality for part *a* means in the context of a real-world situation. Students will need to understand that there cannot be a half or fourth of a person.
- When graphing the solution set, discuss with students that the set will not include values less than zero, and have them explain why based on the context of the situation.
- Students will use the number of 6th graders and 6th grade teachers at their school to help make groups. A discussion of district policy on the number of chaperones (not including the teachers) may need to be held, so students can determine if they need to add more adults to their groups.
- Some students may decide that only one adult and one student would be in a group—discuss with students why that may not be a good plan.
- As you circulate, probe further by asking what would happen if the chaperone number increased or decreased.
- A discussion may also need to be held about whether the total for the benches is included or not included in the weight limit of 1,800 pounds.

After the Task

This task shows students how math is useful in their own lives. Using inequalities can help us determine how many of an item we need or the constraints we have on certain tasks. Extensions of the task can be done by including discussion of what would happen to the groups if the chaperone or student number changed. What would happen if four students were absent that day? Would you decide to change your groups? Why or why not?

Student Instructional Task

All of the 6th graders and all of the 6th grade teachers at your school are visiting the Kilgore Oil Museum in Kilgore, Texas. During the visit, the students have the opportunity to get on an elevator that brings them below the surface of the earth to see different layers of soil and rock. The elevator can hold at most 15 people and has a weight limit of no more than 1,800 pounds. The elevator contains five rows of benches, and the five benches weigh 175 pounds together. The average adult weighs 180 pounds and the average 6th grader weighs 100 pounds.

1. Write an inequality that shows approximately how many people can ride the elevator. Be sure to define your variable. Model the solution on a number line.

2. The students and adults will be split into groups so that everyone attending can see the exhibit. The groups must be created to ensure that everyone can ride the elevator safely. Using the average weights listed above, determine the number of students and the number of adults that each group could contain. Explain how you decided how many adults and how many students should be in each group.

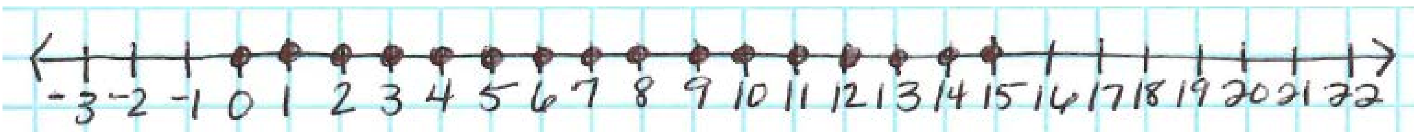
3. Write an inequality that represents the number students in each of your groups. Define the variable.

Instructional Task Exemplar Response

All of the 6th graders and all of the 6th grade teachers at your school are visiting the Kilgore Oil Museum in Kilgore, Texas. During the visit, the students have the opportunity to get on an elevator that brings them below the surface of the earth to see different layers of soil and rock. The elevator can hold at most 15 people and has a weight limit of no more than 1,800 pounds. The elevator contains five rows of benches, and the five benches weigh 175 pounds together. The average adult weighs 180 pounds and the average 6th grader weighs 100 pounds.

1. Write an inequality that shows how many people can ride the elevator. Be sure to define your variable. Model the solution on a number line.
   P = the number of people that can ride the elevator

   $p \leq 15$



2. The students and adults will be split into groups so that everyone attending can see the exhibit. The groups must be created so that everyone can ride the elevator safely. Using the average weights listed above, determine the number of students and the number of adults that each group could contain. Explain how you decided how many adults and how many students should be in each group.
   Sample answer based on 72 6th graders, 3 teachers, 3 adult chaperones, and the weight of the benches must be subtracted from the 1,800-pound limit.

   1,800 lbs – 175 lbs = 1,625 lbs, so the people on the elevator can total no more than 1,625 pounds.

   If 1 adult rides with each group, then 1,625 – 180 = 1,445 lbs. The 72 students can be split into 6 groups of 12 kids each. 12 kids will weigh approximately 1,200 pounds.

   1 adult and 12 kids makes 13 people riding the elevator, which is less than the 15-person limit.

   1(180) + 12(100) = 1,380 pounds

   1,380 lbs ≤ 1,625 lbs

   That weight does not exceed the total allowable weight of 1,625 pounds.

   Split the 6th graders into 6 groups with 1 adult and 12 kids in each group.

3. Write an inequality that represents the number of students in each of your groups. Define the variable.
   This answer will be dependent upon the answer the students get for question 2.

Sample Answer: $s \leq 12$; s represents the number of students who would be able to ride the elevator if only one adult rode with the group.

Crawfish Boil (IT)

Overview

Students will use their understandings of ratios and proportional reasoning to recommend how many pounds of crawfish and how many other items would need to be purchased for a party.

Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the birdhouse at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
6.RP.A.1	• 4.OA.A.2 • 5.NF.B.5 • 5.OA.B.3	1. The ratio of boys to girls in a classroom is 2:3. Describe the relationship between the number of boys and girls in the classroom based on the given ratio. a. *There are 2 boys in the classroom for every 3 girls in the classroom.* 2. http://www.illustrativemathematics.org/illustrations/76	• http://www.illustrativemathematics.org/illustrations/263 • http://www.illustrativemathematics.org/illustrations/22 • http://www.illustrativemathematics.org/illustrations/150 • http://www.illustrativemathematics.org/illustrations/143 • http://learnzillion.com/lessonsets/133-understand-ratios-and-using-ratio-language-to-describe-a-ratio-relationship-1 • http://learnzillion.com/lessonsets/114-understand-ratios-and-using-ratio-language-to-describe-a-ratio-relationship-2

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items:</th></tr>
<tr><td>6.RP.A.2</td><td>• 4.MD.A.1
• 5.NF.B.3
• 5.NF.B.7
• 6.RP.A.1</td><td>1. Collin paid $56 for 7 boxes of cookies. What is the unit rate?
a. This is a rate of $8 per box of cookies.
2. http://www.illustrativemathematics.org/illustrations/549
3. http://www.illustrativemathematics.org/illustrations/1175</td><td>• http://www.illustrativemathematics.org/illustrations/1508
• http://www.illustrativemathematics.org/illustrations/858
• http://www.illustrativemathematics.org/illustrations/292
• http://www.illustrativemathematics.org/illustrations/12
• http://www.illustrativemathematics.org/illustrations/829
• http://www.illustrativemathematics.org/illustrations/1196
• http://learnzillion.com/lessonsets/152-understand-unit-rate-and-use-rate-language-in-the-context-of-a-ratio-relationship</td></tr>
<tr><td>6.RP.A.3a</td><td>• 5.G.A.2
• 6.RP.A.1</td><td>1. Matthew can walk at a rate of 4 miles per hour. Complete the table below based on this rate.
<table>
<tr><th>Hours Walked</th><th>Miles Walked</th></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td>16</td></tr>
<tr><td>5</td><td></td></tr>
</table>
a.
<table>
<tr><th>Hours Walked</th><th>Miles Walked</th></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>8</td></tr>
<tr><td>3</td><td>12</td></tr>
<tr><td>4</td><td>16</td></tr>
<tr><td>5</td><td>20</td></tr>
</table>
2. http://www.illustrativemathematics.org/illustrations/711</td><td>• http://learnzillion.com/lessonsets/164-solve-ratio-problems-using-tables-and-the-coordinate-plane-1
• http://learnzillion.com/lessonsets/156-solve-ratio-problems-using-tables-and-the-coordinate-plane-2
• http://learnzillion.com/lessonsets/86-find-ratio-values-and-compare</td></tr>
<tr><td>6.RP.A.3b</td><td>• 6.RP.A.2
• 6.RP.A.3a</td><td>1. If it costs $3 per person to go to the volleyball game, how many people could A'vial pay for if she has $35?
a. She could pay for 11 people.
2. http://www.illustrativemathematics.org/illustrations/193</td><td>• http://learnzillion.com/lessonsets/157-solve-unitrate-problems</td></tr>
</table>

Real-World Preparation: The following questions will prepare students for some of the real world components of this task:

- What is a crawfish boil? Typically it is a get-together where people boil crawfish for everyone to share and enjoy.
- Why would someone need more than one pound of crawfish? One pound of crawfish includes the weight of the shells and other inedible parts of the crawfish. Once peeled to be eaten, one pound of crawfish produces only about 2 ounces of meat.
- What is catering? Catering is when a restaurant prepares food and/or drink for a person, organization, or an event.

During the Task

- Students may create two separate number lines rather than one number line that shows the relationship between the pounds of crawfish and the number of people. It may help to provide students with the number line already drawn and they would only have to fill in the numbers.
- Students may be tempted to multiply 3 by 150 for part c in the first section. Ask students to verify that work by plotting the point on the graph and make the connection to the proportional relationship.
- Students will need to make some decisions about the types and quantities of beverages as well as other items in the menu of options.
- Attention will need to be given to students who use an incorrect answer from question 1 with a correct procedure or correct reasoning to answer this portion of the task.

After the Task

Students can connect this task to planning for a party at their house to celebrate their birthday or some other event. Have students plan for the number of people they would like to invite and determine how much food and drink they would need to purchase to make sure everyone they would invite would be able to eat and drink.

Student Instructional Task

Your group has been selected to represent the 6th grade class at meetings to help prepare for an end-of-year celebration. The school has decided to have a crawfish boil to celebrate the end of the school year for the 6th grade students. They would like your group to help them plan the event and determine how much food and drink they will need to order.

1. A local restaurant uses the chart below to recommend the number of pounds of crawfish for takeout and catering orders based on the number of people to be served.

Number of People	Pounds of Crawfish
10	30
20	60
30	90
40	120
50	150
60	
70	
80	
90	
100	

a. Using the table above, create a double number line diagram to represent the number of pounds of crawfish needed for groups up to 100 people, then fill in the missing quantities on the table in the Pounds of Crawfish column.

b. Based on your work in part a, describe the relationship between the number of pounds of crawfish and the number of people to be served as a unit rate. How can this unit rate be used to find out how many pounds of crawfish to order if there are 45 people to be served?

c. Plot the values from the table in part a on a coordinate plane, and draw a straight line through the points. Label the axes. Then use the graph to find the quantity of crawfish that would be needed for 150 people.

2. The same restaurant uses the price list below to charge for other menu items.

Menu Item	Price
Boiled Crawfish	$2.75 per lb.
Corn and Potatoes	$1.50 per lb.
Sweet and Unsweet Tea	$10.00 per gallon
Water (16 oz. bottles)	$6.00 per case of 24
Coke products (12 oz. cans)	$4.50 per case of 12

Use the chart from question 1 and the price list above to help answer the following. Also, consider the following:

- 3 pounds of corn and potatoes will feed four people.
- One gallon of tea (sweet or unsweet) will provide 10 drinks.
- Each person will drink at least 2 beverages.

The school has told your group that they want to spend no more than $12.00 per person. What can be purchased to be prepared for a group of 100 people that stays within the per person budget? Your choices should include an appropriate amount of crawfish, corn and potatoes, and beverages. Your final product should include a narrative explaining your choices with justifications for those decisions. Be prepared to share your narrative with the class.

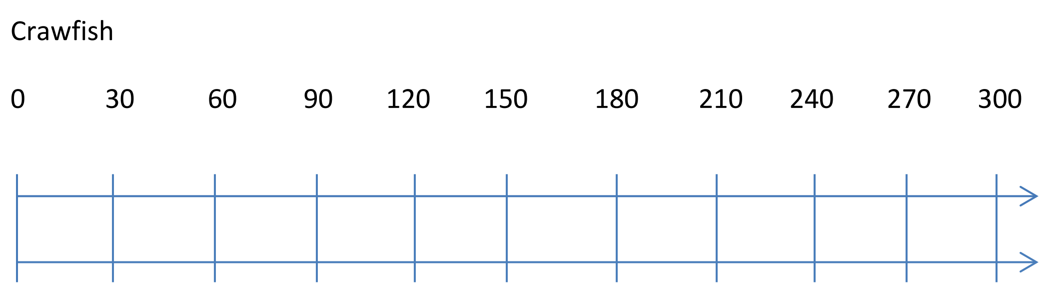
Instructional Task Exemplar Response

Your group has been selected to represent the 6th grade class at meetings to help prepare for an end-of-year celebration. The school has decided to have a crawfish boil to celebrate the end of the school year for the 6th grade students. They would like your group to help them plan the event and determine how much food and drink they will need to order.

1. A local restaurant uses the chart below to recommend the number of pounds of crawfish for takeout and catering orders based on the number of people to be served.

Number of People	Pounds of Crawfish
10	30
20	60
30	90
40	120
50	150
60	*180*
70	*210*
80	*240*
90	*270*
100	*300*

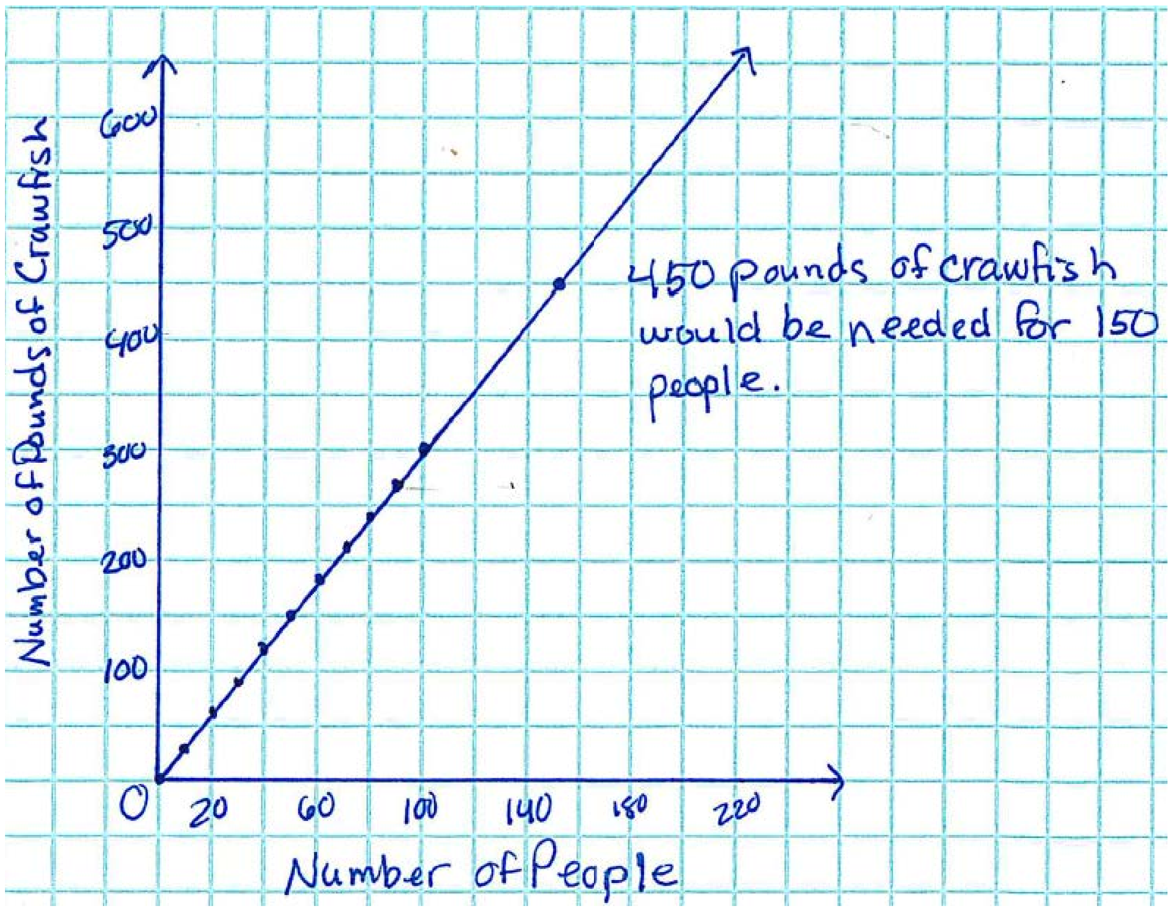
a. Using the table above, create a double number line diagram to represent the number of pounds of crawfish needed for groups up to 100 people, then fill in the missing quantities on the table in the Pounds of Crawfish column.



b. Based on your work in part a, describe the relationship between the number of pounds of crawfish and the number of people to be served as a unit rate. How can this unit rate be used to find out how many pounds of crawfish to order if there are 45 people to be served?

The unit rate of number of pounds of crawfish per person is 3 pounds of crawfish per person. To find the number of pounds of crawfish needed to feed 45 people, multiply 45 by 3. To serve 45 people, 135 pounds of crawfish would be needed.

c. Plot the values from the table on a coordinate plane, and draw a straight line through the points. Label the axes. Then use the graph to find the quantity of crawfish that would be needed for 150 people.



2. The same restaurant uses the price list below to charge for other menu items.

Menu Item	Price
Boiled Crawfish	$2.75 per lb.
Corn and Potatoes	$1.50 per lb.
Sweet and Unsweet Tea	$10.00 per gallon
Water (16 oz. bottles)	$6.00 per case of 24
Coke products (12 oz. cans)	$4.50 per case of 12

Use the chart from question 1 and the price list above to help answer the following. Also, consider the following:

- 3 pounds of corn and potatoes will feed four people.
- One gallon of tea (sweet or unsweet) will serve 10 drinks.
- Each person will drink at least 2 beverages.

The school has told your group that they want to spend no more than $12.00 per person. What can be purchased to be prepared for a group of 100 people that still stays within the per person budget? Your choices should include an appropriate amount of crawfish, corn and potatoes, and beverages. Include a narrative explaining your choices with justifications for those decisions in your final product. Be prepared to share your narrative with the class.

Sample Response:

We recommend the following be purchased for the end-of-year crawfish boil based on 150 people:

- *At a rate of 3 pounds of crawfish per person, we need to buy 3 x 100 = 300 pounds of crawfish. At $2.75 per pound, the cost of the crawfish would be 300 x $2.75 = $825.*
- *If 3 pounds of corn and potatoes feeds 4 people, then 1 person will eat 0.75 pounds of corn and potatoes. For 100 people, this would mean 100 x 0.75 = 75 pounds of corn and potatoes. The cost for the corn and potatoes will be 75 x $1.50 per pound = $112.50.*
- *Based on our group preferences, we decided that we would order enough water for 50 people, Coke products for 30 people, and tea for 20 people. If each person drinks at least two beverages, then we will need to buy at least 100 waters, at least 60 coke products, and enough tea for 40 drinks.*
    - *Waters come 24 to a case, so 100/24 is about 4.2 cases, but since we can't buy a part of a case, we will need to buy 5 cases. 5 cases of water at $6.00 per case will cost $30.*
    - *Coke products are packaged 12 per case, so to get 60 Coke products we will need 5 cases because 60/12 = 5.5 cases of Coke products at $4.50 per case. The total will be $22.50 (5 x 4.50 = 22.50).*
    - *To have enough tea for 40 drinks, we will need to have 4 gallons of tea (40/10 = 4). We suggest buying 2 gallons of sweet tea and 2 gallons of unsweet tea. The cost of 4 gallons of tea can be found by multiplying 4 x $10, which gives a cost of $40.*
    - *The total cost for the end-of-year celebration is found by adding the total costs of each of the menu options: $825 + $112.50 + $30 + $22.50 + $40 = $1,030.00. To find the total cost per person, divide $1,030 by 100 people, and the cost per person is $10.30.*

Teacher note: This portion of the task will take on many different looks. Students have multiple options to fulfill the requirements listed above. They will need to make some decisions about the types and quantities of beverages. Attention will need to be given to students who use an incorrect answer from question 1 with a correct procedure or correct reasoning to answer this portion of the task.

Marathon Prep (IT)

Overview

This instructional task requires students to use an equation in the form of *px* = *q* to represent and solve word problems.

Standards

Reason about and solve one-variable equations and inequalities.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p*, *q,* and *x* are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

<table>
<tr><th>Grade-Level Standards</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items:</th></tr>
<tr><td>6.EE.B.7</td><td>• 5.NF.B.4</td><td>1. Solve the following equation: $23.7 = 2x$
a. $x = 11.85$
2. http://www.illustrativemathematics.org/illustrations/1107</td><td>• http://www.illustrativemathematics.org/illustrations/321
• http://www.illustrativemathematics.org/illustrations/965
• http://learnzillion.com/lessonsets/577-solve-problems-by-writing-and-solving-equations-of-the-form-x-p-q-and-px-q
• http://learnzillion.com/lessonsets/269-solve-problems-with-equations-xpq-and-pxq</td></tr>
<tr><td>6.EE.C.9</td><td>• 5.OA.B.3</td><td>1. Complete the following table:
<table><tr><td>X</td><td>Y</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td></td></tr></table>
a.</td><td>• http://learnzillion.com/lessonsets/675-use-variables-to-relate-two-quantities-in-a-real-world-problem
• http://learnzillion.com/lessonsets/346-use-variables-to-represent-quantities-that-change-in-relationship-to-one-another</td></tr>
</table>

<table>
<tr><th>Grade-Level Standards</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items:</th></tr>
<tr><td></td><td></td><td>
<table>
<tr><td>X</td><td>Y</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>6</td></tr>
<tr><td>4</td><td>8</td></tr>
<tr><td>5</td><td>10</td></tr>
</table>
2. Write an equation to represent the information in the table in question 1.
a. $y = 2x$
3. http://www.illustrativemathematics.org/illustrations/806
</td><td></td></tr>
</table>

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a marathon? This question is designed to make sure that students understand the purpose of this task. A marathon is a 26.21875-mile race. For many running and/or fitness enthusiasts, completing a marathon is an important milestone.
Why do you have to train for a marathon? Since a marathon is such a long race, you must prepare your body.

During the Task

Students may struggle with creating their own race preparation schedule. To help interest them in the task, you may want to mention specific races in specific towns. If students have trouble creating a realistic plan, you could work with the physical education teacher to help students get an idea of their current fitness levels and how that would affect any preparation plan they would create.

After the Task

This task shows students how math is useful in their own lives. Encourage students to follow through with their plans. The class could follow through with a plan together. The class could vote on one plan and complete the training and race together.

Student Instructional Task

Lee is training for a marathon. A marathon is 26.21875 miles. She is going to run four days a week to prepare for the marathon. Lee will run the same distance for four days, and then each week she is going to increase her total distance for the week by 1.5 miles. Below is a partial table to represent her training schedule.

Training Week *(w)*	Weekly Distance *(d)*
1	1.5 miles
2	
3	
4	
5	
6	9 miles
7	

1. Use the table to answer the questions below.

    a. Complete the table above for values of *d*.

    b. Graph the ordered pairs from the table above.

    c. Write an equation to find the weekly distance, *d*, that will be run if she is on week *w* of training. Which variable is the independent variable and which is the dependent variable?

    d. During which week of training will the total distance Lee runs for the week be equivalent to a marathon?

2. Lee decides that she wants to shorten her training schedule. Write an equation that would allow Lee to prepare for her race in less time. Explain how your equation would get Lee ready sooner, and provide support for your explanation.

3. In a group, research local races in your area. Pick one and plan your group's race preparation schedule. Keep the following points in mind:

- What is the date of the race?
- What is the length of the race?
- How many days will the group have to train?
- Has anyone in the group ever run a race before? Does anyone in the group run on a regular basis?

Your group's plan should include a table, an equation, and a graph. Be sure to include the start date and end date of your training. Your group's plan should also include a short narrative. In the narrative, be sure to explain how your group chose the race and planned the training. Your group will present the plan to the class when it is finished.

Instructional Task Exemplar Response

Lee is training for a marathon. A marathon is 26.21875 miles. She is going to run four days a week to prepare for the marathon. Lee will run the same distance for four days, and then each week she is going to increase her total distance for the week by 1.5 miles. Below is a partial table to represent her training schedule.

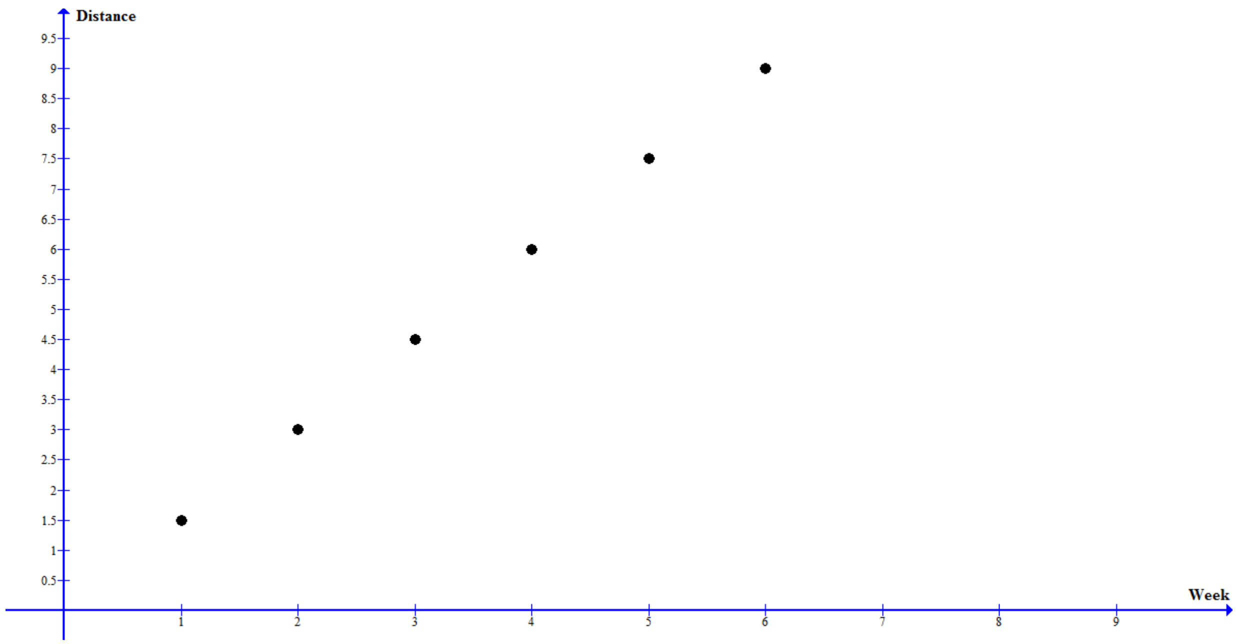
Training Week *(w)*	Weekly Distance *(d)*
1	1.5 miles
2	
3	
4	
5	
6	9 miles
7	

1. Use the table to answer the questions below.

   a. Complete the table above for values of *d*.

Training Week (w)	*Distance Run (d)*
1	*1.5 miles*
2	*3 miles*
3	*4.5 miles*
4	*6 miles*
5	*7.5 miles*
6	*9 miles*
7	*10.5 miles*

b. Graph the ordered pairs from the table above.



c. Write an equation to find the weekly distance, *d*, that will be run if she is on week *w* of training. Which variable is the independent variable and which is the dependent variable?

$$d = 1.5w; \text{independent variable} = w \text{ and dependent variable} = d$$

d. During which week of training will the total distance Lee runs for the week be equivalent to a marathon?

$$26.21875 = 1.5t$$

$$t = \frac{26.21875}{1.5}$$

$$t = 17.479167$$

Lee would run a total distance equivalent to a marathon during week 18.

2. Lee decides that she wants to shorten her training schedule. Write an equation that would allow Lee to prepare for her race in less time. Explain how your equation would get Lee ready sooner, and provide support for your explanation.
Sample Answer:

$$d = 2w; independent\ varirable = w\ and\ dependent\ variable = d$$

This equation models a training schedule that would prepare her more quickly. The slope of this equation is 2 instead of the 1.5 in the original equation. The 2 would make the line rise more quickly, so she would be prepared sooner. The slope of 2 would mean that Lee would need to increase the total number of miles for each week by 2 miles per week. The table below shows that the distance Lee would run each week is longer than the distance she would run each week in the table for the original equation.

Training Week (w)	*Weekly Distance (d)*
1	*2 miles*
2	*4 miles*
3	*6 miles*
4	*8 miles*
5	*10 miles*
6	*12 miles*
7	*14 miles*

We can solve the equation to see how long it would take Lee to finish her training.

$$26.21875 = 2t$$

$$t = \frac{26.21875}{2}$$

$$t = 13.109375$$

We see that it would take her 14 weeks. The original plan required 18 weeks of training.

3. In a group, research local races in your area. Pick one and plan your group's race preparation schedule. Keep the following points in mind:

- What is the date of the race?
- What is the length of the race?
- How many days will the group have to train?
- Has anyone in the group ever run a race before? Does anyone in the group run on a regular basis?

Your group's plan should include a table, an equation, and a graph. Be sure to include the start date and end date of your training. Your group's plan should also include a short narrative. In the narrative, be sure to explain how your group chose the race and planned the training. Your group will present the plan to the class when it is finished.

This portion of the task will take on many different looks. Students have multiple options to fulfill the requirements listed above. They will need to make choices about lengths of races and dates of races.

Sample Response:

Race: Crowley Rice Festival Run

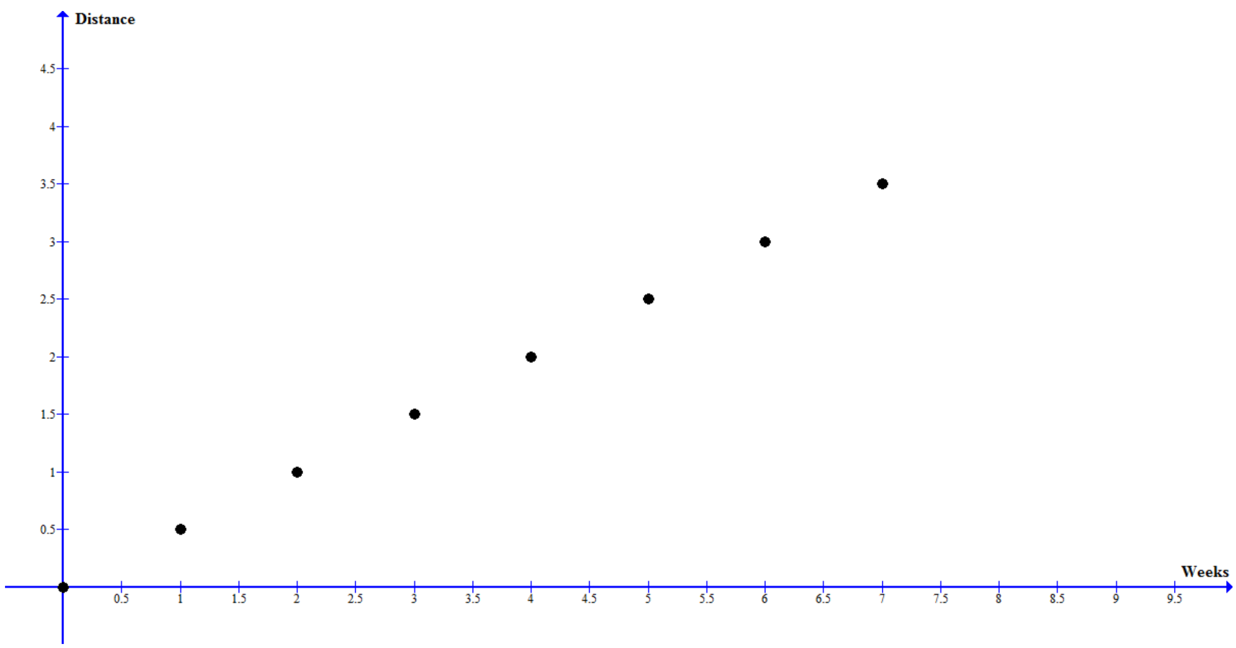
Distance: 5k or 3.1 miles

Date of Race: Saturday, October 18, 2014

We will train for 7 weeks.

Training Week (w)	Weekly Distance (d)
1	.5 miles
2	1 miles
3	1.5 miles
4	2 miles
5	2.5 miles
6	3 miles
7	3.5 miles

$$d = 0.5; independent\ varirable = w\ and\ dependent\ variable = d$$



We decided to prepare for the Rice Festival Run because we attend this festival each year. None of the group members had running experience, so we picked a shorter race. We decided to start off running short distances, so we added on .5 miles to our distance each week. In 7 weeks we would be prepared for our race. In order to be prepared, we will start training August 17, 2014. This will allow us to rest the week before the race.

7TH GRADE TOOLS

7th Grade Remediation Guide

As noted in "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every seventh grade math standard[6].

7th Grade Standard	Previous Grade Standards	7th Gr. Stand. Taught in Advance	7th Gr. Stand. Taught Concurrently
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*	• 6.RPA.2		
7.RP.A.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.	• 6.RP.A.2 • 6.RP.A.3	• 7.RP.A.1	• 7.EE.B.4a *(Not the fluency portion of the standard)*
7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	• 6.RP.A.3	• 7.RP.A.2	
7.NS.A.1a Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	• 6.NS.C.5 • 6.NS.C.6a		• 7.NS.A.1b

[6] This content comes from the work of the math standards' authors found here: http://www.edutron.com/0/Math/ccssmgraph.htm

7th Grade Standard	Previous Grade Standards	7th Gr. Stand. Taught in Advance	7th Gr. Stand. Taught Concurrently
7.NS.A.1b Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	• 6.NS.C.6a • 6.NS.C.7c		• 7.NS.A.1a
7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	• 6.NS.C.7c	• 7.NS.A.1b	
7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.	• 5.NF.A.1	• 7.NS.A.1b • 7.NS.A.1c	
7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.		• 7.NS.A.1d	• 7.NS.A.2b • 7.NS.A.2c
7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real world contexts.			• 7.NS.A.2a • 7.NS.A.2c
7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.	• 5.NF.B.4 • 6.NS.A.1		• 7.NS.A.2a • 7.NS.A.2b
7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	• 5.NF.B.3		
7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.	• 4.OA.A.3 • 6.NS.B.3	• 7.NS.A.2c • 7.NS.A.2d • 7.NS.A.1d	
7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	• 6.EE.A.3 • 6.EE.A.4		• 7.EE.A.2
7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."			• 7.EE.A.1

7th Grade Standard	Previous Grade Standards	7th Gr. Stand. Taught in Advance	7th Gr. Stand. Taught Concurrently
7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9¾ inches long in the center of a door that is 27½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.		• 7.NS.A.3	
7.EE.B.4a (Not the fluency portion of the standard) Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x* + *q*) = *r*, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?	• 6.EE.B.6 • 6.EE.B.7	• 7.NS.A.3	• 7.RP.A.2
7.EE.B.4a Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x* + *q*) = *r*, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?		• 7.EE.B.4a *(Not the fluency portion of the standard)* • 7.NS.A.3	
7.EE.B.4b Solve word problems leading to inequalities of the form *px* + *q* > *r* or *px* + *q* < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.	• 6.EE.B.6 • 6.EE.B.8	• 7.EE.B.4a	
7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	• 6.G.A.1	• 7.RP.A.2	
7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	• None *Introduced in 7th Grade*		

7th Grade Standard	Previous Grade Standards	7th Gr. Stand. Taught in Advance	7th Gr. Stand. Taught Concurrently
7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	• None *Introduced in 7th Grade*		
7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	• 6.G.A.1		
7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	• 4.MD.C.7		
7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	• 6.G.A.1 • 6.G.A.2 • 6.G.A.4		
7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	• 6.SP.A.1 • 6.SP.A.2	• 7.SP.C.5	
7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.		• 7.SP.A.1	
7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	• 5.NF.B.4 • 6.NS.A.1 • 6.SP.A.2		
7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.		• 7.SP.A.2 • 7.SP.B.3	

7th Grade Standard	Previous Grade Standards	7th Gr. Stand. Taught in Advance	7th Gr. Stand. Taught Concurrently
7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	• None *Introduced in 7th Grade*		
7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.		• 7.SP.C.5 • 7.RP.A.3	
7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?		• 7.SP.C.6 • 7.RP.A.3	
7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?		• 7.SP.C.7 • 7.RP.A.3	

7th Grade Tasks At a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for seventh grade include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standard. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the seventh grade tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Sleep Survey Page 84	ECR	• 7.EE.B.3	• 7.NS.A.3
Anna's Room Page 89	ECR	• 7.EE.A.1 • 7.EE.B.3 • 7.EE.B.4a • 7.G.B.6	• 6.G.A.1 • 6.G.A.2 • 6.G.A.4 • 6.EE.A.3 • 6.EE.A.46. • EE.B.6 • 6.EE.B.7 • 7.NS.A.3 • 7.NS.A.3
Cookies for the Bake Sale Page 94	ECR	• 7.EE.B.3 • 7.EE.B.4b • 7.RP.A.2b • 7.RP.A.2c • 7.RP.A.3	• 6.RP.A.2 • 6.RP.A.3 • 6.EE.B.6 • 6.EE.B.8 • 7.EE.B.4a • 7.NS.A.3 • 7.RP.A.1 • 7.RP.A.2
The Equation Competition Page 99	ECR	• 7.RP.A.2	• 6.RP.A.2 • 6.RP.A.3 • 7.RP.A.1

Title	Type	Task Standards	Task Remedial Standards
Distance Between Houses Page 104	ECR	• 7.NS.A.1c • 7.NS.A.3	• 4.OA.A.3 • 6.NS.B.3 • 6.NS.C.7c • 7.NS.A.1b • 7.NS.A.1d • 7.NS.A.2c • 7.NS.A.2d
Video Games Page 108	IT	• 7.NS.A.2 • 7.NS.A.3	• 4.OA.A.3 • 5.NF.B.3 • 5.NF.B.4 • 6.NS.A.1 • 6.NS.B.3
Club Budget Page 115	IT	• 7.NS.A.1 • 7.NS.A.3	• 4.OA.A.3 • 5.NF.A.1 • 6.NS.B.3 • 6.NS.C.5 • 6.NS.C.6a • 6.NS.C.7c
Birthday Shopping Page 121	IT	• 7.EE.A.2 • 7.EE.B.3	• 7.NS.A.3
Field Trip Page 127	IT	• 7.NS.A.1d • 7.NS.A.3	• 4.OA.A.3 • 5.NF.A.1 • 6.NS.B.3 • 7.NS.A.1b • 7.NS.A.1c • 7.NS.A.1d • 7.NS.A.2c • 7.NS.A.2d
Park Area Page 132	IT	• 7.RP.A.1 • 7.G.A.1	• 6.RP.A.2 • 6.G.A.1 • 7.RP.A.2

Sleep Survey (ECR)

Overview

Students will work with rational numbers including decimals, fractions, and percentages to answer questions about a survey conducted by students.

Standards

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 an hour. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Prior to the Task

Students must be able to convert fractions to decimals, decimals to fractions, percentages to decimals, decimals to percentages, fractions to percentages, and percentages to fractions. Students need to be able to round to the nearest whole number.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.EE.B.3	• 7.NS.A.3	1. What is the decimal equivalent of 45%? a. 0.45 2. Write 25% as a fraction in its simplest form. a. ¼ 3. Write a percentage for 35/100. a. 35% 4. Write 4% as a decimal. a. 0.04 5. What is 12% of 73? a. 8.76 6. http://www.illustrativemathematics.org/illustrations/108 7. http://www.illustrativemathematics.org/illustrations/478 8. http://www.illustrativemathematics.org/illustrations/1588	• http://www.illustrativemathematics.org/illustrations/298 • http://learnzillion.com/lessonsets/680-solve-complex-problems-with-positive-and-negative-rational-numbers-in-all-forms-converting-between-forms-and-assessing-the-reasonableness-of-answers • http://learnzillion.com/lessonsets/135-solve-multistep-reallife-and-mathematical-problems-with-positive-and-negative-rational-numbers-in-any-form

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a survey? It is a question, or set of questions, that is asked of a large group to find out how the group feels about a topic.

What does the term *interview* mean with regard to a survey? When someone is interviewed for a survey, he or she is asked questions for which responses are noted.

After the Task

Students may have difficulty with question 3. When multiplying, they will get fractions of students. They will have to realize that this number must be rounded to the nearest whole number to represent an accurate number of students.

Student Extended Constructed Response Task

Mr. Browning asked students in his science class to survey their schoolmates to find out how many hours of sleep students at their school get each night. To keep his class from asking the same students more than once, each group was given a certain grade to interview. The following chart displays the results of the survey.

Grade	Fewer Than 8 Hours	8 Hours	More Than 8 Hours
6^{th}	23%	26%	51%
7^{th}	3/10	¼	9/20
8^{th}	0.28	0.2	0.52

1. Which grade has the largest number of students getting more than 8 hours of sleep? What is the difference between the two largest groups? Show all of your work.

2. Which grade has the largest number of students getting fewer than 8 hours of sleep? What is the difference between the two largest groups? Show all of your work.

3. If there are 70 sixth graders, 68 seventh graders, and 76 eighth graders, complete the table below to show approximately how many students are in each category by grade.

Grade	Fewer Than 8 Hours	8 Hours	More Than 8 Hours
6^{th}			
7^{th}			
8^{th}			

4. Find the percentage of **the total number** of students surveyed across all three grades for each category: Fewer Than 8 Hours, 8 Hours, and More Than 8 Hours. Show all work or explain your reasoning.

Exemplar Constructed Response Task Exemplar Response

Mr. Browning asked members of his science class to survey their schoolmates to find out how many hours of sleep students at their school get each night. To keep his class from asking the same students more than once, each group was given a certain grade to interview. The following chart displays the results of the survey.

Grade	Fewer Than 8 Hours	8 Hours	More Than 8 Hours
6th	23%	26%	51%
7th	3/10	¼	9/20
8th	0.28	0.2	0.52

1. Which grade has the largest number of students getting more than 8 hours of sleep? What is the difference between the two largest groups? Show all of your work.

   More 8th graders get more than 8 hours of sleep. There are 1%, 0.01, or 1/100 more 8th graders than 6th graders that get more than 8 hours of sleep.
   Work should include changing numbers to percentages, decimals, or fractions in order to compare. There are three sample answers below. Any one is acceptable—not all three are needed.

   Sample answer 1: 6th graders: 51%; 7th graders: 45% because 9/20 is equal to 45/100, which is 45%; 8th graders: 52% because 0.52 is 52/100, which is 52%

   Sample answer 2: 8th graders: 0.52; 7th graders: 9/20 is equal to 45/100, which is 0.45; 6th graders: 51% is equal to 51/100, which is 0.51

   Sample answer 3: 7th graders: 9/20, which equals 45/100; 8th graders: 0.52, which is 52/100; 6th graders 51%, which is 51/100

2. Which grade has the largest number of students getting fewer than 8 hours of sleep? What is the difference between the two largest groups? Show all of your work.

   More 7th graders get fewer than 8 hours of sleep. There are 2%, 2/100, or 0.02 more 7th graders than 8th graders that get fewer than 8 hours of sleep.
   Work should include changing numbers to percentages, decimals, or fractions in order to compare. There are three sample answers below. Any one is acceptable—not all three are needed.

   Sample answer 1: 7th graders: 3/10, which equals 30/100; 6th graders: 0.23, which is 23/100; 8th graders: 51%, which is 51/500

   Sample answer 2: 6th graders: 0.23; 7th graders: 3/10 equals 30/100, which is 0.30; 8th graders: 51%, which is 0.51
   Sample answer 3: 8th graders: 51%; 7th graders: 3/10 equals 30/100, which is 30%; 6th graders: 0.23, which is 23%

If there are 70 sixth graders, 68 seventh graders, and 76 eighth graders, complete the table below to show approximately how many students are in each category by grade.

Grade	**Fewer Than 8 Hours**	**8 Hours**	**More Than 8 Hours**
6th	*16*	*18*	*36*
7th	*20*	*17*	*31*
8th	*21*	*15*	*40*

3. Find the percentage of **the total number** of students surveyed across all three grades for each category: Fewer Than 8 Hours, 8 Hours, and More Than 8 Hours. Show all work or explain your reasoning.

There are 214 students surveyed from all three grades. The group of fewer than 8 hours has 57 students, 8 hours has 50 students, and more than 8 hours has 107 students.
57 ÷ 214 = 0.266 so about 27% of students surveyed get fewer than 8 hours of sleep.
51 ÷ 214 = 0.234 so about 23% of students surveyed get 8 hours of sleep.
107 ÷ 214 = 0.50 so about 50% of students surveyed get more than 8 hours of sleep.

Other explanations/work may be given and should be given credit if the reasoning is correct and complete.

Anna's Room (ECR)

Overview

In this extended constructed response students will have to apply properties of operations to write an expression that represents the area of a figure. Students will be asked to find the value of an unknown by applying their knowledge of equations. Students must also add, subtract, multiply, and divide rational numbers.

Standards

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

7.EE.B.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

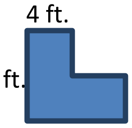
Solve real-life and mathematical problems involving angle measure, surface area, and volume.

7.G.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.EE.A.1	• 6.EE.A.3 • 6.EE.A.4	1. Simplify the following expression: a. $4x + 2(3x + 8) + x$ i. $11x + 16$ b. $\frac{3}{4}x + \frac{1}{2}(5x + 10) - 2x$ i. $1\frac{1}{4}x + 5$ 2. http://www.illustrativemathematics.org/illustrations/541	• http://www.illustrativemathematics.org/illustrations/542 • http://www.illustrativemathematics.org/illustrations/461 • http://learnzillion.com/lessonsets/141-apply-properties-of-operations-to-linear-expressions-with-rational-coefficients-1 • http://learnzillion.com/lessonsets/126-apply-properties-of-operations-to-linear-expressions-with-rational-coefficients-2

7.EE.B.3	• 7.NS.A.3	1. Simplify: $9\frac{4}{5} - 6.25 + 3\frac{1}{3}$ a. $6\frac{53}{60}$ 2. http://www.illustrativemathematics.org/illustrations/108 3. http://www.illustrativemathematics.org/illustrations/478 4. http://www.illustrativemathematics.org/illustrations/1588	• http://www.illustrativemathematics.org/illustrations/298 • http://learnzillion.com/lessonsets/680-solve-complex-problems-with-positive-and-negative-rational-numbers-in-all-forms-converting-between-forms-and-assessing-the-reasonableness-of-answers • http://learnzillion.com/lessonsets/135-solve-multistep-reallife-and-mathematical-problems-with-positive-and-negative-rational-numbers-in-any-form
7.EE.B.4a	• 6.EE.B.6 • 6.EE.B.7 • 7.NS.A.3	1. Solve the following equation: a. *x + 8x + 24 = 60* *x = 4*	• http://www.illustrativemathematics.org/illustrations/425 • http://www.illustrativemathematics.org/illustrations/1107 • http://learnzillion.com/lessonsets/323-solving-word-problems-with-equations-and-inequalities
7.G.B.6	• 6.G.A.1 • 6.G.A.2 • 6.G.A.4	1. Find the area of the figure:  a. *Area: 73.25 ft^2* 2. http://www.illustrativemathematics.org/illustrations/266	• http://www.illustrativemathematics.org/illustrations/647 • http://www.illustrativemathematics.org/illustrations/534 • http://learnzillion.com/lessonsets/452-find-the-area-volume-and-surface-area-of-two-and-three-dimensional-objects

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

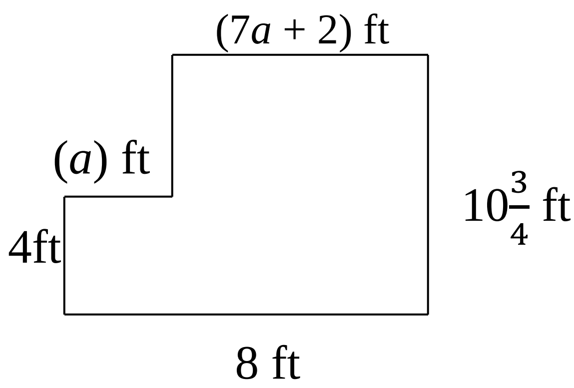
In what measurement is tile sold? Most of the time tile is sold in square feet.

After the Task

This task relates expressions and rational number operations to a real-life context. Students could go further by calculating the area of their own bedroom, and then researching the cost of tiling or carpeting their bedroom.

Student Extended Constructed Response

Use the drawing below of Anna's room to answer all of the questions.



1. Write an expression, in simplest form, that represents the area of Anna's room. Show all work.

2. Using the diagram, find the value of *a*.

3. Anna is getting a new dog, so she has decided that she needs to replace her carpet with tile. She goes to the local hardware store to figure out how many tiles she needs and how much it will cost her to tile her room. Anna finds a tile she likes and learns that each tile covers 1.25 ft^2. Each tile will cost Anna \$1.75. Determine how many tiles Anna will need to buy in order to cover her floor. Then find out how much Anna will spend on her new floor. Justify your answer by showing all of your work or explaining your reasoning.

Extended Constructed Response Exemplar Response

1. Write an expression in simplest form that represents the area of Anna's room. Show all work.

$$\left[10\frac{3}{4}(7a+2)+4a\right] \; square\ feet$$
$$\left[75\frac{1}{4}a+21\frac{1}{2}+4a\right] \; square\ feet$$
$$\left(79\frac{1}{4}a+21\frac{1}{2}\right) \; square\ feet$$

Students may convert the fractions to decimals, or write the expression using mixed numbers. In this sample response, students would divide the complex figure into two rectangles and calculate the area of each rectangle. There is more than one way to divide the figure, but students should all end with equivalent expressions. Students might also find the area of the "whole" rectangle and subtract the "missing" rectangle in the upper left corner. This expression would be $\left(86-6\frac{3}{4}a\right)$ *square feet.*

2. Using the diagram, find the value of *a*.

Students should set up an equation. $a + 7a + 2 = 8$

$$8a + 2 = 8$$
$$-2 \qquad -2$$
$$8a = 6$$
$$8 \qquad 8$$
$$a = \frac{3}{4}$$

3. Anna is getting a new dog, so she has decided that she needs to replace her carpet with tile. She goes to the local hardware store to figure out how many tiles she needs, and how much it will cost her to tile her room. Anna finds a tile she likes and learns that each tile covers 1.25 ft². Each tile will cost Anna $1.75. Determine how many tiles Anna will need to buy in order to cover her floor. Then find out how much Anna will spend on her new floor. Justify your answer by showing all of your work or explaining your reasoning.

**Note: If students get the previous questions wrong, it could cause them to have an incorrect answer to this portion. Student work should be checked to determine if the correct procedures were used with incorrect values.*

Students should substitute ¾ in. for a and simplify the expression they wrote in question 1. Students should get an area of $80\frac{15}{16}$ *ft². Students should then divide that amount by 1.25, converting fractions to decimals (or vice versa) as needed. Students will need to round up to a whole number of tiles, so Anna will need to buy 65 tiles. If each tile costs $1.75, students should multiply $1.75 by 65. Anna will spend $113.75 to tile her room.*

$$\left(79\frac{1}{4}\left(\frac{3}{4}\right)+21\frac{1}{2}\right) \; square\ feet$$
$$59\frac{7}{16}+21\frac{1}{2} \; square\ feet$$

$$80\frac{15}{16}\ square\ feet$$

Number of tiles:

$80\frac{15}{16} \div 1.25$

$80.9375\ square\ feet \div 1.25\ square\ feet\ per\ tile = 64.75\ tiles$

Because we can only buy whole tiles, Anna needs to buy 65 tiles.

Cost of tiles:

65 tiles x $1.75 per tile = $113.75

Cookies for the Bake Sale (ECR)

Overview

Students are asked to use information from an ingredient list to answer questions about making bags of cookies for a school bake sale. They will work with multiplying fractions.

Standards

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 hour. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation*.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions*.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.EE.B.3	• 7.NS.A.3	1. What is $18.25 divided by 25? a. $0.73 2. How do you interpret 0.549 as a money amount? a. $0.55 3. http://www.illustrativemathematics.org/illustrations/108 4. http://www.illustrativemathematics.org/illustrations/478 5. http://www.illustrativemathematics.org/illustrations/1588	• http://www.illustrativemathematics.org/illustrations/298 • http://learnzillion.com/lessonsets/680-solve-complex-problems-with-positive-and-negative-rational-numbers-in-all-forms-converting-between-forms-and-assessing-the-reasonableness-of-answers • http://learnzillion.com/lessonsets/135-solve-multistep-reallife-and-mathematical-problems-with-positive-and-negative-rational-numbers-in-any-form
7.EE.B.4b	• 6.EE.B.6 • 6.EE.B.8 • 7.EE.B.4a	1. Write an inequality for a number that is no more than 8. a. $n < 8$ 2. Write an inequality to model that 5 times a number is at least 25. a. $5n \geq 25$ 3. Write an inequality to model an amount of money divided by 24 that is no more than $15. a. $n \div 24 \leq 15$ 4. http://www.illustrativemathematics.org/illustrations/986	• http://www.illustrativemathematics.org/illustrations/425 • http://www.illustrativemathematics.org/illustrations/642 • http://www.illustrativemathematics.org/illustrations/643 • http://learnzillion.com/lessonsets/323-solving-word-problems-with-equations-and-inequalities
7.RP.A.2b	• 6.RP.A.2 • 6.RP.A.3 • 7.RP.A.1	1. What is the constant of proportionality of $3x = y$? a. 3 2. What is the constant of proportionality of $2x = y$? a. 2	• http://www.illustrativemathematics.org/illustrations/77 • http://www.illustrativemathematics.org/illustrations/549 • http://www.illustrativemathematics.org/illustrations/131 • http://www.illustrativemathematics.org/illustrations/470 • http://learnzillion.com/lessonsets/367-identifying-the-constant-of-proportionality-unit-rate • http://learnzillion.com/lessonsets/136-identify-the-constant-of-proportionality-unit-rate-1
7.RP.A.2c	• 6.RP.A.2 • 6.RP.A.3 • 7.RP.A.1	1. Write an equation for the proportional relationship of 5 miles per hour with the variable *h* representing any hour and *m* representing total miles. a. $5h = m$ 2. Write an equation for the proportional relationship of 60 words per minute with *m* representing the minutes and *w* representing the total number of words. a. $60m = w$ 3. http://www.illustrativemathematics.org/illustrations/1527	• http://www.illustrativemathematics.org/illustrations/1611 • http://www.illustrativemathematics.org/illustrations/134 • http://www.illustrativemathematics.org/illustrations/498 • http://www.illustrativemathematics.org/illustrations/828 • http://learnzillion.com/lessonsets/325-represent-proportional-relationships-by-equations
7.RP.A.3	• 6.RP.A.3 • 7.RP.A.2	1. What is 5 times ½? a. 2 ½	• http://www.illustrativemathematics.org/illustrations/1175

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		2. What is 4 times ⅓? a. 1 ⅓ 3. http://www.illustrativemathematics.org/illustrations/106 4. http://www.illustrativemathematics.org/illustrations/105	• http://www.illustrativemathematics.org/illustrations/118 • http://www.illustrativemathematics.org/illustrations/100 • http://learnzillion.com/lessonsets/658-use-proportional-relationships-to-solve-multistep-ratio-and-percent-problems • http://learnzillion.com/lessonsets/608-use-proportional-relationships-to-solve-ratio-and-percent-problems • http://learnzillion.com/lessonsets/224-use-proportional-relationships-to-solve-multistep-ratio-and-percent-problems • http://learnzillion.com/lessonsets/55-solve-proportional-problems

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a profit? It is the money made after the price of materials is subtracted.

What information does an ingredient list of a recipe provide? It includes the items needed to make the recipe and the amount of each item needed.

After the Task

Students may misinterpret the 36 bags of cookies as 36 cookies and may adjust their ingredient list by multiplying by 1½ instead of 9. Students may try to divide the amount spent by the number of batches instead of the number of bags they are creating. They may even try to divide it per cookie. Students may need further explanation of profit in order to get started on the final question.

Student Extended Constructed Response

Suzy has the following recipe for her favorite cookies. She is going to make them for an upcoming class bake sale. The ingredients are shown in the list below.

½ cup butter	½ cup sugar
⅓ cup brown sugar	1 egg
½ tsp. vanilla	1 ½ cup flour
¼ tsp. salt	½ tsp. baking soda

1. This ingredient list allows Suzy to make 24 cookies. If 1 batch of cookies makes 4 bags, write an equation to find how many batches, *c*, are needed to fill a total number of bags, *b*. Identify the constant of proportionality in the equation you wrote.

2. Suzy has decided to make 36 **bags** of cookies for the bake sale. Rewrite the ingredient list so Suzy has the correct quantities of ingredients to make the correct number of cookies.

3. If Suzy purchases her items at a local grocery store for $15.48, how much does it cost Suzy to make one bag of cookies?

4. Suzy needs to make a profit of at least $10 for her part of the bake sale. What is the least amount she can sell each bag of her cookies for in order to make a profit of at least $10? A *profit* is the amount of money remaining after Suzy sells the cookies and takes out the money used to purchase the items to make the cookies. Write and solve an inequality for the situation. Be sure to define the variable.

Extended Constructed Response Exemplar Response

Suzy has the following recipe for her favorite cookies. She is going to make them for an upcoming class bake sale. The ingredients are shown in the list below.

½ cup butter	½ cup sugar
⅓ cup brown sugar	1 egg
½ tsp. vanilla	1 ½ cup flour
¼ tsp. salt	½ tsp. baking soda

1. This ingredient list allows Suzy to make 24 cookies. If 1 batch of cookies makes 4 bags, write an equation to find how many batches, *c*, are needed to fill a total number of bags, *b*. Identify the constant of proportionality in the equation you wrote.

   b = 4c ; the constant of proportionality is 4

2. Suzy has decided to make 36 **bags** of cookies for the bake sale. Rewrite the ingredient list so Suzy has the correct quantities of ingredients to make the correct number of cookies.

   4c = 36, c = 9; she needs to bake 9 batches
   4 ½ cups butter 4 ½ cups sugar
   3 cups brown sugar 9 eggs
   4 ½ tsp. vanilla 13 ½ cups flour
   2 ¼ tsp. salt 4 ½ tsp. baking soda

3. If Suzy purchases her items at a local grocery store for $15.48, how much does it cost Suzy to make one bag of cookies?

   $15.48 ÷ 36 = $0.43; each bag costs $0.43 to make

4. Suzy needs to make a profit of at least $10 for her part of the bake sale. What is the least amount she can sell each bag of her cookies for in order to make a profit of at least $10? A profit is the amount of money remaining after Suzy sells the cookies and takes out the money used to purchase the items to make the cookies. Write and solve an inequality for the situation. Be sure to define the variable.

   36p - $15.48 ≥ $10; p = the price per bag of cookies
   36p ≥ $10 + $15.48
   36p ≥ $25.48
   p ≥ $25.48 ÷36
   p ≥ $0.71
   She needs to sell her cookies for at least 71 cents per bag to make at least $10.

The Equation Competition (ECR)

Overview

In this task, students will be asked to determine if a set of data is proportional using a coordinate plane and a table. Students must identify the constant of proportionality when given a set of data in table form. Students will explain the meaning of ordered pairs within the context of a given situation, as well as write and solve equations based on the given information.

Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items:</th></tr>
<tr><td>7.RP.A.2</td><td>• 6.RP.A.2
• 6.RP.A.3
• 7.RP.A.1</td><td>• Identify the constant of proportionality for each situation.
o<table><tr><th># of People</th><th>Total Cost</th></tr><tr><td>6</td><td>14</td></tr><tr><td>12</td><td>28</td></tr><tr><td>24</td><td>56</td></tr></table>▪ $k = \frac{7}{3}$
o Sarah biked a constant rate of 14 mph.
▪ $k = 14$
o $y = \frac{1}{4}x$
▪ $k = \frac{1}{4}$
• http://www.illustrativemathematics.org/illustrations/181
• http://www.illustrativemathematics.o</td><td>• http://www.illustrativemathematics.org/illustrations/82
• http://www.illustrativemathematics.org/illustrations/828
• http://www.illustrativemathematics.org/illustrations/1175
• http://www.illustrativemathematics.org/illustrations/193
• http://www.illustrativemathematics.org/illustrations/137
• http://learnzillion.com/lessonsets/612-explain-what-point-xy-on-the-graph-of-a-proportional-relationship-means
• http://learnzillion.com/lessonsets/590-recognize-and-represent-proportional-relationships-interpret-a-point-on-the-graph-of-a-proportional-relationship</td></tr>
</table>

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		rg/illustrations/1527 • http://www.illustrativemathematics.org/illustrations/100 • http://www.illustrativemathematics.org/illustrations/1526	

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a Mathcounts Team? A Mathcounts Team is a group of students who attend math competitions around the state and country. The students usually practice after school with one of the math teachers at their school.

After the Task

Students may have trouble determining how to label the coordinate plane. Discuss with students the wording of the problem. If it states that one value is proportional to another value, then the first value is *y* and the second value is *x*. However, this would change if the wording discussed a ratio of two quantities (it would then be *x*:*y*). Students need to understand that a proportional relationship is the value of *y* to *x*, and need to relate this to the constant of proportionality and the unit rate.

Students may also have trouble describing the different points on the graph and how they relate to the context of the situation. Remind them that the ordered pair represents *(x, y)*, so they can use the labels of the graph or table to help determine the relationship.

Students may also struggle when trying to write an equation. Be sure students understand which value is *y* and which value is *x.* If they know the equation, $y = kx$, and they understand that *k* is the constant of proportionality, it will help them set up the equation.

Student Extended Constructed Response

Markeyah and Cameron are on the Reagan Middle School Mathcounts team. During one of their weekly practices, their teacher Mrs. Bratlie challenged them to a competition. Markeyah and Cameron had to see how many equations they could each solve correctly in 10 minutes. After the 10 minutes were up, it was discovered that Markeyah had solved four equations correctly for every three equations Cameron had solved correctly.

1. Is the number of equations Markeyah solved correctly proportional to the number of equations Cameron solved correctly? Explain your reasoning using a graph on the coordinate plane. Be sure to label the axes.

Mrs. Bratlie decided to have another pair of students try the equation competition for 20 minutes. Drake and Lakeisha were asked to solve as many equations as they each could in 20 minutes. Mrs. Bratlie recorded the number of equations each student solved correctly after 10 minutes and again after 20 minutes.

	Total Number of Questions Solved Correctly After 10 minutes	Total Number of Questions Solved Correctly After 20 minutes
Drake	18	30
Lakeisha	21	35

2. Determine the constant of proportionality if Drake's values represent the *x*-coordinates and Lakeisha's values represent the *y*-coordinates.

3. What ordered pair would represent the number of equations solved correctly by both Drake and Lakeisha when Drake solves one equation correctly?

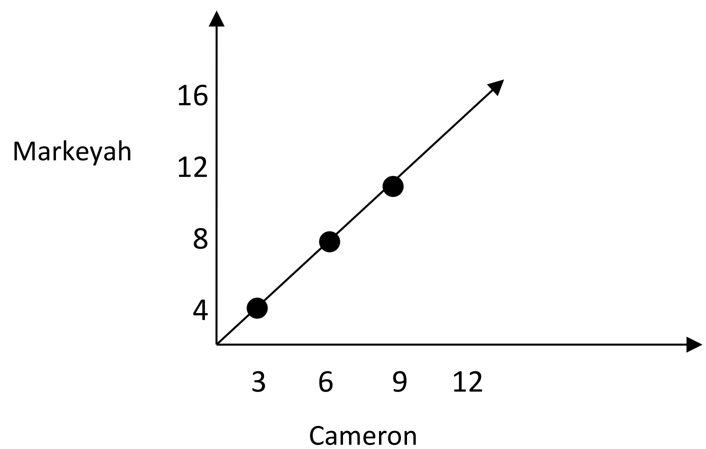
4. Explain what the point (0, 0) represents in the context of this situation.

5. In 40 minutes, Lakeisha solved a total of 63 equations. Write and solve an equation that can be used to find the total number of equations Drake solved correctly.

Extended Constructed Response Exemplar Response

Markeyah and Cameron are on the Reagan Middle School Mathcounts team. During one of their weekly practices, their teacher Mrs. Bratlie challenged them to a competition. Markeyah and Cameron had to see how many equations they could each solve correctly in 10 minutes. After the 10 minutes were up, it was discovered that Markeyah had solved four equations correctly for every three equations Cameron had solved correctly.

1. Is the number of equations Markeyah solved correctly proportional to the number of equations Cameron solved correctly? Explain your reasoning using a graph on the coordinate plane. Be sure to label the axes.



YES, the relationship is proportional. The graph is a straight line, and it goes through the origin.

Mrs. Bratlie decided to have another pair of students try the equation competition for 20 minutes. Drake and Lakeisha were asked to solve as many equations as they each could in 20 minutes. Mrs. Bratlie recorded the number of equations each student solved correctly after 10 minutes and again after 20 minutes.

	Total Number of Questions Solved Correctly After 10 minutes	Total Number of Questions Solved Correctly After 20 minutes
Drake	18	30
Lakeisha	21	35

2. Determine the constant of proportionality if Drake's values represent the *x*-coordinates and Lakeisha's values represent the *y*-coordinates.

   $k = \frac{7}{6}$ if Lakeisha's values are graphed on the y-axis and Drake's values are graphed on the x-axis.

3. What ordered pair would represent the number of equations solved correctly by both Drake and Lakeisha when Drake solves one equation correctly?

(1, $\frac{7}{6}$); students need to know that if the x-coordinate is 1, the y-coordinate represents the unit rate (1, r).

4. Explain what the point (0, 0) represents in the context of this situation.

Sample answer: Zero problems had been solved by Drake, so Lakeisha had not solved any problems.

5. In 40 minutes, Lakeisha solved a total of 63 equations. Write and solve an equation that can be used to find the total number of equations Drake solved correctly.

y=$\frac{7}{6}x$; 63 =$\frac{7}{6}x$; x = 54 equations; Drake solved 54 equations in 40 minutes.

Distance Between Houses (ECR)

Overview

Students will represent the locations of the houses of friends relative to the school on a number line and use that information to help find the distances between the friends' houses.

Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Understand subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.NS.A.1c	• 6.NS.C.7c • 7.NS.A.1b	1. What is the difference between 3 and -5? *a. 8* 2. What is the distance between $-2\frac{1}{3}$ and $-5\frac{3}{8}$? a. $3\frac{1}{24}$ *units* 3. http://www.illustrativemathematics.org/illustrations/314 4. http://www.illustrativemathematics.org/illustrations/317	• http://learnzillion.com/lessonsets/659-understand-subtraction-as-addition-of-additive-inverses-and-differences-in-terms-of-distance-on-the-number-line • http://learnzillion.com/lessonsets/150-understand-subtraction-of-rational-numbers-as-adding-the-additive-inverse • http://learnzillion.com/lessonsets/137-apply-properties-of-operations-to-add-and-subtract-rational-numbers-and-understanding-subtraction-of-rational-numbers-as-adding-the-additive-inverse
7.NS.A.3	• 4.OA.A.3 • 6.NS.B.3 • 7.NS.A.2c • 7.NS.A.2d • 7.NS.A.1d	1. Jonathan lives $18\frac{2}{3}$ miles from school. His mom has already driven $4\frac{1}{4}$ miles to bring him to school. How much farther does his mom have to drive before Jonathan will be at school? a. $14\frac{5}{12}$ *miles* 2. http://www.illustrativemathematics.org/illustrations/298	• http://www.illustrativemathematics.org/illustrations/1289 • http://www.illustrativemathematics.org/illustrations/374 • http://www.illustrativemathematics.org/illustrations/274 • http://learnzillion.com/lessonsets/193-solve-realworld-problems-involving-the-four-operations-with-rational-numbers-1

After the Task

Students may have difficulty using differences to find the answers to parts b and c. Have students use the number line they created in part a to help them find the distance. Then have students explain how the distance they found on the number line can be expressed as a sum or difference, making the connection to the absolute value of the difference between the numbers.

Also, the task found at **http://www.illustrativemathematics.org/illustrations/591** can be used after this task for additional practice.

Student Extended Constructed Response

Aakash, Bao Ying, Chris, and Donna all live on the same street as their school. The street runs from east to west. .

- Aakash lives 5 ½ blocks to the west of the school.
- Bao Ying lives 4 ¼ blocks to the east of the school
- Chris lives 2 ¾ blocks to the west of the school.
- Donna lives 6 ½ blocks to the east of the school.

Use this information to complete the following.

a. Represent the relative position of the houses on a number line with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.

b. How far does Bao Ying live from Aakash? Show how you arrived at your answer using sums or differences.

c. Donna says she lives 3 ¾ blocks away from Chris. Is she correct? Explain your reasoning using the number line or by using sums or differences.

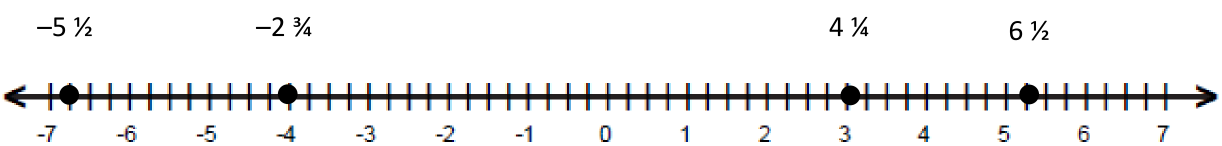
Extended Constructed Response Exemplar Response

Aakash, Bao Ying, Chris, and Donna all live on the same street as their school. The street runs from east to west. .

- Aakash lives 5 ½ blocks to the west of the school.
- Bao Ying lives 4 ¼ blocks to the east of the school
- Chris lives 2 ¾ blocks to the west of the school.
- Donna lives 6 ½ blocks to the east of the school.

Use this information to complete the following.

a. Represent the relative position of the houses on a number line with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.



b. How far does Bao Ying live from Aakash? Show how you arrived at your answer using sums or differences.

$4\frac{1}{4} - (-5\frac{1}{2}) = 9\frac{3}{4}$

Bao Ying lives 9 ¾ blocks from Aakash.

c. Donna says she lives 3 ¾ blocks away from Chris. Is she correct? Explain your reasoning using the number line or by using sums or differences.

Donna is not correct. Donna lives 9 ¼ blocks from Chris. To find the distance between two points on the number line, you can find the absolute value of the difference between the values of the two points.

$|6\frac{1}{2} - (-2\frac{3}{4})| = |6\frac{1}{2} + 2\frac{3}{4}| = 9\frac{1}{4}$.

Other valid explanations may also be accepted.

Video Games (IT)

Overview

This instructional task requires students to use multiplication and division of rational numbers to purchase a video game system.

Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.NS.A.2	• 5.NF.B.3 • 5.NF.B.4 • 6.NS.A.1	1. -46.28×4 a. -185.12 2. $-234.8 \div -23.2$ Round your answer to two decimal places. a. 10.12 3. http://www.illustrativemathematics.org/illustrations/604 4. http://www.illustrativemathematics.org/illustrations/593	• http://www.illustrativemathematics.org/illustrations/858 • http://www.illustrativemathematics.org/illustrations/321 • http://www.illustrativemathematics.org/illustrations/965 • http://www.illustrativemathematics.org/illustrations/50 • http://www.illustrativemathematics.org/illustrations/407 • http://www.illustrativemathematics.org/illustrations/464 • http://learnzillion.com/lessonsets/281-extending-multiplication-of-fractions-to-rational-numbers • http://learnzillion.com/lessonsets/18-multiply-and-divide-improper-fractions • http://learnzillion.com/lessonsets/600-convert-a-rational-number-to-a-decimal-using-long-division
7.NS.A.3	• 4.OA.A.3 • 6.NS.B.3	1. You have to pay $75 to your mom in equal payments over 6 months. If you are paying her from your savings account, how much will your account change each month?	• http://www.illustrativemathematics.org/illustrations/1289 • http://www.illustrativemathematics.org/illustrations/272 • http://www.illustrativemathematics.org/illustrations/275

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		a. -$12.50 2. http://www.illustrativemathematics.org/illustrations/298	• http://www.illustrativemathematics.org/illustrations/374 • http://www.illustrativemathematics.org/illustrations/274 • http://www.illustrativemathematics.org/illustrations/1299 • http://www.illustrativemathematics.org/illustrations/1300 • http://learnzillion.com/lessonsets/193-solve-realworld-problems-involving-the-four-operations-with-rational-numbers-1 • http://learnzillion.com/lessonsets/20-simplify-expressions-with-order-of-operations

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a savings account? This question is designed to make sure that students understand the purpose of this task. A *savings account* is an account at a bank. You deposit money into an account, usually for a long period of time. A savings account is a good way of collecting money for a future purchase.

What is a Statement of Account? A *Statement of Account* is a listing of the activity occurring in an account over a period of time. Statements of account show money withdrawn and money deposited.

What is a balance? Students may be unfamiliar with banking terminology. A *balance* is the amount of money in your account.

What is the difference between a withdrawal and a deposit? Students may confuse these terms. A *deposit* is putting money into an account, and a *withdrawal* is taking money out of an account

During the Task

Students may struggle with the idea of change and negative numbers. Remind students that they are starting with money in the account and taking money out each month. If they are starting with money and then taking out money, they can represent this situation with a negative number.

Students may struggle with creating a monthly statement. You can scaffold this task for them using the situation in problem 3. Have students create monthly statements for six months using a starting account balance of $500. This will help students practice the process of creating a monthly statement before they have to start making decisions about how to spend money.

During the task, circulate around the room and look for groups who may be spending more money than they have in the account. The students may also have trouble calculating their totals with the tax included.

After the Task

This task shows students how math is useful in their own lives. Savings accounts can be useful when they want to make a big purchase. Saving a little each month can allow them to purchase something expensive without having to pay interest.

Student Instructional Task

1. David is purchasing an Xbox One. David's mom will buy the game system, but he will have to pay her back. He will pay her the same amount each month from his savings account. He will take six months to pay her back. If he does not deposit or withdraw any money from the account, the account balance will change by $-499.00.

    a. How much would David's account change each month? Explain.

    b. Describe the change in the account after three months.

2. David wants to purchase three Xbox One games. The total for the three games is $179.88

    a. What is the price of each game if they are all the same price? Show your calculations.

    b. If the games are paid for in equal amounts over six months, how much would David pay each month? Explain.

3. David decides to buy the three games with the Xbox One. His mom will pay for the games and the game console, and he will pay her back in equal payments over six months.

    a. How much would David's account change each month? Show your calculations.

    b. Describe the change in the account after three months.

4. You decide to purchase your own game system. You have $950 saved. Research game systems and game prices. Decide which game system and which games you would like to purchase.

    a. Create a total bill for your purchases. In addition to the game system and games, you must pay sales tax. Use the sales tax rate in the town where you will purchase your game system and games.

b. You are going to pay for your purchases in equal monthly installments over six months. Create an Account Statement for your savings account each month. Each Statement of Account should show how you calculated the balance at the end of each month.

c. Write a short narrative explaining how you chose your game system and games. Assume your beginning balance is $950. Be sure to include your ending balance and a brief explanation of how you calculated your monthly balances. Be prepared to share your project with the class.

Instructional Task Exemplar Response

1. Your friend David is purchasing an Xbox One. David's mom will buy the game system, but he will have to pay her back. He will pay her the same amount each month from his savings account. He will take six months to pay her back. If he does not deposit or withdraw any money from the account, the account balance will change by $-499.00.
   a. How much would your friend's account change each month? Explain.

$$\$ - 499 \div 6$$

$$\$ - 83.17$$

David's account would change by -$83.17 each month. The change is -$83.17 each month because money is being withdrawn from the account.

   b. Describe the change in the account after three months.

$$-\$83.17 \times 3$$

$$-\$249.51$$

The account would change by -$249.51 after three payments over three months.

2. David wants to purchase three Xbox One games. The total for the three games is $179.88
   a. What is the price of each game if they are all the same price? Show your calculations.

$$\$179.88 \div 3$$

$$\$59.96$$

Each game would cost $59.96.

   b. If the games are paid for in equal amounts over six months, how much would David pay each month?

$$\$179.88 \div 6$$

$$\$29.98$$

The total cost of the games is $179.88, so that number must be divided by 6, the number of payments. He would pay $29.98 each month.

3. David decides to buy the three games with the Xbox One. His mom will pay for the games and the game console, and he will pay her back in equal payments over six months.
   a. How much would your friend's account change each month? Show your calculations.

$$\frac{499 + 179.88}{6}$$

$$\frac{678.88}{6}$$

$$\$113.15$$

His account would change by -$113.15 per month.

b. Describe the change in the account after three months.

$$-\$113.15 \times 3$$

$$-\$339.45$$

The account would change by -$339.45 after three payments over three months.

4. You decide to purchase your own game system. You have $950 saved. Research game systems and game prices. Decide which game system and which games you would like to purchase.
   a. Create a total bill for your purchases. In addition to the game system and games, you must pay sales tax. Use the sales tax rate in the town where you will purchase your game system and games.

This portion of the task will take on many different looks. Students have multiple options to fulfill the requirements listed above. They will need to decide on the number of games they want to purchase. They also need to choose a game system.

This is a sample bill. The listed prices are from Walmart.com.

PlayStation 4:	*$458.00*
Lego Marvel Super Heroes:	*$44.98*
Skylanders Swap Force Start Pack:	*$51.65*
Subtotal:	*$554.63*
Tax (9%):	*$49.92*
Total:	*$604.55*

b. You are going to pay for your purchases in equal monthly installments over six months. Create an Account Statement for your savings account each month. Each Statement of Account should show how you calculated the balance at the end of each month.

c. This is a sample of the information needed in the monthly Statement of Account.

$$\$604.55 \div 6 = \$100.76$$

Month 1:

Beginning Balance: $950

Withdrawal: $100.76

$950-$100.76= $849.24

Ending Balance: $849.24

d. Write a short narrative explaining how you chose your game system and games. Assume your beginning balance is $950. Be sure to include your ending balance and a brief explanation of how you calculated your monthly balances. Be prepared to share your project with the class.

This is a sample narrative.

> *I chose to buy a PlayStation 4 for $458.00. I chose this game system because I have an old PlayStation 2, so I thought that it would be nice to have the new PlayStation console. I don't have a lot of time to play games and didn't want to spend all of my money, so I chose two games. <u>Lego Marvel Super Heroes</u> was $44.98, and <u>Skylanders Swap Force Start Pack</u> was $51.65. My subtotal was $554.63. The sales tax in Crowley, Louisiana, where I would make my purchases is 9%, so my sales tax is $49.92. My total amount spent was $604.55. My monthly payment for each of the six months is $100.76. My savings account would have $345.45 left after I finished paying for my PlayStation 4 and games.*

Club Budget (IT)

Overview

This instructional task requires students to use addition and subtraction of rational numbers to create a budget for a school club.

Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
c. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
d. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
e. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.NS.A.1	• 5.NF.A.1 • 6.NS.C.5 • 6.NS.C.6a • 6.NS.C.7c	1. $\$123.64 + \$243.75$ a. $367.39 2. $\$432.76 - \$175.62$ a. $257.14 3. http://www.illustrativemathematics.org/7.NS.A.1	• http://www.illustrativemathematics.org/5.NF.A.1 • http://www.illustrativemathematics.org/6.NS.C.5 • http://learnzillion.com/lessonsets/411-add-and-subtract-rational-numbers-represent-addition-and-subtraction-on-a-horizontal-or-vertical-number-line-diagram • http://learnzillion.com/lessonsets/596-adding-and-subtracting-rational-numbers-using-distance-absolute-value-and-opposites
7.NS.A.3	• 4.OA.A.3 • 6.NS.B.3	1. Your class is considering going on a field trip to a skating ring. The admission fee is $12 per person. You have 25 students in your class. How much will it cost for your class to enter the skating ring? a. $300 2. http://www.illustrativemathematics.org/7.NS.A.3	• http://www.illustrativemathematics.org/4.OA.A.3 • http://www.illustrativemathematics.org/6.NS.B.3 • http://learnzillion.com/lessons/1150-use-addition-and-subtraction-to-solve-realworld-problems-involving-decimals

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a budget? This question is designed to make sure that students understand the purpose of this task. They need to figure out a plan to track how much money is deposited and how much money is spent to ensure that the account always has money.

What is an account? Some students may be unfamiliar with accounts and using banks. You may need to explain that a bank is where you can bring money. The bank holds the money in an account until you are ready to spend it.

What is a balance? Students may be unfamiliar with banking terminology. A *balance* is the amount of money in your account.

What is the difference between a withdrawal and a deposit? Students may confuse these terms. A *deposit* is putting money into an account, and a *withdrawal* is taking money out of an account.

During the Task

Students may struggle with creating a monthly statement. You can scaffold this task for them using the three months in part one. Have students create monthly statements for September, October, and November. This will help students practice the process of creating a monthly statement before they have to start making decisions about how to make and spend money.

During the task, as you circulate around the room, look for groups who may only be adding in unrealistic numbers for their fundraisers.

For example, students may only add in $25 for a car wash fundraiser. Ask students guiding questions like the following:

1. If you made $25 at your car wash, how many cars did you wash?
    a. 5 cars at $5 each
2. Would you have a car wash if you were only going to wash five cars all day?
    a. Students at this point should discuss a car wash and about how many cars they might expect to wash.

After the Task

This task shows students how math is useful in their own lives. Encourage students to think of ways creating a budget might be useful in their own lives. Students may mention creating budgets for clubs or budgets to track their personal money.

Student Instructional Task

The members of your group have been selected as the officers for your school's Junior Beta Club. This executive committee is responsible for tracking all money deposited into the club's account as well as all money spent during the year. The committee also needs to create a budget for the remainder of the year. At the beginning of September, the balance in the account was $253.24.

1. Listed below are the activities the club spent money on or collected money for:

Sept. 1-Sept. 30	Oct. 1–Oct. 31	Nov. 1–Nov. 30
Dues: 25 members at $5 each	Gardening materials: $–124.98	Supplies for Thanksgiving baskets: $–40.43
Induction ceremony: $–100.32	Bake sale: $205.50	

a. Using the table above, create a number line to represent the amount of money added to the account during this three-month period.

b. Using the table above, create a number line to represent the amount of money spent during this three-month period.

c. How much money is in the account at the end of November? Show two different ways to find the balance of the account.

2. This year the club also voted to include some fun activities throughout the year and an end-of-year trip to celebrate the club's success. Below is a list of the suggested activities and fundraisers.

Fun Activities	Fundraisers
Bowling: $60 per lane (up to 6 people per lane)	Candy sale: $60 per member
Skating: $12 per person	Wrapping paper: $50 per member
Laser tag: $15 per person	Holiday wreaths: $65 per member
Water park: $50 per person	Car wash: $5 per car
Zoo visit: $12 per student; $17.50 per adult	Dress-down day: $1 per student

a. Create a monthly budget for the remainder of the year. In your budget, you will propose which fun activity or activities your club should pursue. You will also propose which fundraisers your club should use to raise the money to finish the year. Keep the following points in mind:

- Begin with December 1 through December 31 and end with May 1 through May 31.
- Show the balance at the beginning of each month (use the ending balance from November as the beginning balance for December).
- Show any proposed expenses for the month as well as any proposed fundraisers.

- Budget $75 per month for supplies for service projects.
- The club must also attend the Louisiana Jr. Beta Club Convention in Lafayette, Louisiana, in May. The cost of the trip for the convention is $1,500. This includes transportation and hotel rooms.
- Include at least one Fun Activity from the list above.
- The balance at the end of May should be at least $250 to begin the next school year.

Your budget should include a month by month statement as well as a short narrative explaining why you chose certain activities and fundraisers. Each monthly statement should show how you calculated the balance at the end of each month. In your narrative, be sure to explain how you determined which fundraisers to choose, which activity to choose, and when to conduct certain fundraisers. Also, be sure to state how much money will be remaining at the end of May. Be prepared to share your budget with the class.

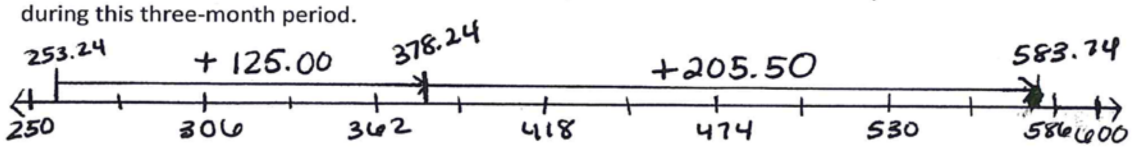
Instructional Task Exemplar Response

The members of your group have been selected as the officers for your school's Junior Beta Club. This executive committee is responsible for tracking all money deposited into the club's account as well as all money spent during the year. The committee also needs to create a budget for the remainder of the year. At the beginning of September, the balance in the account was $253.24.

1. Listed below are the activities the club spent money on or collected money for:

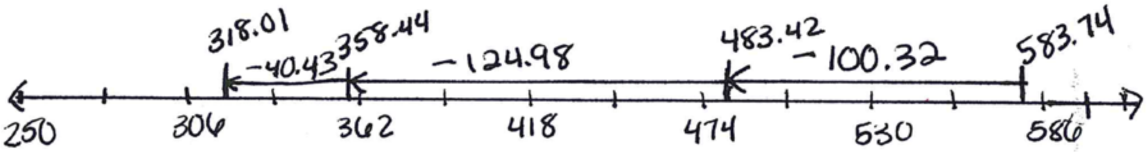
Sept. 1-Sept. 30	**Oct. 1-Oct. 31**	**Nov. 1-Nov. 30**
Dues: 25 members at $5 each	Gardening materials: $–124.98	Supplies for Thanksgiving baskets: $–40.43
Induction ceremony: $–100.32	Bake sale: $205.50	

a. Using the table above, create a number line to represent the amount of money added to the account during this three-month period.



Dues: 25 X 5 = $125

b. Using the table above, create a number line to represent the amount of money spent during this three-month period.



c. How much money is in the account at the end of November? Show two different ways to find the balance of the account.

At the end of November, there is $318.01 in the account.

① $253.24 + 330.50 = 583.74$

$583.74 - 265.73 = 318.01$

② $330.50 - 265.73 = 64.77$

$253.24 + 64.77 = 318.01$

Students may have different number lines for parts a and b as they may not use the beginning balance. The problem asks to show the amount added or spent; students may choose to start at zero. There may also be different methods shown in part c.

2. This year the club also voted to include some fun activities throughout the year and an end-of-year trip to celebrate the club's success. Below is a list of the suggested activities and fundraisers.

Fun Activities	**Fundraisers**
Bowling: $60 per lane (up to 6 people per lane)	Candy sale: $60 per member
Skating: $12 per person	Wrapping paper: $50 per member
Laser tag: $15 per person	Holiday wreaths: $65 per member
Water park: $50 per person	Car wash: $5 per car
Zoo visit: $12 per student; $17.50 per adult	Dress-down day: $1 per student

a. Create a monthly budget for the remainder of the year. In your budget, you will propose which fun activity or activities your club should pursue. You will also propose which fundraisers your club should use to raise the money to finish the year. Keep the following points in mind.

- Begin with December 1 through December 31 and end with May 1 through May 31.
- Show the balance at the beginning of each month (use the ending balance from November as the beginning balance for December).
- Show any proposed expenses for the month as well as any proposed fundraisers.
- Budget $75 per month for supplies for service projects.
- The club must also attend the Louisiana Jr. Beta Club Convention in Lafayette, Louisiana, in May. The cost of the trip for the convention is $1,500. This includes transportation and hotel rooms.
- Include at least one Fun Activity from the list above.
- The balance at the end of May should be at least $250 to begin the next school year.

Your budget should include a month by month statement as well as a short narrative explaining why you chose certain activities and fundraisers. Each monthly statement should show how you calculated the balance at the end of each month. In your narrative, be sure to explain how you determined which fundraisers to choose, which activity to choose, and when to conduct certain fundraisers. Also, be sure to state how much money will be remaining at the end of May. Be prepared to share your budget with the class.

This portion of the task will take on many different looks. Students have multiple options to fulfill the requirements listed above. They will need to make some assumptions about the number of chaperones needed for the activities they may wish to propose. They may also need to make some assumptions about the fundraisers, specifically the number of cars they would need to wash and how many students might buy a dress-down day pass. Students could research the rules about fundraisers in their school/district and check to see how many students participated in past dress-down events to attempt to determine values that make sense with this problem. At minimum, students must include fundraisers to cover $1,950 for the service project supplies and convention. Additional fundraisers would be needed to cover the cost of the activity they chose to propose.

Birthday Shopping (IT)

Overview

Students will demonstrate the ability to rewrite expressions in real-world situations.

Standards

Use properties of operations to generate equivalent expressions.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 an hour. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.EE.A.2		1. What does the variable *d* without a coefficient mean? a. 1 times d 2. What is *d* + 0.14*d*? a. 1.14*d* 3. What is *d* - 0.34*d*? a. 0.66*d* 4. http://www.illustrativemathematics.org/illustrations/1450	• http://learnzillion.com/lessonsets/568-understand-that-rewriting-an-expression-in-different-forms-can-help-solve-the-problem • http://learnzillion.com/lessonsets/204-rewrite-an-expression-to-understand-how-the-quantities-are-related
7.EE.B.3	• 7.NS.A.3	1. What is 8% of $45? a. $3.60 2. How much is 25% off of $34? a. $8.50 3. If a $46 shirt is discounted 30%, what is the sale price of the shirt? a. $32.20 4. http://www.illustrativemathematics.o	• http://www.illustrativemathematics.org/illustrations/298 • http://learnzillion.com/lessonsets/680-solve-complex-problems-with-positive-and-negative-rational-numbers-in-all-forms-converting-between-forms-and-assessing-the-reasonableness-of-answers • http://learnzillion.com/lessonsets/135-solve-

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
		rg/illustrations/108 5. http://www.illustrativemathematics.org/illustrations/478 6. http://www.illustrativemathematics.org/illustrations/1588	multistep-reallife-and-mathematical-problems-with-positive-and-negative-rational-numbers-in-any-form

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What does percent off mean? It is a markdown or amount taken off the original price.

What is sales tax? *Sales tax* is a percentage of the amount of money you pay for a purchase. It is added to the purchase price to create the final cost of an item.

How much is sales tax? It varies, depending on the region where the purchase is made.

During the Task

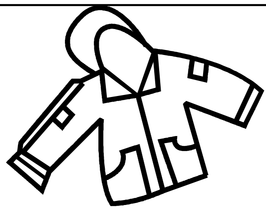
- If students choose only two items to purchase, ask them to find how many items they could buy to spend as close to the total amount of $250 without going over the price.
- Students may find the total savings without adding the sales tax to the original prices.
- When students write the total savings as a percentage, they may start trying to add the discount percentages from the given information in the problem. Discuss with them why adding the percentage values would not give them the correct percent savings.
- Students may struggle with writing two expressions for each situation. Ask students how they could write the expression using one operation rather than two.

After the Task

Students will be able to relate this task to shopping for items that are on sale. Provide students with copies of sales papers from various stores and have students find the percent discount, the sales price, or the original price of a variety of items to provide more practice with this concept.

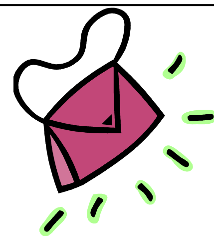
Student Instructional Task

It’s your birthday! You’ve received $250 from your party guests. Your father takes you to a local department store to spend your money. Below are items that can be found, along with the current discount percentage.



Jacket - $45
20% off

Tennis Shoes - $75
15% off



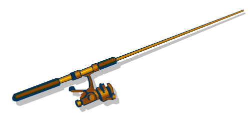
Purse - $65
30% off

Television - $129
25% off

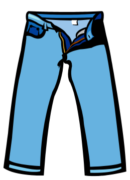
Headphones - $55
10% off

Men’s/Women’s Boots - $109
20% off

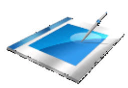
Team Jersey - $85
25% off



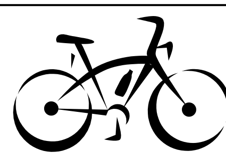
Fishing Pole and Reel - $158
35% off



Men’s/Women’s Jeans $45
20% off



Tablet - $189
40% off



Bicycle - $139
15% off

MP3 Player - $78
30% off

Book Collection - $36
35% off

Laptop - $199
10% off

Use the information above to answer the following.

1. Calculate the sales price for each item.

2. What items could you buy with $250? Choose at least two items. How much of your money would you spend purchasing the items at the discounted price after taxes are added? (Use the current sales tax in your area.) Show all your work.

3. What is your total savings based on what you would have paid if the items were not on sale? State the savings as a dollar amount and as a percentage. Explain your answer.

4. Choose an item from the store. Write two equivalent expressions that can be used to find the sale price. Explain how you know the two expressions are equivalent. Be sure to define your variable.

Instructional Task Exemplar Response

It's your birthday! You've received $250 from your party guests. Your father takes you to a local department store to spend your money. Below are items that can be found, along with the current discount percentage.

Use the information above to answer the following.

1. Calculate the sales price for each item.

Jacket-$36	*Fishing Pole and Reel-$102.70*
Tennis Shoes-$63.75	*Men's/Women's Jeans-$36*
Purse-$45.50	*Tablet-$113.40*
Television-$96.75	*Bicycle-$118.15*
Headphones-$49.50	*MP3 Player-$54.60*
Men's/Women's Boots-$87.20	*Book Collection-$23.40*
Team Jersey-$63.75	*Laptop-$179.10*

2. What items could you buy with $250? Choose at least two items. How much of your money would you spend purchasing the items at the discounted price after taxes are added? (Use the current sales tax in your area.) Show all your work.

This answer will vary by student. The explanation should include:

- *The items the student would purchase*
- *The total spent after taxes (the tax should be correct for your area—you may need to provide this)*

Sample response:

I decided to buy:

Jacket: $36

Jeans: $36

Tablet: $113.40

Books: $23.40

Total amount with discounts before taxes: $208.80

Sales tax: 9%

$208.80 x 0.09 = $18.79

Total with sales tax: $208.80 + $18.79 = $227.59

3. What is your total savings based on what you would have paid if the items were not on sale? State the savings as a dollar amount and as a percentage. Explain your answer.

This answer will vary by student. The explanation should include:

- *The original price with the total spent subtracted to show the total savings.*

Sample response:

Jacket: $45

Jeans: $45

Tablet: $189

Books: $36

Total amount before taxes: $315

Sales tax: 9%

$315 x 0.09 = $28.35

Total with sales tax: $315 + $28.35 = $343.35

Total savings in dollars: $343.35 - $227.59 = $115.76

Total savings as a percentage: $115.76 ÷ $343.35 x 100 = 33.714, so approximately 34% savings

4. Choose an item from the store. Write two equivalent expressions that can be used to find the sale price. Explain how you know the two expressions are equivalent. Be sure to define your variable.

This item will vary by student.

Example: Jacket original price $45 discounted 20%

j = the original price of the jacket

j – 0.20j and 0.80 j

I know these two expressions are equivalent because 1 whole minus 0.20 leaves 0.80.

***Students may also explain that they substituted the original price of the jacket ($45) and found the same value for both expressions ($36).*

Field Trip (IT)

Overview

Students are asked to make several decisions about a class trip, including where to go and which bus company to use. They are then asked to justify these decisions.

Standards

Apply and extend previous understandings of operations with fractions.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.NS.A.1d	• 5.NF.A.1 • 7.NS.A.1b • 7.NS.A.1c	1. What is 25 x 4.75? a. 118.75 2. If a van carries 15 students, how many vans are needed for 48 students? a. 4	• http://www.illustrativemathematics.org/illustrations/855 • http://www.illustrativemathematics.org/illustrations/861 • http://learnzillion.com/lessonsets/137-apply-properties-of-operations-to-add-and-subtract-rational-numbers-and-understanding-subtraction-of-rational-numbers-as-adding-the-additive-inverse • http://learnzillion.com/lessonsets/17-add-and-subtract-mixed-numbers
7.NS.A.3	• 4.0A.A.3 • 6.NS.B.3 • 7.NS.A.1d • 7.NS.A.2c • 7.NS.A.2d	1. If one chaperone can go on a trip free with six paid students, how many can go with 62 students? a. 10 2. How much would the rental fee for a van be for 36 miles, if the van company charges $15 for the rental fee and $5 for each mile? a. $195 3. http://www.illustrativemathematics.org/illustrations/298	• http://www.illustrativemathematics.org/illustrations/1289 • http://www.illustrativemathematics.org/illustrations/273 • http://www.illustrativemathematics.org/illustrations/604 • http://www.illustrativemathematics.org/illustrations/604 • http://www.illustrativemathematics.org/illustrations/593 • http://learnzillion.com/lessonsets/193-solve-realworld-problems-involving-the-four-operations-with-rational-numbers-1

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What factors do you take into account when deciding where to go in order to plan a trip? Factors to take into account are the places people would like to go, the distance to drive, the cost of admission, time for travel and to visit the attraction, and how many people are going.

What is group pricing? Group pricing is a special discount given when there is a large group being admitted to an attraction. Usually, it is a few dollars cheaper to encourage groups to visit.

During the Task

- This task is designed for students to complete in a group. A group of three or four is optimal.
- When students are calculating costs, they should consider the 28 students in the class **plus** the number of students in the group (three or four).
- Students have numerous choices to make. They may have a hard time deciding where to start. They need to understand that they are going to have to figure out the pricing for each place based on the total number of students going, plus the cost of travel. They will also need to figure in the cost for the chaperones to attend.
- When students are considering the distance to the different attractions, they may assume that the given distance is the round-trip distance. Other students may assume the given distance is one-way. Ask probing questions to have students explain their reasoning about this concept. Technically the distance given is meant to be the distance to the attraction only—they would double the distance to include the return trip.

After the Task

A follow-up to this task may be the planning of a real field trip in which the students get to help determine the destination based on several factors. It can also be related to the choices their parents have to make in going on a summer vacation trip.

Student Instructional Task

Mr. Falting, a teacher at Roosevelt Middle School, is planning to take his middle school students on a field trip to some nearby attractions. Help Mr. Falting decide where he should take his class.

Here are some of the choices.

Place	Price Per Person	Price Per Student for Groups over 10	Chaperones and Teachers
Aquarium	\$12.00	\$10.00	One free for every 10 students; \$5.00 each after that
Zoo	\$13.00	\$9.00	One free for every eight students; \$6.00 each after that
Space Museum	\$15.00	\$11.00	One free for every five students; \$5.00 each after that

Below are the results of a poll of each class member's top two choices.

Attraction	Number of Students First Choice	Number of Students Second Choice
Aquarium	10	7
Zoo	10	11
Space Museum	8	10

Here are some other things Mr. Falting must take into account when planning the trip:

- Bus company A charges a \$20 rental fee and \$7 per mile for each bus. Each bus holds 25 people.
- Bus company B charges \$7.75 per mile for each bus. Each bus holds 20 people.
- Distance from the school:
  - Zoo: 32.9 miles
  - Space museum: 22.7 miles
  - Aquarium: 28.3 miles
- The school fund will pay the first \$250 of the trip.
- In addition to Mr. Falting, there will be two teachers and five chaperones. Any costs for the teachers and chaperones will be divided equally among the students.
- Each student will pay the same amount.

1. Taking all factors into account, where should the class go for the field trip? Give supporting evidence for your choice. Remember to account for all students in the class (including you and your group members).

2. How much will each person need to pay to go on the trip you have chosen? Explain carefully how you decided.

Instructional Task Exemplar Response

Mr. Falting, a teacher at Roosevelt Middle School, is planning to take his middle school students on a field trip to some nearby attractions. Help Mr. Falting decide where he should take his class.

Here are some of the choices.

Place	Price Per Person	Price Per Student for Groups over 10	Chaperones and Teachers
Aquarium	$12.00	$10.00	One free for every 10 students; $5.00 each after that
Zoo	$13.00	$9.00	One free for every eight students; $6.00 each after that
Space Museum	$15.00	$11.00	One free for every five students; $5.00 each after that

Below are the results of a poll of each class member's top two choices.

Attraction	Number of Students First Choice	Number of Students Second Choice
Aquarium	10	7
Zoo	10	11
Space Museum	8	10

Here are some other things Mr. Falting must take into account when planning the trip.

- Bus company A charges a $20 rental fee and $7 per mile for each bus. Each bus holds 25 people.
- Bus company B charges $7.75 per mile for each bus. Each bus holds 20 people.
- Distance from the school:
  - Zoo: 32.9 miles
  - Space museum: 22.7 miles
  - Aquarium: 28.3 miles
- The school fund will pay the first $250 of the trip.
- In addition to Mr. Falting, there will be two teachers and five chaperones. Any costs for the teachers and chaperones will be divided equally among the students.
- Each student will pay the same amount.

1. Taking all factors into account, where should the class go for the field trip? Give supporting evidence for your choice. Remember to account for all students in the class (including you and your group members).

   Sample response (there will likely be different responses based on personalities in the class):
   ***Note: This sample response is based on the 28 students polled (see table above) plus four students in the group working this problem.*

   The teacher should take the class to the space museum. It is the second favorite choice. I chose it over the favorite choice of the zoo, because it is the cheaper trip.

For 32 students to attend the museum at $11.00 a person for admission, the admission cost would be $352.00. The cost for chaperones would be 2 times $5.00, which is $10.00. The first six would go for free, because for every five students one teacher/chaperone would be free. Thirty-two divided by 5 equals 6.4—therefore only six chaperones would go for free because there can't be 0.4 of a chaperone. They would need to take two buses from Bus Company A. The charge would be $675.60. One bus from Bus Company A would cost $337.80 because the total mileage would be 2 times 22.7 miles(to go there and back) multiplied by $7 per mile and added to the $20 rental fee. This is cheaper than Company B, which would be $7.75 times 45.4 miles, which is equal to $351.85 for one bus. The total for admission and the two buses would be $1,037.60. The school fund would pay the first $250, leaving $787.60.

1. How much will each person need to pay to go on the trip you have chosen? Explain carefully how you decided.
   Sample response (students' responses will need to be based on their decision for the first part):

The amount left to pay by the class is $787.60. This total should be divided by the 32 students, since they are sharing all costs equally. Each student will pay $24.61 to go to the space museum.

Park Area (IT)

Overview

Students will apply their understanding of scale drawings and unit rate to solve a problem involving finding the area of a park and deciding which company to use for the proposed work on the park.

Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items:
7.RP.A.1	• 6.RP.A.2	1. What is the unit rate in miles per hour if Mandy walks ¼ mile in 12 minutes? *a. $1\frac{1}{4}$ miles per hour* *2.* If the ratio of scaled length to actual length is 1 inch to 35 feet, what is the ratio of scaled area (square inches) to actual area (square feet)? *a. 1 sq. in. to 1225 sq. ft.* 3. http://www.illustrativemathematics.org/illustrations/470 4. http://www.illustrativemathematics.org/illustrations/828	• http://www.illustrativemathematics.org/illustrations/77 • http://www.illustrativemathematics.org/illustrations/549 • http://learnzillion.com/lessonsets/521-compute-unit-rates-associated-with-ratios-of-fractions • http://learnzillion.com/lessonsets/459-compute-unit-rates-using-fractions • http://learnzillion.com/lessonsets/107-compute-unit-rates-associated-with-ratios-of-fractions
7.G.A.1	• 6.G.A.1 • 7.RP.A.2	1. The given ratio on a scaled drawing of the Eiffel Tower is 1 centimeter:50 meters. If the height of the drawing is 6.48 centimeters, what is the actual height of the Eiffel Tower? *a. 324 meters* 2. http://www.illustrativemathematics.org/illustrations/107 3. http://www.illustrativemathematics.org/illustrations/1082	• http://www.illustrativemathematics.org/illustrations/647 • http://learnzillion.com/lessonsets/604-apply-scale-factor-to-realworld-problems • http://learnzillion.com/lessonsets/451-solve-problems-involving-scale-drawings-of-geometric-figures • http://learnzillion.com/lessonsets/199-solve-problems-involving-scale-drawings-of-geometric-figures

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

What is a landscape contractor? A *landscape contractor* is a company people might hire to improve the appearance of a piece of land by adding features like gravel, pavement, flower beds, etc.

What does it mean to "bid out" a project? "Bidding out" is a process where a purchaser looks for a company to complete a specific project. The companies interested in completing the work create a price list based on the work to be done for the project then present their proposals to the purchaser in order to convince the purchaser to use their services.

What are "labor costs"? Labor costs are how much a company pays their workers for the amount of time their workers would be working on a project.

How many hours do people normally work in a day? A week? Discuss with students that this really depends on the type of job. Students will need to think about the work that would happen for this project, the time of day, and possibly time of year. This will be one assumption students would have to make for this task.

During the Task

- This task could be completed in groups or as individuals. Completing the task in groups would allow for more discussion among students. In their groups, students can determine how to start working the problem and what other factors need to be considered.
- Students may stop at finding the area of the park in square inches rather than finding the actual area. Ask probing questions to help students understand the connection between the scaled area and the actual area.
- Students may think that the cost for materials for Parks Plus, LLC, is based on the area rather than the linear measure. Ask students to identify the difference between the costs of the materials for the two companies.
- Students may wish to find the cost for both companies to complete the project for both park designs. Ask students to identify ways they might be able to identify which company would be better regardless of the park design.
- There are multiple ways students can find the area of the figures. Encourage students to share with each other their plans for finding the area.

After the Task

Have students present their recommendations to the class. Have students keep track of which options each group/student recommends. Discuss as a class which seems to be the best option.

Teachers can also ask students to find the total cost per square foot for this task based on the option and company they recommended.

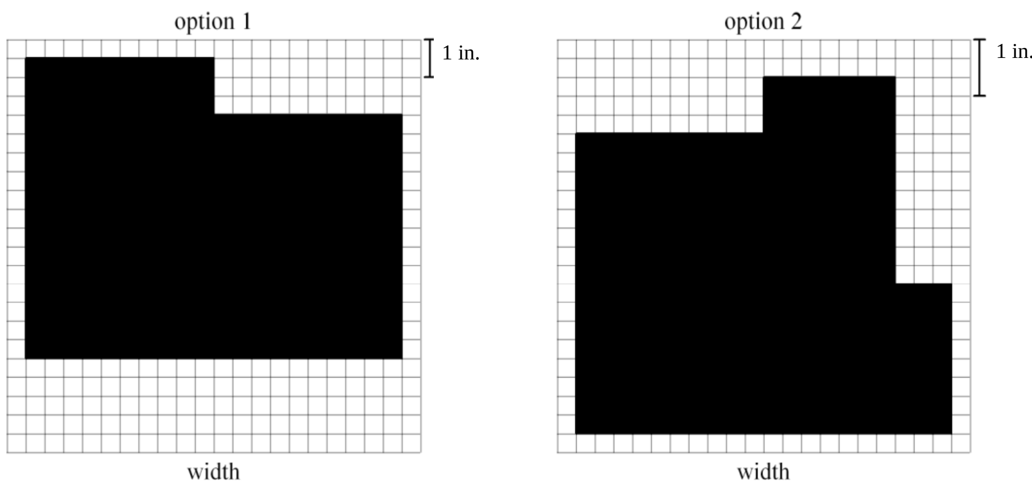
Students can connect this task to work that might be happening on the school grounds or at their own homes. Teachers may wish to choose an area of the school grounds that students can plan to landscape (an existing flower bed, a new flower bed, planting trees, etc.) and use the concepts here to determine how much it would cost. Then they could complete the project.

Student Instructional Task

The Parks and Recreation Committee plans to build a new park for young children. The members are determining where the park should be located; however, the committee members disagree on the design of the park. Two designs have been presented to the committee to vote on, but no decision could be reached; thus, the committee is seeking your help. The two options for the design of the new park are shown below:

Scale: 1 inch : 50 feet

Scale: 1 inch : 75 feet



The committee has to select a landscape contractor to cover the ground of the park with a combination of grass, gravel, mulch, and other child-friendly play surfaces. After bidding out the project to multiple landscape contractors, the committee narrows it down to two contractors. The bids are shown below:

	Parks Plus, LLC	Masterscapers, LLC
Costs for Materials	\$25 per $\frac{1}{2}$ scaled inch	\$100 for every $\frac{1}{3}$ scaled square inch
Labor Costs	\$175 per hour	\$1,000 per day
Estimated Completion Time	45 days	60 days

1. Which park design should the committee choose? Be sure to explain your choice and show all work that would support your choice. In your explanation, include the actual area of the park design you are recommending and any other factors you considered when making your choice.

2. Based on the design you chose in part 1, which contractor should the committee choose? Be sure to explain your choice and show all work that would support your choice. In your explanation, include all factors you considered when making your choice.

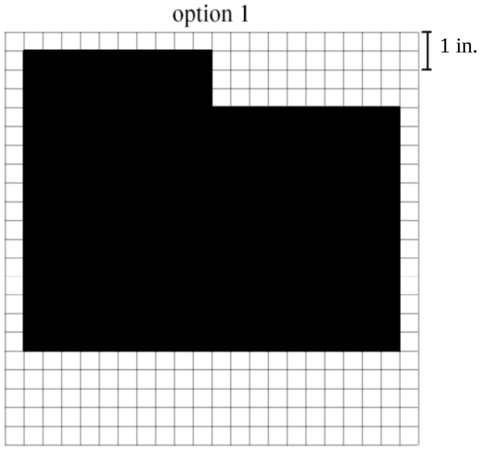
Task adapted with permission from Universal Achievement, LLC

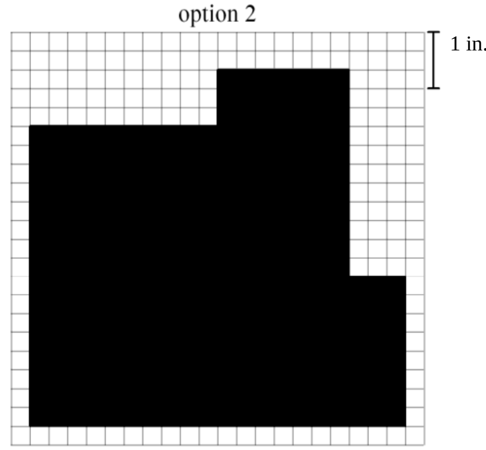
Instructional Task Exemplar Response

The Parks and Recreation Committee plans to build a new park for young children. The members are determining where the park should be located; however, the committee members disagree on the design of the park. Two designs have been presented to the committee to vote on, but no decision could be reached; thus, the committee is seeking your help. The two options for the design of the new park are shown below:

Scale: 1 inch : 50 feet

Scale: 1 inch : 75 feet





The committee has to select a landscape contractor to cover the ground of the park with a combination of grass, gravel, mulch, and other child-friendly play surfaces. After bidding out the project to multiple landscape contractors, the committee narrows it down to two contractors. The bids are shown below:

	Parks Plus, LLC	Masterscapers, LLC
Costs for Materials	\$25 per $\frac{1}{2}$ scaled inch	\$100 for every $\frac{1}{3}$ scaled square inch
Labor Costs	\$175 per hour	\$1,000 per day
Estimated Completion Time	45 days	60 days

1. Which park design should the committee choose? Be sure to explain your choice and show all work that would support your choice. In your explanation, include the actual area of the park design you are recommending and any other factors you considered when making your choice.

Students should consider many factors including the size of the park and the cost by both contractors to complete the work for each park. There will be some assumptions that students may need to make, and those assumptions should be stated in the students' explanations.

Sample response:

Park Option #1:
Scale: 1 inch to 50 feet
Scaled area:
(1 in)(1 in) = 1 in^2; (50 ft)(50 ft) = 2500 ft^2
So 1 in^2 on the drawing represents 2500 ft^2 in actual area.

Area of Park Option #1:
(8 in)(10 in) – (1.5 in)(5 in)
80 in^2 – 7.5 in^2
72.5 in^2

$$\frac{1\ in^2}{2500\ ft^2} = \frac{72.5\ in^2}{(2500\ ft^2)(72.5)} = \frac{72.5\ in^2}{181{,}250\ ft^2}$$

Area of Park Option #1 is 181,250 ft^2.

Park Option #2:
Scale: 1 inch to 75 feet
Scaled area:
(1 in)(1 in) = 1 in^2; (75 ft)(75 ft) = 5625 ft^2
So 1 in^2 on the drawing represents 5625 ft^2 in actual area.

Area of Park Option #2:

$$\left(\frac{16}{3}\ in\right)\left(\frac{20}{3} in\right) - (1\ in)\left(\frac{1}{3}\ in\right)$$

$$\frac{320}{9}\ in^2 - \frac{1}{3} in^2$$

$$\frac{317}{9}\ in^2$$

$$\frac{1\ in^2}{5625\ ft^2} = \frac{{}^{317}/_{9}\ in^2}{(5625\ ft^2)({}^{317}/_{9})} = \frac{{}^{317}/_{9}\ in^2}{198{,}125\ ft^2}$$

Area of Park Option #2 is 198,125 ft^2.

I would advise the committee to choose park option #1 because it has the smaller area. While that may reduce some of the play area, I think it would make it easier for parents to watch their children in a smaller area.

2. Which contractor should the committee choose? Be sure to explain your choice and show all work that would support your choice. In your explanation, include all factors you considered when making your choice.

Sample response: (Answers here will likely be based on work from the first question. If students had incorrect work in question one, the previous work should be taken into account when assessing their understanding in this question.)

My work is based on my recommendation of park option #1.

Parks Plus, LLC, cost:

Materials: \$25 per scaled ½ inch—I have to find the price per square inch in order to find the price per square foot. $\frac{\$25}{{}^{1}/_{2}\ in} = \frac{\$50}{1\ in}$ *so this would mean the cost for 1 in^2 would be (\$50)(\$50), which is \$2,500 per square inch.*

From the work to find the area I did in question 1, I know that 1 square inch represents 2,500 square feet. So, $\frac{\$2500}{1\ in^2} = \frac{\$2500}{2500\ ft^2} = \frac{\$1}{1\ ft^2}$. This means it costs \$1 per square foot. The area of the first park is 181,250 square feet so the cost for materials is \$181,250.

Labor Costs for Parks Plus, LLC:

\$175 per hour and 45 days to complete the work—I'll assume they will work 8 hours a day.

45 days x 8 hours per day = 360 hours total

360 hours x \$175 per hour = \$63,000

Total costs for Parks Plus, LLC: \$181,250 + \$63,000 = \$244,250.

Masterscapers, LLC, Costs:

Materials: \$100 per $\frac{1}{3}$ scaled square inch—I have to find the price per square inch in order to find the price per square foot. $\frac{\$100}{1/3\ in^2} = \frac{\$300}{1\ in^2}$. Since 1 in^2 represents 2500 ft^2, $\frac{\$300}{1\ in^2} = \frac{\$300}{2500\ ft^2}$; this is equal to \$3 per 25 square feet.

$$\frac{\$3}{25\ ft^2} = \frac{x}{181{,}250\ ft^2}$$

$$181{,}250\ ft^2\left(\frac{\$3}{25\ ft^2}\right) = \left(\frac{x}{181{,}250\ ft^2}\right)181{,}250\ ft^2$$

$$\frac{\$543{,}750}{25} = x$$

$$\$21{,}750 = x$$

So, materials for Masterscapers, LLC, will cost \$21,750.

Labor Costs: \$1,000 per day and 60 days to complete the work.

60 x \$1000 = \$60,000

Total costs for Masterscapers, LLC: \$21,750 + \$60,000 = \$81,750.

Even though Parks Plus, LLC, seemed cheaper in the chart, after finding the actual costs, Parks Plus will cost more. So, I would recommend the committee choose Masterscapers, LLC.

8TH GRADE TOOLS

8th Grade Remediation Guide

As noted in the "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every eighth grade math standard[7].

8th Grade Standard	Previous Grade Standards	8th Gr. Stand. Taught in Advance	8th Gr. Stand. Taught Concurrently
8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	• 7.NS.A.2d		• 8.NS.A.2 • 8.EE.A.2
8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.			• 8.NS.A.1 • 8.EE.A.2
8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3–5 = 3–3 = 1⁄33 = 1⁄27.	• 6.EE.A.1		
8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	• 6.EE.B.5 • 7.NS.A.3		• 8.NS.A.1 • 8.NS.A.2 • 8.G.B.6
8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.	• 4.OA.A.2 • 5.NBT.A.2	• 8.EE.A.1	• 8.EE.A.4
8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	• 7.EE.B.3	• 8.EE.A.1	• 8.EE.A.3

[7] This content comes from the work of the math standards' authors found here: http://www.edutron.com/0/Math/ccssmgraph.htm

8th Grade Standard	Previous Grade Standards	8th Gr. Stand. Taught in Advance	8th Gr. Stand. Taught Concurrently
8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	• 7.RP.A.2		• 8.EE.B.6
8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	• 7.G.A.1 • 7.RP.A.2	• 8.G.A.5	• 8.EE.B.5
8.EE.C.7a Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).			• 8.EE.C.7b
8.EE.C.7b Solve linear equations in one variable. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	• 7.EE.A.1 • 7.EE.B.4a		• 8.EE.C.7a • 8.SP.A.3
8.EE.C.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	• 6.EE.B.5 • 7.EE.B.4a	• 8.EE.B.6	
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	• 7.RP.A.2		

8th Grade Standard	Previous Grade Standards	8th Gr. Stand. Taught in Advance	8th Gr. Stand. Taught Concurrently
8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	• 7.RP.A.2	• 8.EE.B.5 • 8.EE.B.6 • 8.F.A.1	
8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.		• 8.F.A.2 • 8.EE.B.6 • 8.F.A.1	
8.F.B.4 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	• 7.RP.A.2	• 8.F.A.3	• 8.SP.A.2 • 8.SP.A.3 • 8.F.B.5
8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		• 8.F.A.1 • 8.F.A.2 • 8.F.A.3	• 8.F.B.4
8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	• 7.G.A.2 • 7.G.B.5		
8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.		• 8.G.A.1	
8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	• 6.G.A.3	• 8.G.A.1	

8th Grade Standard	Previous Grade Standards	8th Gr. Stand. Taught in Advance	8th Gr. Stand. Taught Concurrently
8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.		• 8.G.A.2 • 8.G.A.3	
8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.		• 8.G.A.2 • 8.G.A.4	
8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.	• 7.G.B.6		• 8.EE.A.2 • 8.G.B.7
8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.			• 8.G.B.6
8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	• 6.G.A.3	• 8.G.B.7	
8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.		• 8.EE.A.2	
8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	• 6.NS.C.8		
8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		• 8.SP.A.1	• 8.F.B.4
8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.		• 8.SP.A.2	• 8.EE.C.7b • 8.F.B.4

8th Grade Standard	Previous Grade Standards	8th Gr. Stand. Taught in Advance	8th Gr. Stand. Taught Concurrently
8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	• None *Introduced in 8th Grade*		

8th Grade Tasks At a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for eighth grade include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standards. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the eighth grade tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Scientific Notation Page 146	ECR	• 8.EE.A.3 • 8.EE.A.4	• 4.OA.A.2 • 5.NBT.A.2 • 7.EE.B.3
Dan's Leaking Bottle Page 152	ECR	• 8.F.B.4	• 7.RP.A.2 • 8.F.A.3
Summer Jobs Page 155	ECR	• 8.F.A.2 • 8.F.B.4 • 8.F.B.5	• 7.RP.A.2 • 8.F.A.1 • 8.F.A.2 • 8.F.A.3 • 8.EE.B.5 • 8.EE.B.6
Party Zone Palace Page 161	ECR	• 8.F.A.1 • 8.F.A.2 • 8.F.A.3	• 7.RP.A.2 • 8.EE.B.5 • 8.EE.B.6 • 8.F.A.1 • 8.F.A.2
Congruence Transformations Page 165	ECR	• 8.G.A.1 • 8.G.A.2 • 8.G.A.3	• 6.G.A.3 • 7.G.A.2 • 7.G.B.5 • 8.G.A.1
Connecting Proportional Relationships, Lines, and Linear Equations Page 174	IT	• 8.EE.B.5 • 8.EE.B.6	• 7.G.A.1 • 7.RP.A.2 • 8.G.A.5
T-Shirt Fundraiser Page 181	IT	• 8.EE.C.8c	• 6.EE.B.5 • 7.EE.B.4a • 8.EE.B.6

Title	Type	Task Standards	Task Remedial Standards
Game Design Page 188	IT	• 8.G.A.1 • 8.G.A.2 • 8.G.A.3	• 6.G.A.3 • 7.G.A.2 • 7.G.B.5 • 8.G.A.1
Tank Volume Page 196	IT	• 8.NS.A.2 • 8.EE.A.2 • 8.G.C.9	• 7.NS.A.2d • 6.EE.B.5 • 7.NS.A.3 • 8.EE.A.2
Pythagorean Theorem Proof Page 210	IT	• 8.G.B.6	• 7.G.B.6

Scientific Notation (ECR)

Overview

This instructional task requires students to simplify expressions and answer questions with scientific notation.

Standards

Work with radicals and integer exponents

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.A.3	• 4.OA.A.2 • 5.NBT.A.2	1. Write 0.0045 in scientific notation. a. 4.5×10^{-3} 2. http://www.illustrativemathematics.org/illustrations/476 3. http://www.illustrativemathematics.org/illustrations/1593	• http://www.illustrativemathematics.org/illustrations/263 • http://www.illustrativemathematics.org/illustrations/1524 • http://www.illustrativemathematics.org/illustrations/1620 • http://learnzillion.com/lessonsets/272-estimate-and-compare-with-integers-to-the-power-of-10
8.EE.A.4	• 7.EE.B.3	1. $4 \times 10^{-3} \cdot 2 \times 10^{6}$ a. 8×10^{3} 2. http://www.illustrativemathematics.org/illustrations/113	• http://www.illustrativemathematics.org/illustrations/108 • http://www.illustrativemathematics.org/illustrations/478 • http://learnzillion.com/lessonsets/276-perform-operations-with-numbers-expressed-in-scientific-notation-including-decimals

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a roundtrip? A roundtrip means that you will travel from one location to another and then return to the original location, generally following the same route.

After the Task:

Be sure to remind students that radical and integer exponents continue to be used throughout 8th grade and high school math courses.

Common Errors/Misunderstandings by Problem Number:

- Some students may forget to multiply each distance by 2 to account for the roundtrip.

- Some students may divide the smaller number by the larger number instead of the larger number by the smaller number.

- Remind students to treat 10^5 as like terms.

- Students may make a mistake when subtracting the exponents. Remind them that $5 - 8 = -3$. Discuss with students what this answer means in terms of the situation described (the trip would take less than one second).

- Students may leave their answers as 25×10^{-5}. Remind them that they should only have one non-zero digit in front of the decimal in scientific notation.

- Students may give their answers as 1×10^2 drops instead of 100 drops. Although this answer is correct, ask them how many drops that would be. Ask them which representation of the number of drops would be easier for most people to understand.

- Students may leave their answers as 9462×10^{-5} liters. Remind them that they should only have one non-zero digit in front of the decimal in scientific notation.

Student Extended Constructed Response

City 1	City 2	Approximate Distance Between Cities
New Orleans, Louisiana	Washington, DC	2×10^6 meters
New Orleans, Louisiana	Nashville, Tennessee	9×10^5 meters
New Orleans, Louisiana	Seattle, Washington	4×10^6 meters
New Orleans, Louisiana	Shreveport, Louisiana	5×10^5 meters

1. What is the combined distance that you would travel if you completed a roundtrip from New Orleans, Louisiana, to Washington, DC, and a roundtrip from New Orleans, Louisiana, to Seattle, Washington? Write your answer in scientific notation.

2. How many times farther is the trip from New Orleans, Louisiana, to Washington, DC, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

3. How much shorter is the trip from New Orleans, Louisiana, to Shreveport, Louisiana, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

4. If a car were invented that could travel at the speed of light, approximately 3×10^8 meters per second, how long would it take to drive from Shreveport, Louisiana, to New Orleans, Louisiana?

The volume of a drop of gasoline is approximately 5×10^{-5} liter.

5. You stopped to put gasoline in your car. As you finished filling your gas tank, five drops of gas dripped to the ground. What is the total volume of leaked gasoline? Write your answer in scientific notation.

6. How many drops would have to fall for 5×10^{-3} liter of gas to be lost?

7. You already have a $9457 \times 10^{-5}\ l$ puddle of gasoline. If one more drop falls into the puddle, how much gasoline would be in the puddle? Write your answer in scientific notation.

Extended Constructed Response Exemplar Response

City 1	City 2	Approximate Distance Between Cities
New Orleans, Louisiana	Washington, DC	2×10^6 meters
New Orleans, Louisiana	Nashville, Tennessee	9×10^5 meters
New Orleans, Louisiana	Seattle, Washington	4×10^6 meters
New Orleans, Louisiana	Shreveport, Louisiana	5×10^5 meters

1. What is the combined distance that you would travel if you completed a roundtrip from New Orleans, Louisiana, to Washington, DC, and a roundtrip from New Orleans, Louisiana, to Seattle, Washington? Write your answer in scientific notation.

Roundtrip from New Orleans, Louisiana, to Washington, DC: $2 \times 2 \times 10^6 = 4 \times 10^6$*meters*

Roundtrip from New Orleans, Louisiana, to Seattle, Washington: $2 \times 4 \times 10^6 = 8 \times 10^6$ *meters*

Combined Travel: $4 \times 10^6 meters + 8 \times 10^6\ meters$

12×10^6 *meters*

$1.2 \times 10 \times 10^6$ *meters*

1.2×10^7 *meters*

The combined distance is 1.2×10^7 *meters.*

2. How many times farther is the trip from New Orleans, Louisiana, to Washington, DC, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

$$\frac{2 \times 10^6}{9 \times 10^5}$$

$$\frac{2}{9} \times \frac{10^6}{10^5}$$

$$\frac{2}{9} \times 10^1$$

$$.22 \times 10$$

$$2.2$$

The trip from New Orleans, Louisiana, to Washington, DC, is about twice as long as the trip from New Orleans, Louisiana, to Nashville, Tennessee.

3. How much shorter is the trip from New Orleans, Louisiana, to Shreveport, Louisiana, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

$$9 \times 10^5 - 5 \times 10^5$$
$$4 \times 10^5 \textit{ meters}$$

The trip from New Orleans, Louisiana, to Shreveport, Louisiana, is 4×10^5 *meters shorter than the trip from New Orleans, Louisiana, to Nashville, Tennessee.*

4. If a car were invented that could travel at the speed of light, approximately 3×10^8 meters per second, how long would it take to drive from Shreveport, Louisiana, to New Orleans, Louisiana?

$$\frac{5 \times 10^5 \; meters}{3 \times 10^8 meters \; per \; second}$$
$$\frac{5}{3} \times \frac{10^5}{10^8} \; seconds$$
$$\frac{5}{3} \times 10^{-3} \; seconds$$
$$1.67 \times 10^{-3} \; seconds$$

It would take 1.67×10^{-3} *seconds to drive from Shreveport to New Orleans.*

The volume of a drop of gasoline is approximately 5×10^{-5} liter.

5. You stopped to put gasoline in your car. As you finished filling your gas tank, five drops of gas dripped to the ground. What is the total volume of leaked gasoline? Write your answer in scientific notation.

$$5 \times 5 \times 10^{-5}$$
$$25 \times 10^{-5}$$
$$2.5 \times 10^{-4}$$

2.5×10^{-4} *liter of gasoline leaked.*

6. How many drops would have to fall for 5×10^{-3} liter of gas to be lost?

$$\frac{5 \times 10^{-3} \; liter}{5 \times 10^{-5} \; liter \; per \; drop}$$
$$\frac{5}{5} \times \frac{10^{-3}}{10^{-5}} \; drops$$
$$\frac{5}{5} \times 10^{2} \; drops$$

$$1 \times 10^2 \; drops$$

$$100 \; drops$$

100 drops would have to fall for 5×10^{-3} *liter of gas to be lost.*

7. You already have a 9457×10^{-5} liter puddle of gasoline. If one more drop falls into the puddle, how much gasoline would be in the puddle? Write your answer in scientific notation.

$$9457 \times 10^{-5} liter + 5 \times 10^{-5} \; liter$$

$$9462 \times 10^{-5} \; liter$$

$$9.462 \times 10^3 \times 10^{-5} \; liter$$

$$9.462 \times 10^{-2} \; liter$$

The puddle would be 9.462×10^{-2} *liter.*

Dan's Leaking Bottle (ECR)

Overview

Students write a function from a given situation. They must graph the function and determine the rate of change and initial value.

Standards

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.F.B.4	• 7.RP.A.2 • 8.F.A.3	1. State the slope and y-intercept of the following equation: $$y = \frac{2}{3}x + 5$$ a. *Slope:* $\frac{2}{3}$ *y-intercept: (0, 5)* 2. http://www.illustrativemathematics.org/illustrations/120 3. http://www.illustrativemathematics.org/illustrations/247	• http://www.illustrativemathematics.org/illustrations/813 • http://www.illustrativemathematics.org/illustrations/180 • http://www.illustrativemathematics.org/illustrations/181 • http://learnzillion.com/lessonsets/686-construct-functions-determine-slope-and-initial-value-and-interpret-in-terms-of-a-situation • http://learnzillion.com/lessonsets/357-construct-functions-to-model-linear-relationships-between-two-quantities • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a water dispenser? A water dispenser is a type of large bottle that holds water. You can use a nozzle to get the water out. If there is a hole in the dispenser, the water will leak out. The rate at which the water leaks will depend on the size of the hole.

After the Task:

Students may struggle with graphing the function. They may want to graph the points by how much water has leaked out rather than subtracting from the original 8-inch water line. Students who struggle to find the rate of change may need additional practice in finding the unit rate.

Student Extended Constructed Response

Dan has an automatic water dispenser he uses to give his dogs water. The bottle holds 64 ounces of water when it is full. Lately, Dan notices that the bottle is leaking. Dan measures the height of the water in the dispenser bottle and finds that the level of water is 8 inches from the bottom of the bottle. After 3 minutes he finds that the water level has dropped ¾ of an inch because of the leak. After another 5 minutes, he finds that the water level has dropped another 1 ¼ inches. If the water is leaking at a constant rate, answer the following questions.

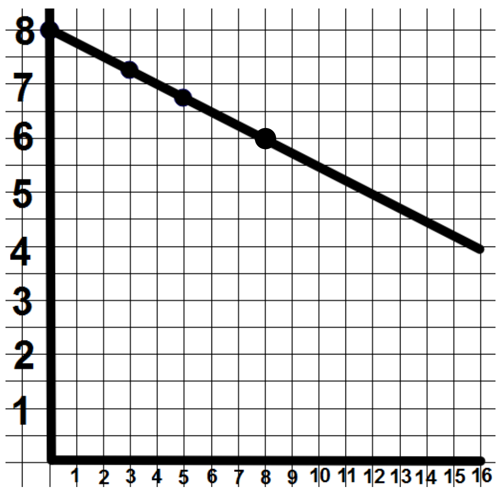
1. Using the description above, write a function with defined variables to show the relationship between the time that passes and the water level in the bottle. Graph the function.

2. Identify the rate of change and the initial value. Explain what the rate of change and initial value mean in the context of Dan's leaking bottle.

Extended Constructed Response Exemplar Response

Dan has an automatic water dispenser he uses to give his dogs water. The bottle holds 64 ounces of water when it is full. Lately, Dan notices that the bottle is leaking. Dan measures the height of the water in the dispenser bottle and finds that the level of water is 8 inches from the bottom of the bottle. After 3 minutes he finds that the water level has dropped ¾ of an inch because of the leak. After another 5 minutes, he finds that the water level has dropped another 1 ¼ inches. If the water is leaking at a constant rate, answer the following questions.

1. Using the description above, write a function with defined variables to show the relationship between the time that passes and the water level in the bottle. Graph the function.



8 – ¼ m = w; where m is the number of minutes that have passed, w is the amount of water left in the bottle

The graph should be similar to this. The line must go through the points (0, 8), (3, 7 ¼), and (8, 6). These are the points they can solve for with the provided information.

2. Identify the rate of change and the initial value. Explain what the rate of change and initial value mean in the context of Dan's leaking bottle.
   The rate of change is ¼ of an inch per minute. This means the water level drops ¼ of an inch every minute of time that passes.
   The initial value is 8. 8 inches; this is the original height of the water from the bottom of the bottle at which Dan begins checking the level.

Summer Jobs (ECR)

Overview

Students are asked to write functions when given the pay for two different people. They must also look at a graph of a person's pay and describe the features of the graph.

Standards

Define, evaluate, and compare functions.

8.F.A.2 Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*.

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.F.A.2	• 7.RP.A.2 • 8.EE.B.5 • 8.EE.B.6 • 8.F.A.1	1. Which has a greater rate of change? 5x + 9 = y or 9x + 9 = y a. 9x + 9 = y 2. Which would be cheaper for x = $6? 4x + 8 = y or 3x + 12 = y a. 3x + 12 = y 3. http://www.illustrativemathematics.org/illustrations/641	• http://www.illustrativemathematics.org/illustrations/129 • http://www.illustrativemathematics.org/illustrations/1537 • http://www.illustrativemathematics.org/illustrations/184 • http://www.illustrativemathematics.org/illustrations/1165 • http://learnzillion.com/lessonsets/271-understand-and-compare-functions • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.F.B.4	• 7.RP.A.2 • 8.F.A.3	1. What is the initial value and rate of change for 3x + 8 = y? a. Initial value: 8; rate of change: 3 2. What is the rate of change for the ratio $18 for 3 hours? a. 6 3. http://www.illustrativemathematics.org/illustrations/120 4. http://www.illustrativemathematics.org/illustrations/247	• http://www.illustrativemathematics.org/illustrations/813 • http://www.illustrativemathematics.org/illustrations/180 • http://www.illustrativemathematics.org/illustrations/181 • http://learnzillion.com/lessonsets/686-construct-functions-determine-slope-and-initial-value-and-interpret-in-terms-of-a-situation • http://learnzillion.com/lessonsets/357-construct-functions-to-model-linear-relationships-between-two-quantities • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions
8.F.B.5	• 8.F.A.1 • 8.F.A.2 • 8.F.A.3	1. Describe what a graph for the linear equation 2x + 9 = y would look like. a. The slope would be 2, meaning it would go up two units for every unit moved to the right. 9 is the initial value and would be the y-intercept. 2. Describe what a graph for 9.25x = y looks like. a. The initial value is 0. The graph would begin at the origin. The slope is 9.25, meaning it would go up 9.25 units for every unit moved to the right. 3. http://www.illustrativemathematics.org/illustrations/628 4. http://www.illustrativemathematics.org/illustrations/632 5. http://www.illustrativemathematics.org/illustrations/633 6. http://www.illustrativemathematics.org/illustrations/674	• http://www.illustrativemathematics.org/illustrations/1165 • http://www.illustrativemathematics.org/illustrations/641 • http://www.illustrativemathematics.org/illustrations/813 • http://learnzillion.com/lessonsets/705-describe-functions-by-analyzing-and-building-graphs • http://learnzillion.com/lessonsets/358-describe-the-functional-relationship-between-two-quantities-by-analyzing-a-graph

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What does “earnings per hour” mean? It is the amount of money a person earns for each hour worked.

After the Task:

Students may struggle with the initial value for the two functions. This could be from not really understanding the parts of a function written in y-intercept form. Additional practice may be required in writing equations in y-intercept form and then defining the parts.

Student Extended Constructed Response

Lonnie and Tony get summer jobs. Lonnie makes $9.25 per hour. The table below shows how Tony is paid.

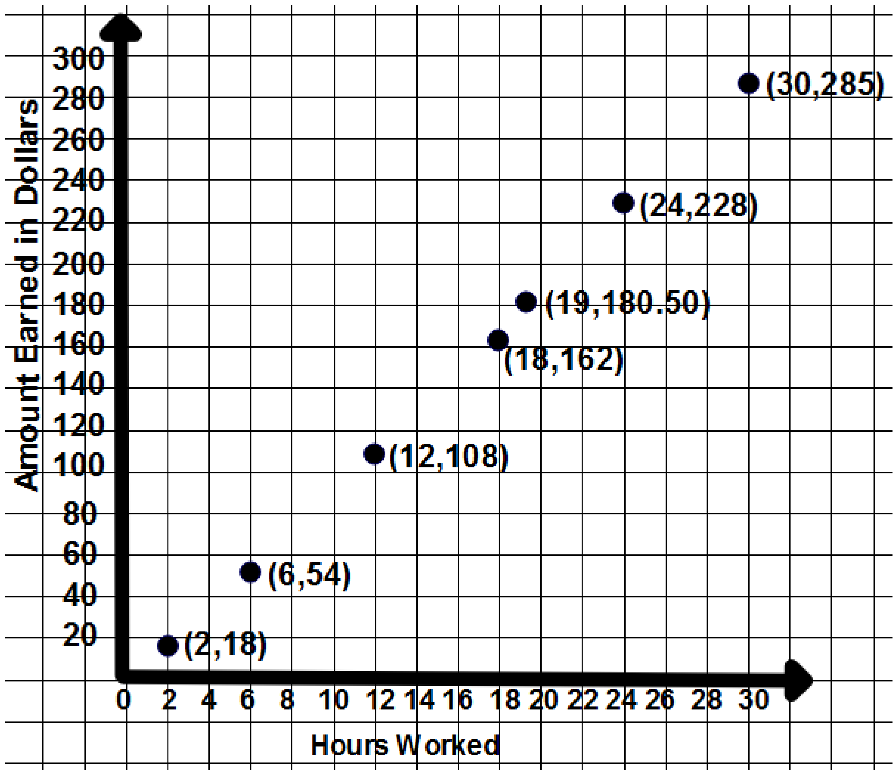
Tony's hours worked	4	9	13	18
Tony's earnings (dollars)	$35	$78.75	$113.75	$157.50

1. Who earns more money per hour? How much more does that person make per hour?

2. Write a function to model the relationship for Tony's pay shown in the table. Explain the rate of change and initial value of the function in terms of the given situation.

3. Is it possible for Lonnie and Tony to work the same number of hours and make the same amount of money? Explain.

4. Marcus works for the same company. The graph represents his pay for a 30-hour work week.



Explain whether this graph represents a linear or non-linear function. Justify your answer with information from the graph.

Extended Constructed Response Exemplar Response

Lonnie and Tony get summer jobs. Lonnie makes \$9.25 per hour. The table below shows how Tony is paid.

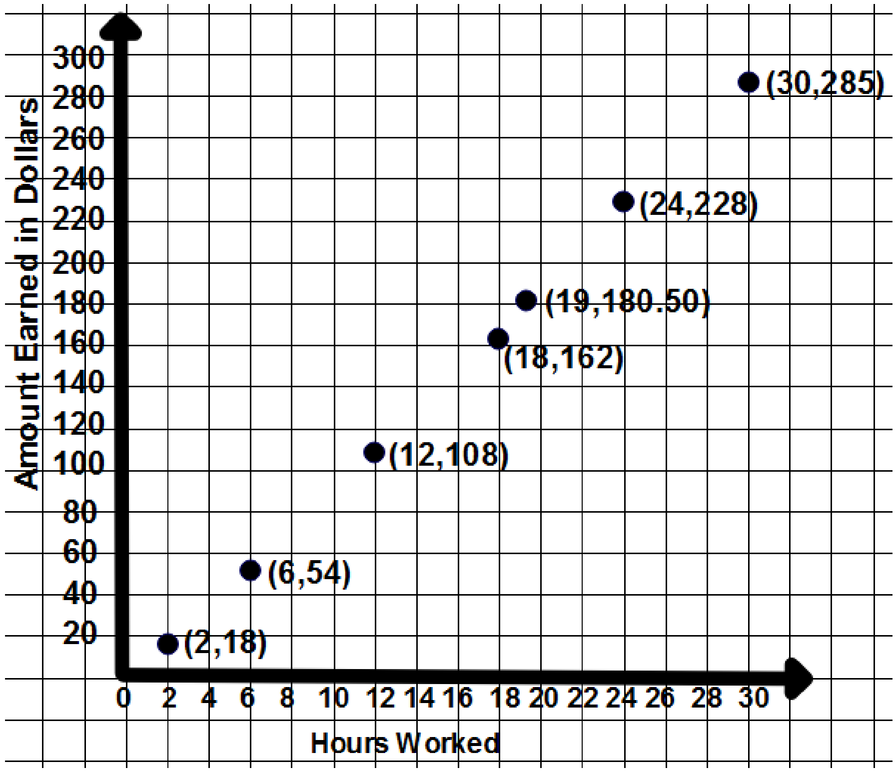
Tony's hours worked	4	9	13	18
Tony's earnings (dollars)	\$35	\$78.75	\$113.75	\$157.50

1. Who earns more money per hour? How much more does that person make per hour?
   Lonnie earns more money per hour. He earns \$0.50 per hour more than Tony.

2. Write a function to model the relationship for Tony's pay shown in the table. Explain the rate of change and initial value of the function in terms of the given situation.
   Tony y = 8.75x + 0 or y = 8.75x; the rate of change is 8.75 representing the amount of money Tony earns per hour. The initial value is 0, which means that at 0 hours Tony makes \$0.

3. Could Lonnie and Tony work the same number of hours and make the same amount of money? Explain.
   They could never make the same amount of money, because they earn different rates. They both earn \$0 for working 0 hours.

1. Marcus works for the same company. The graph represents his pay for a 30-hour work week.



Explain whether this graph represents a linear or non-linear function. Justify your answer with information from the graph.

For the first 18 hours, Marcus makes $9 per hour. The rate of change for the points (2, 18) through (18, 162) is $9. At 19 hours, the rate of change is no longer $9. It is $9.50. This rate of change continues through 30 hours. This graph is non-linear since the rate of change is not the same for all points.

Party Zone Palace (ECR)

Overview

In this task, students are applying the definition of a function, identifying linear functions, and comparing the rates of change on different functions to plan a birthday party.

Standards

Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

8.F.A.2 Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*.

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

<table>
<tr><th>Grade Level Standard</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items :</th></tr>
<tr><td>8.F.A.1</td><td>• 7.RP.A.2</td><td>1. Does this table represent a function?
<table><tr><th>Time (hours)</th><th>Distance (miles)</th></tr><tr><td>2</td><td>65</td></tr><tr><td>3</td><td>97.5</td></tr><tr><td>4</td><td>130</td></tr><tr><td>8</td><td>260</td></tr></table>a. Yes, because there is exactly one output for each input.
2. http://www.illustrativemathematics.org/illustrations/715
3. http://www.illustrativemathematics.org/illustrations/713</td><td>• http://www.illustrativemathematics.org/illustrations/104
• http://learnzillion.com/lessonsets/420-understand-functions-and-their-graphs
• http://learnzillion.com/lessonsets/271-understand-and-compare-functions</td></tr>
<tr><td>8.F.A.2</td><td>• 8.EE.B.5
• 8.EE.B.6
• 8.F.A.1
• 7.RP.A.2</td><td>1. Determine the unit rate for each situation.
a.
<table><tr><th># of People</th><th>Total Cost</th></tr><tr><td>6</td><td>24</td></tr><tr><td>12</td><td>48</td></tr><tr><td>24</td><td>96</td></tr></table></td><td>• http://www.illustrativemathematics.org/illustrations/129
• http://www.illustrativemathematics.org/illustrations/1537
• http://www.illustrativemathematics.org/illustrations/184
• http://www.illustrativemathematics.org/illustrations/1165</td></tr>
</table>

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		i. *Unit Rate: $4 per person* b. *d = 65t,* where *t* is the time in hours and *d* is the distance in miles. i. *Unit rate is 65 miles per hour* 2. http://www.illustrativemathematics.org/illustrations/641	• http://learnzillion.com/lessonsets/271-understand-and-compare-functions • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions
8.F.A.3	• 8.F.A.2 • 8.EE.B.6 • 8.F.A.1	1. Determine which functions are linear. A. $y = 4x + 7$ B. $y = 3x^2$ C. $y = \frac{1}{5}x - 12$ i. *A and C are linear. B is not linear because x is squared.* 2. http://www.illustrativemathematics.org/illustrations/813	• http://www.illustrativemathematics.org/illustrations/713 • http://www.illustrativemathematics.org/illustrations/641 • http://learnzillion.com/lessonsets/561-interpret-the-equation-y-mx-b • http://learnzillion.com/lessonsets/277-interpret-the-equation-y-mx-b-as-defining-a-linear-function

Real-world preparation: The following discussion will prepare students for some of the real-world components of this task:

When reserving a birthday party, many venues offer different packages for people to choose from. These packages are usually based on the number of people attending the party.

After the Task:

Students may struggle when trying to find the unit rate for each package. If students do struggle, they may need remediation with 7.RP.2 in determining unit rates from equations, tables, and graphs. Package C does not have a unit rate, as the price is based upon groups of people. This causes package C to not represent a *linear function*. Students may need help remembering what determines if a function is linear or not.

This problem could be extended by adding in different elements for students to consider like whether a package includes the cake, balloons, invitations, or tickets for the games. Then students could make their own assumptions when determining the better deal for Benjamin's parents.

Student Extended Constructed Response

Benjamin is turning 9 and wants to have his birthday party at Party Zone Palace. Party Zone Palace offers several different birthday packages. The pricing for three packages are displayed below.

<table>
<tr><th>Package A</th><th>Package B</th><th>Package C</th></tr>
<tr>
<td>$C = 8p + 12$
C = Total cost
P = Number of people</td>
<td>
<table>
<tr><th>Number of people</th><th>Total cost</th></tr>
<tr><td>3</td><td>$33</td></tr>
<tr><td>4</td><td>$44</td></tr>
<tr><td>5</td><td>$55</td></tr>
<tr><td>9</td><td>$99</td></tr>
</table>
</td>
<td>
<table>
<tr><th>Number of people</th><th>Cost per person</th></tr>
<tr><td>1-15</td><td>$15</td></tr>
<tr><td>16-30</td><td>$12</td></tr>
<tr><td>more than 30</td><td>$9</td></tr>
</table>
</td>
</tr>
</table>

1. Does Package C represent a function? Explain your reasoning.

2. Explain how you know that Package A and Package B represent linear functions.

3. Does package A or Package B have the greater cost per person? Explain your reasoning.

4. Benjamin wants to invite 36 people to his party. Determine which package his parents should buy for his birthday party in order to get the better deal. Justify your reasoning with equations and/or tables.

Extended Constructed Response Exemplar Response

Benjamin is turning 9 and wants to have his birthday party at Party Zone Palace. Party Zone Palace offers several different birthday packages. The pricing for three packages are displayed below.

<table>
<tr><th>Package A</th><th>Package B</th><th>Package C</th></tr>
<tr>
<td>$C = 8p + 12$
C = Total cost
p = Number of people</td>
<td><table><tr><th>Number of people</th><th>Total cost</th></tr><tr><td>3</td><td>$33</td></tr><tr><td>4</td><td>$44</td></tr><tr><td>5</td><td>$55</td></tr><tr><td>9</td><td>$99</td></tr></table></td>
<td><table><tr><th>Number of people</th><th>Cost per person</th></tr><tr><td>1-15</td><td>$15</td></tr><tr><td>16-30</td><td>$12</td></tr><tr><td>more than 30</td><td>$9</td></tr></table></td>
</tr>
</table>

1. Does Package C represent a function? Explain your reasoning.
   a. *Yes, Package C represents a function because there is exactly one output for each input.*

2. Explain how you know that Package A and Package B represent linear functions.
   a. *Package A and Package B represent linear functions because they both have a constant rate of change.*

3. Does package A or Package B have the greater cost per person? Explain your reasoning.
   a. *Package A: $C = 8p + 12$* *Package B: Find the Unit Rate: $33 \div 3 = 11$*

   b. *Package A charges \$8 per person, while Package B costs \$11 per person. Therefore, Package B has a greater price per person.*

4. Benjamin wants to invite 36 people to his party. Determine which package his parents should buy for his birthday party in order to get the better deal. Justify your reasoning with equations and/or tables.

   Package A: $C = 8(36) + 12$ *Package B: $C = 11(36)$* *Package C: $C = 9(36)$*

   $C = \$288$* *$C = \$396$* *$C = \$324$

   Package A is the better deal costing only \$288 for 36 people.

**Students may choose to extend the table for Package B to show their reasoning.*

Congruence Transformations (ECR)

Overview

Students will practice performing various congruence transformations and describing the effects of the transformations.

Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments, of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :

8.G.A.1	• 7.G.A.2
	• 7.G.B.5
		1. $\overline{AB}$ is reflected over line *l*. What must be true about the resulting image $\overline{A'B'}$?
		  a. *The resulting image, $\overline{A'B'}$, is the same length as $\overline{AB}$.*
		2. $\Delta\, ABC$ is rotated around point *A*. What is true about angle *B'C'A'*?
		  a. *Angle B'C'A' is congruent to angle BCA.*
			• http://learnzillion.com/lessonsets/473-verify-properties-of-rotations-reflections-and-translations

8.G.A.2	• 8.G.A.1	1. Two triangles are drawn on a coordinate plane. How can you tell if they are congruent? a. The two triangles are congruent if one triangle can be obtained from the other through a sequence of rotations, reflections, and/or translations. 2. http://www.illustrativemathematics.org/illustrations/646	• http://learnzillion.com/lessonsets/528-understand-congruency-in-twodimensional-figures • http://learnzillion.com/lessonsets/466-assess-congruence-using-rotations-reflections-and-translations

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		3. http://www.illustrativemathematics.org/illustrations/1228 4. http://www.illustrativemathematics.org/illustrations/1231	
8.G.A.3	• 6.G.A.3 • 8.G.A.1	1. Graph rectangle *RSTU* with vertices $R(2,3), S(3,1), T(-1,-1)$, and $U(-2,1)$. Rotate rectangle *RSTU* 90° clockwise around the origin. What are the coordinates of *R'S'T'U'*? a. *R'(3, -2), S'(1, -3), T'(-1, 1), and U'(1, 2)* 2. http://www.illustrativemathematics.org/illustrations/1243	• http://www.illustrativemathematics.org/illustrations/1188 • http://learnzillion.com/lessonsets/534-describe-the-effect-of-dilations-translations-rotations-and-reflections-on-twodimensional-figures-using-coordinates • http://learnzillion.com/lessonsets/476-describe-the-effects-of-dilations-translations-rotations-and-reflections-using-coordinates

After the Task:

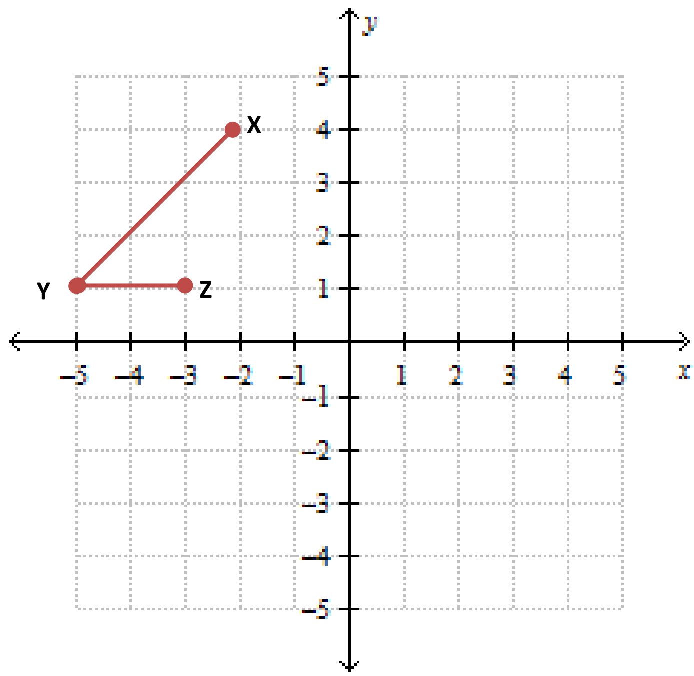
For Part c in Question 1, students may use the original figure when performing the reflection. This would affect their answer to Part e as well.

In the second question, students may struggle with finding a starting point to map one triangle to the other. This requires a higher cognitive demand than performing given transformations. Have students practice performing various transformations on triangle *ABC* and describe how the transformations move the triangle to help students understand which transformations would be best to use. The following are sample questions that may help students with this portion of the task:

1. In which quadrant is triangle *PQR*? Which transformations would move a figure from one quadrant to another?
2. What do you notice about the orientation of the triangle? Which transformations would change the orientation of the triangle?

Student Extended Constructed Response

1. Complete all parts. You may use a ruler and/or protractor to complete the task.



a. Rotate ∠XYZ 90˚counterclockwise about the origin. Draw and label the image ∠X'Y'Z'.

b. Name the ordered pairs for: X' ____________
Y' ____________
Z' ____________

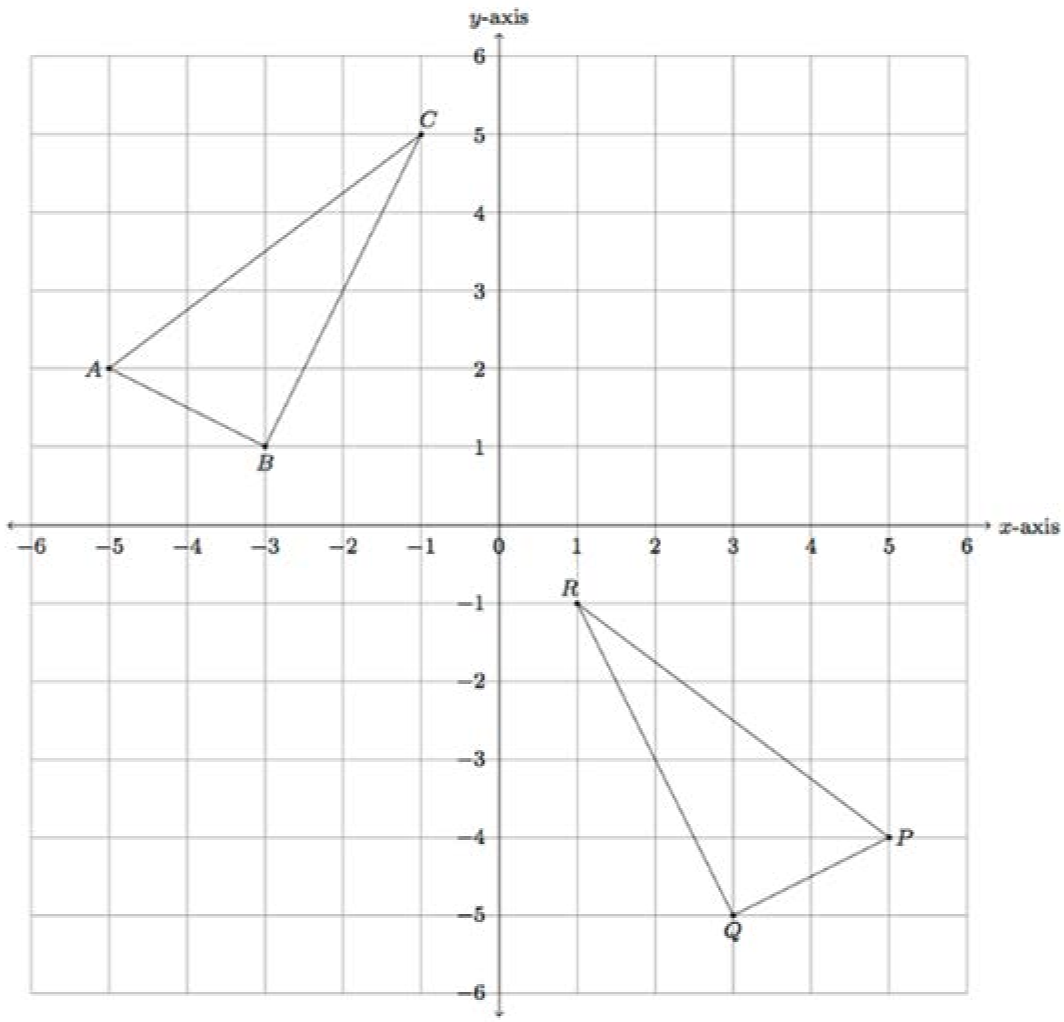
c. Reflect ∠X'Y'Z' over the y-axis. Draw and label ∠X"Y"Z".

d. What is the *m*∠XYZ? _______________ What is the *m*∠X"Y"Z"?_______________

How do the measures of these angles compare?
Explain why this is reasonable for an angle that is rotated and reflected.

e. Sarah thinks that she can produce the ∠X"Y"Z" from ∠XYZ with one transformation, a 180˚ rotation about the origin. Is she correct? Explain why or why not.

2. Triangles ABC and PQR are shown below in the coordinate plane.



a. Show that ΔABC is congruent to ΔPQR with a reflection followed by a translation.

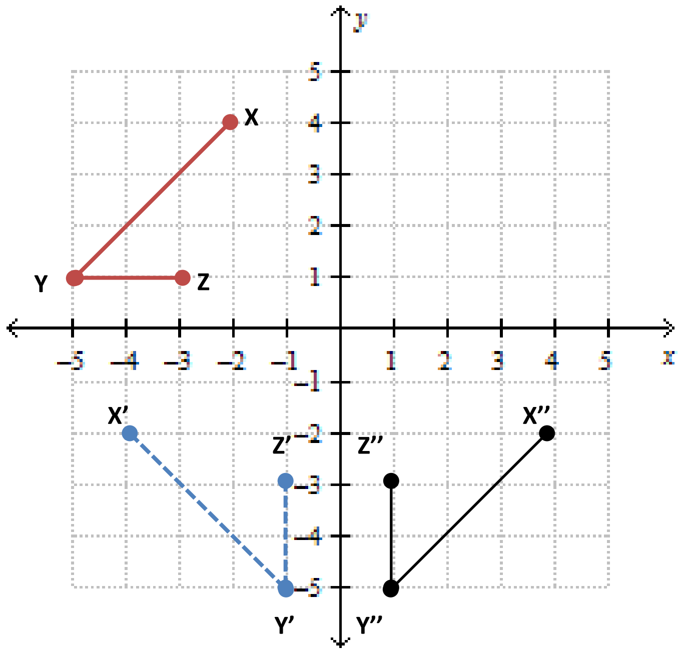
b. If you reverse the order of your reflection and translation in Part a does it still map ΔABC to ΔPQR?

c. Find a second way, different from your work in Parts a or b, to map ΔABC to ΔPQR using translations, rotations, and/or reflections.

Task adapted from http://www.illustrativemathematics.org/illustrations/1232.

Extended Constructed Response Exemplar Response

1. Complete all parts. You may use a ruler and/or protractor to complete the task.



a. Rotate ∠XYZ 90˚counterclockwise about the origin. Draw and label the image ∠X'Y'Z'.

b. Name the ordered pairs for: X' __*(-4, -2)*_______
Y' __*(-1, -5)*_______

Z' __*(-1, -3)*_______

c. Reflect ∠X'Y'Z' over the y-axis. Draw and label ∠X"Y"Z".

d. What is the *m*∠XYZ? ______45˚_________ What is the *m*∠X"Y"Z"?_______45˚ _____

How do the measures of these angles compare? *The angles are congruent.*
Explain why this is reasonable for an angle that is rotated and reflected. *Rotations and reflections produce congruent transformations, so the angles should be congruent after transformation.*

e. Sarah thinks that she can produce the ∠X"Y"Z" from ∠XYZ with one transformation, a 180˚ rotation about the origin. Is she correct? Explain why or why not.

No, the coordinates for ∠X"Y"Z" are X"(4, -2), Y"(1, -5), and Z"(1, -3), but the coordinates of the angle if it is rotated 180˚ are X(2, -4), Y(5, -1), and Z(3, -1).

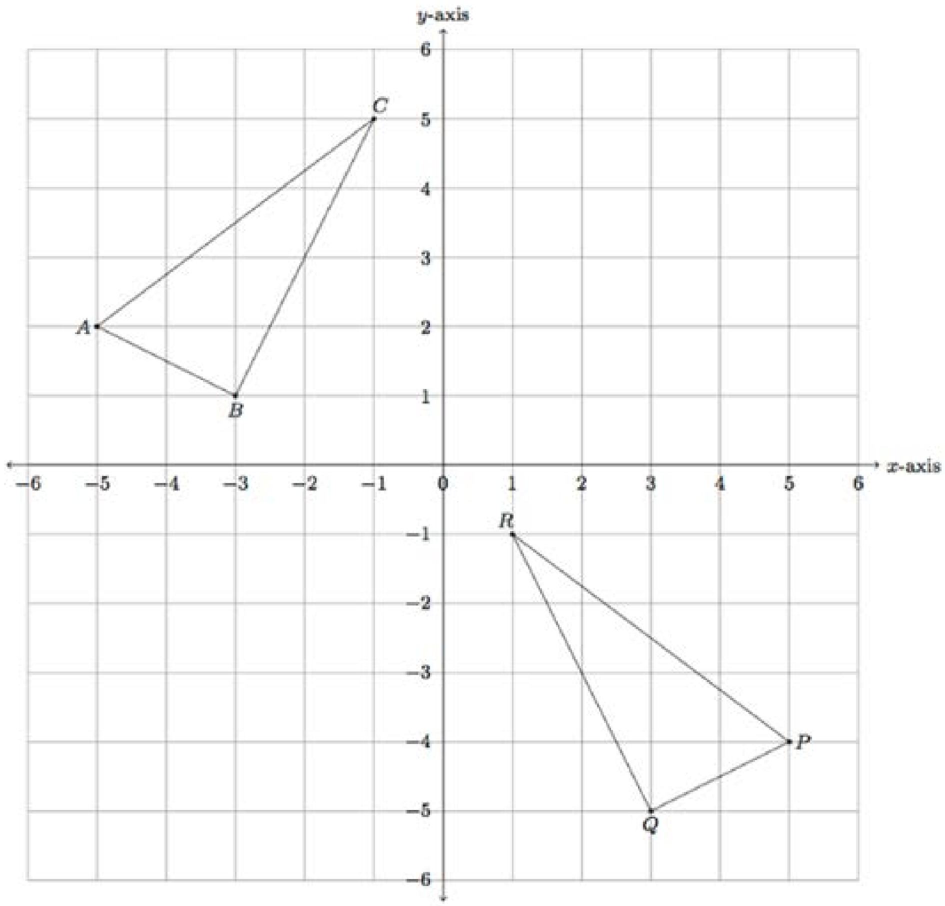
Or

In order to produce 180˚ rotation, two reflections are necessary.

Or

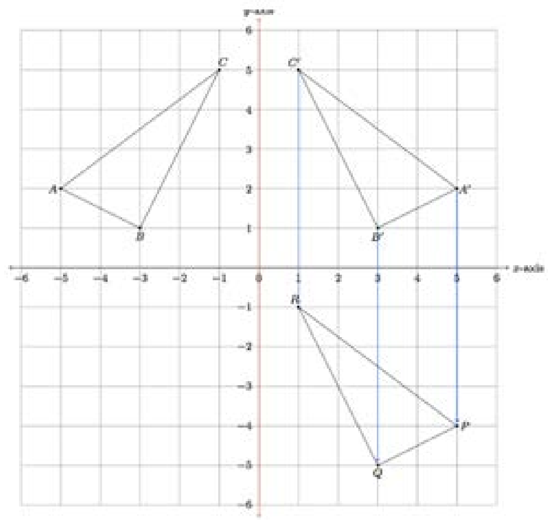
Any other student responses that are accurate and demonstrate strong mathematical reasoning.

2. Triangles ABC and PQR are shown below in the coordinate plane.



a. Show that ΔABC is congruent to ΔPQR with a reflection followed by a translation.

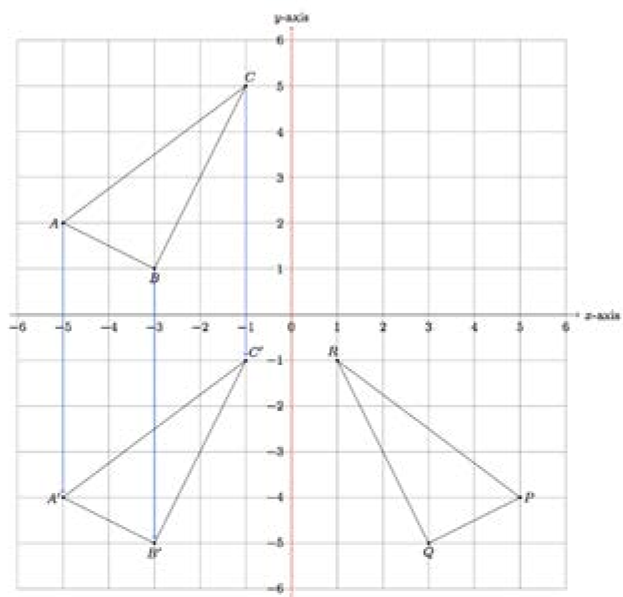
Reflect across the y-axis, then translate down 6 units.



There are other possible combinations of reflections and translations, but this is the one the students will most likely choose. However, other solutions will need to be checked for accuracy.

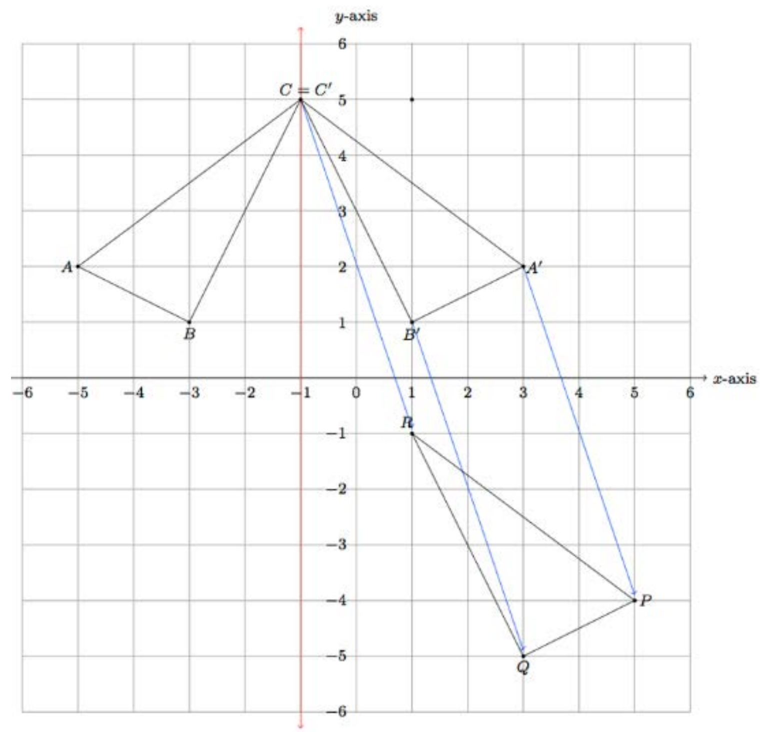
b. If you reverse the order of your reflection and translation in Part a does it still map ΔABC to ΔPQR?

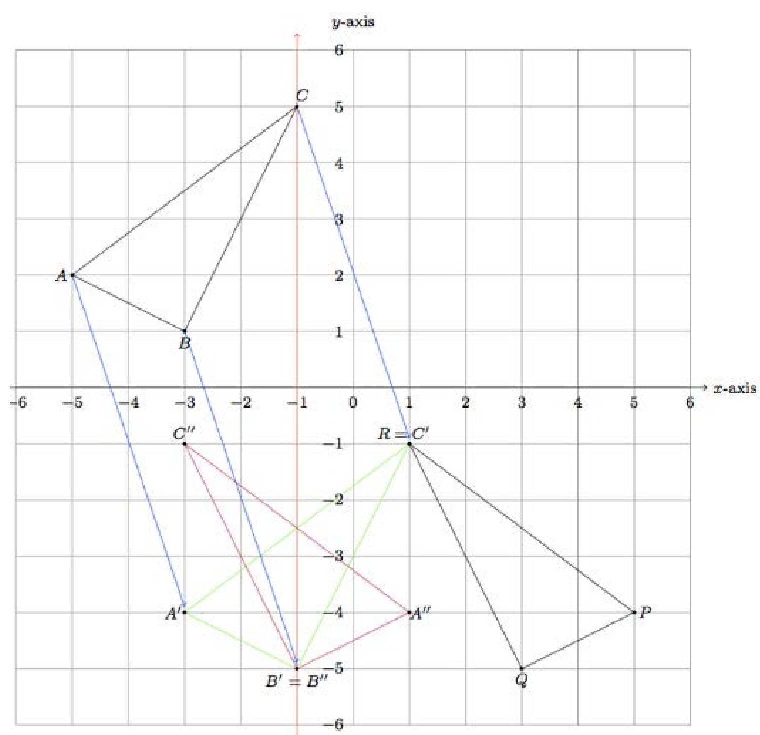
This answer is dependent on the answer in Part a. Using the reflection and translation chosen above, the answer would be yes. ΔABC translated down 6 units and then reflected over the y-axis would result in ΔPQR. See below.

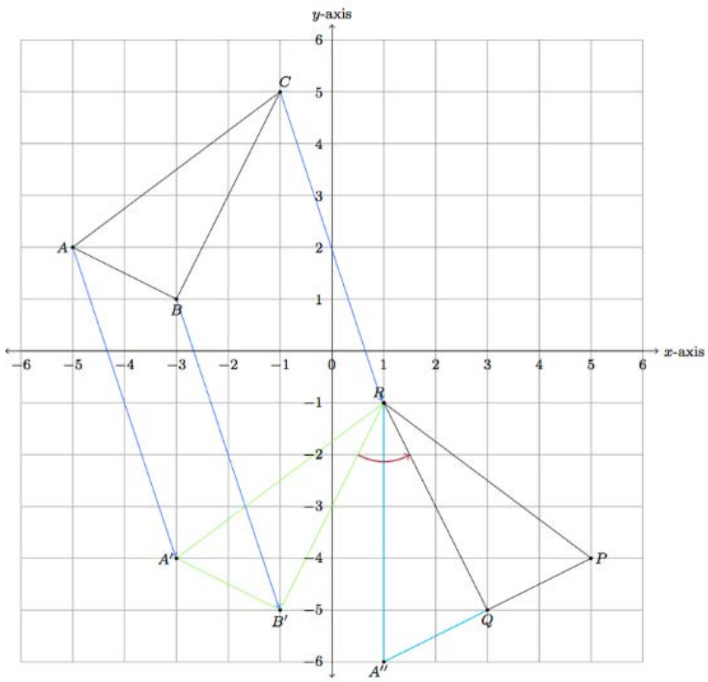


c. Find a second way, different from your work in Part a, to map ΔABC to ΔPQR using translations, rotations, and/or reflections.

There are several possible answers. Below are a few samples.


y-axis
x-axis
C = C'
A
B
A'
B'
R
P
Q


y-axis
x-axis
C
A
B
C''
R = C'
A'
A''
B' = B''
P
Q



y-axis
x-axis
A
B
C
R
P
Q
A′
B′
A″

Connecting Proportional Relationships, Lines, and Linear Equations (IT)

Overview

This instructional task requires students to understand the connections between proportional relationships, lines, and linear equations.

Standards

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.B.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.B.5	• 7.RP.A.2	1. What is the slope between the points (2, 3) and (5, 9)? a. 2 2. http://www.illustrativemathematics.org/illustrations/129 3. http://www.illustrativemathematics.org/illustrations/55 4. http://www.illustrativemathematics.org/illustrations/184	• http://www.illustrativemathematics.org/illustrations/104 • http://www.illustrativemathematics.org/illustrations/1186 • http://www.illustrativemathematics.org/illustrations/1526 • http://learnzillion.com/lessonsets/275-graph-interpret-and-compare-proportional-relationships
8.EE.B.6	• 7.G.A.1 • 7.RP.A.2 • 8.G.A.5	1. How can you tell if two triangles are similar? a. Two triangles are similar if a series of transformations can take one triangle to the other one. 2. http://www.illustrativemathematics.org/illustrations/1537	• http://www.illustrativemathematics.org/illustrations/1082 • http://www.illustrativemathematics.org/illustrations/101 • http://www.illustrativemathematics.org/illustrations/1527 • http://learnzillion.com/lessonsets/274-use-similar-triangles-to-explain-why-the-slope-m-is-the-same-between-two-points-on-a-nonvvertical-line-in-the-coordinate-plane

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

Why would I need to know how much fresh fruit costs by the pound? Fresh fruit and vegetables are often sold by the pound. Sometimes they come in already-weighed bundles with the weight and cost indicated. Sometimes you bag your own and weigh it yourself to estimate how much it will cost.

Why do I need to look at prices for the same item packed differently? Sometimes stores offer the same item in different sizes. Each size will have a different price. Oftentimes it is cheaper by the unit to buy a bigger item. Sometimes, however, it is not, so it is important to figure out the unit price when shopping in order to save money.

During the Task:

Make sure that students are being precise when they graph. They should label the axes and indicate the units. Students may need help plotting points.

Students may struggle with proving that the triangles are similar. If needed, briefly review similarity transformations, congruence transformations, and the angle-angle criterion for similarity.

After the Task:

Have students bring a favorite recipe. Then have them go to the store and figure out how much it would cost to make it buying the smallest available containers of their ingredients and the biggest available containers. Then discuss how much of each ingredient would be left after making your recipe using each. Then have them decide which size container they should buy for each item.

Student Instructional Task

You are shopping for bananas.

<u>Option 1:</u>

Your first option at your local grocery store is purchasing 5 lbs. of bananas for $1.45.

Use this information to complete the following:

1. Fill in the following chart.

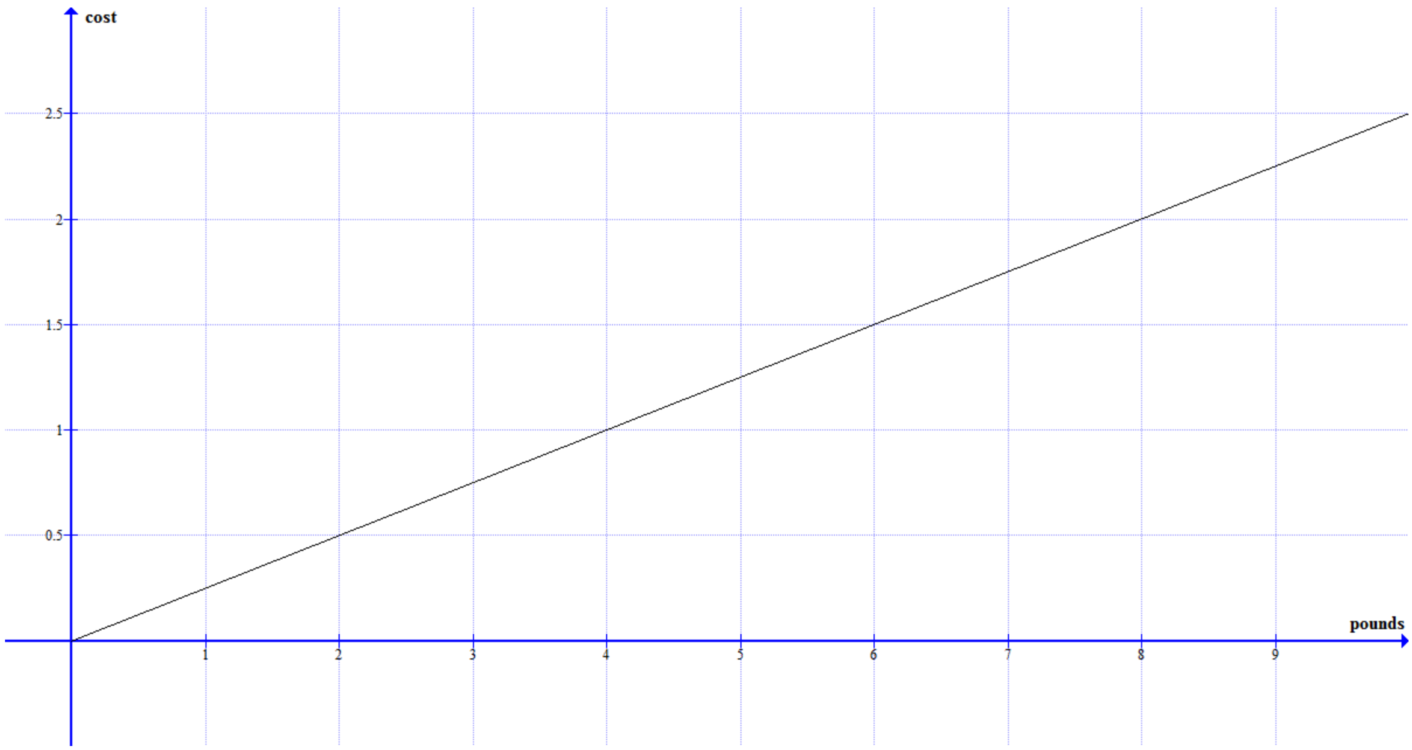
Pounds	Cost
1	
2	
3	
4	
5	

2. On the same coordinate plane:
    a. Plot the ordered pairs representing 0 pounds and 1 pound. Connect these points with a line segment.
    b. Plot the ordered pairs representing 2 pounds and 4 pounds. Connect these points with a line segment.
    c. Using similar triangles, explain why the slopes of both line segments are the same.
    d. What is the cost per pound of bananas? How does the cost per pound of bananas relate to the slope of your line segments?

3. Write an equation to represent how much money (y) you would spend for (x) pounds of bananas if 5 lbs. of bananas cost $1.45.

Option 2:

Your second option for purchasing bananas at the grocery store is shown in the graph below.



4. Compare the two options available at your grocery store. Which store offers bananas at the cheaper price? Explain your reasoning.

5. You are going shopping for your favorite fresh fruit or vegetable. Either visit a local store or use the Internet to research the price of the fresh fruit or vegetable. Use your research to complete the following:
    a. Create a table to represent the price of your fruit or vegetable for 0-5 pounds purchased.
    b. Create a graph to represent the price of your fruit or vegetable for 0-5 pounds purchased.
    c. Write an equation based on the graph you created.
    d. What does the slope represent in terms of your fruit or vegetable?
    e. Swap your table with one classmate, your graph with a different classmate, and your equation with a third classmate. Whose fruit or vegetable was cheaper? How do you know? Write a short paragraph to summarize all three sets of information.

Instructional Task Exemplar Response

You are shopping for bananas.

Option 1:

Your first option at your local grocery store is purchasing 5 lbs. of bananas for $1.45.

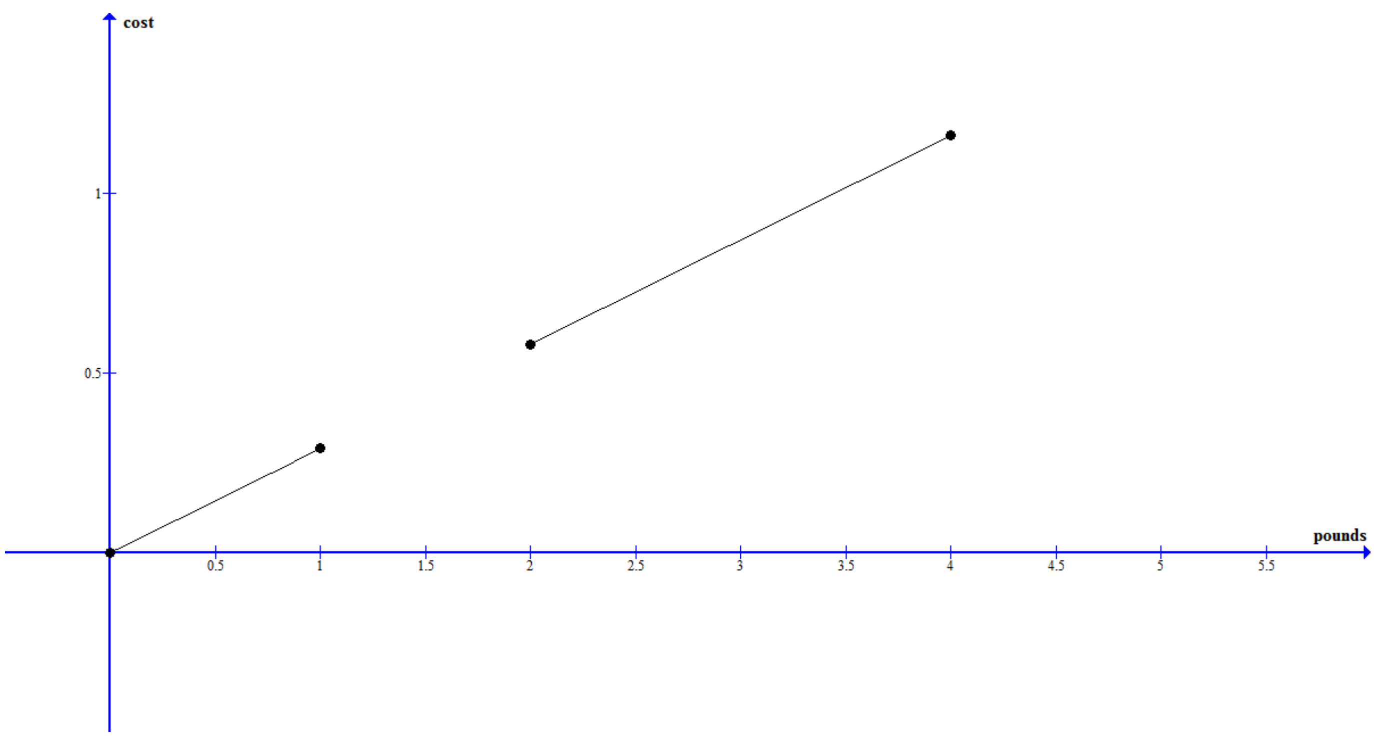
Use this information to complete the following:

1. Fill in the following chart.

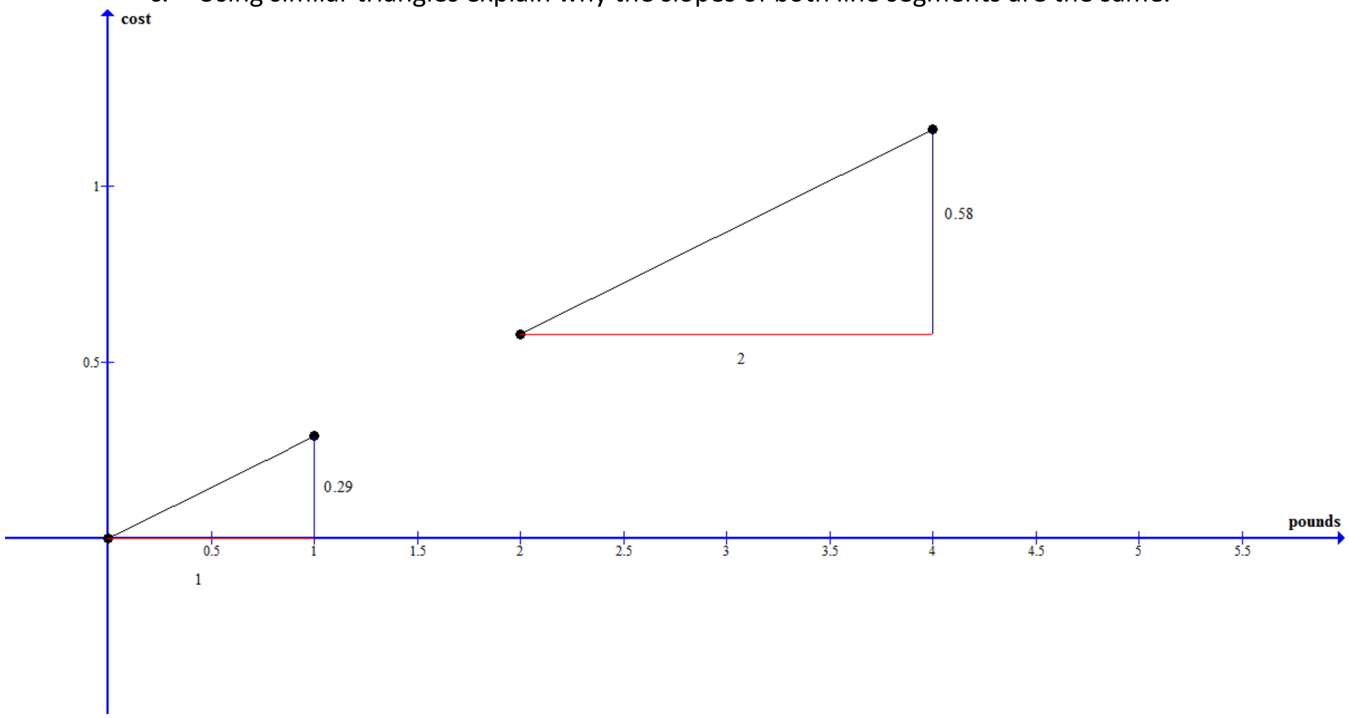
Pounds	Cost
1	*$0.29*
2	*$0.58*
3	*$0.87*
4	*$1.16*
5	*$1.45*

2. On a coordinate plane:
   a. Plot the ordered pairs representing 0 pounds and 1 pound. Connect these points with a line segment.

   b. Plot the ordered pairs representing 2 pounds and 4 pounds. Connect these points with a line segment.



c. Using similar triangles explain why the slopes of both line segments are the same.



The first triangle is translated to the right 2 units and up 0.58 units. Then the triangle is dilated by a factor of 2. Since we can use a translation and a dilation to make one triangle from the other, we know that they are similar triangles. To find the slope between two points, we find the ratio of the lengths of the legs in a right triangle. The lengths of corresponding sides in similar triangles are also proportional. Therefore, the ratio of the lengths of the legs of the two triangles must be the same, and the two triangles would have the same slope.

We can see this algebraically. Slope is equal to $\frac{y_2-y_1}{x_2-x_1}$*.For the first line segment that would be* $\frac{0.29}{1}$*=0.29, and for the second line segment it would be*$\frac{0.58}{2} = 0.29$*. These line segments have the same slope.*

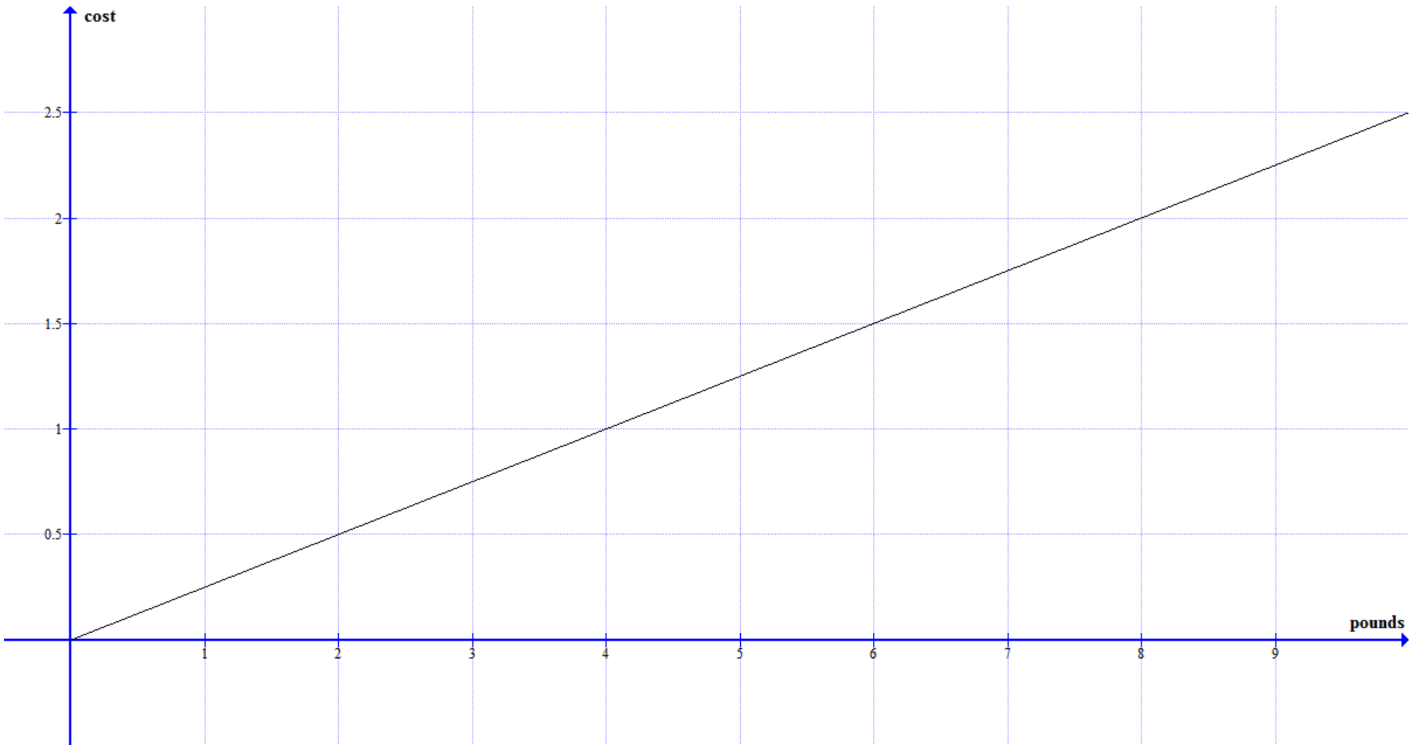
d. What is the cost per pound of bananas? How does the cost per pound of bananas relate to the slope of your line segments?

The bananas cost $0.29 per pound. The cost per pound is the same as the slope of the two line segments.

3. Write an equation to represent how much money (*y*) you would spend for (*x*) pounds of bananas if 5 lbs. of bananas cost $1.45.
y=0.29x

Option 2:

Your second option for purchasing bananas at the grocery store is shown in the graph below.



4. Compare the two options available at your grocery store. Which store offers bananas at the cheaper price? Explain your reasoning.

The slope of the graph for Option 2 is 0.25, so the bananas in Option 2 cost $0.25 a pound. The bananas in Option 1 cost $0.29 a pound. As a result, the bananas in Option 2 are cheaper.

5. You are going shopping for your favorite fresh fruit or vegetable. Either visit a local store or use the Internet to research the price of the fresh fruit or vegetable. Use your research to complete the following:
   a. Create a table to represent the price of your fruit or vegetable for 0-5 pounds purchased.
   b. Create a graph to represent the price of your fruit or vegetable for 0-5 pounds purchased.
   c. Write an equation based on the graph you created.
   d. What does the slope represent in terms of your fruit or vegetable?
   e. Swap your table with one classmate, your graph with a different classmate, and your equation with a third classmate. Whose fruit or vegetable was cheaper? How do you know? Write a short paragraph to summarize all three sets of information.

The answers for this section will vary according to the items chosen and their prices. The results should look much like the results from the previous exercises.

T-Shirt Fundraiser (IT)

Overview

Students will determine the best place to order T-shirts for an upcoming fundraiser and help a new business determine prices for the T-shirts they will sell.

Standards

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.C.8c	• 6.EE.B.5 • 7.EE.B.4a • 8.EE.B.6	1. Write an equation to represent 8 times a number of dollars plus 7 is equal to the total number of dollars. a. 8x + 7 = y 2. Solve the equation 5x + 9 = y when x is equal to 9. a. 54 3. Solve the system $x + y = 27$ $x - y = 9$ a. (18, 9) 4. http://www.illustrativemathematics.org/illustrations/934 5. http://www.illustrativemathematics.org/illustrations/1362	• http://www.illustrativemathematics.org/illustrations/673 • http://www.illustrativemathematics.org/illustrations/1537 • http://learnzillion.com/lessonsets/777-analyze-and-solve-pairs-of-simultaneous-linear-equations-solve-systems-in-two-equations-algebraically • http://learnzillion.com/lessonsets/776-solve-pairs-of-simultaneous-linear-equations-understand-why-solutions-correspond-to-points-of-intersection • http://learnzillion.com/lessonsets/50-graphing-to-solve-systems-of-equations

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a fundraiser? A fundraiser is an activity held to raise money for an organization or cause.

Why might an organization have a fundraiser? An organization may hold a fundraiser to pay for a special activity.

Why would a company not want to have the highest priced item? A company wants to have prices that are competitive in order for customers to choose them over their competition. If their price is too low, they will make not make a large profit but may be chosen often. If their price is too high, they will not be chosen often and will lose business.

During the Task:

- Look for students struggling with graphing the two situations.
- Students may get stuck determining how to find the price for the new business to sell their T-shirts. Students may need help seeing the area between the two lines as the area the new business wants to fall between. However, some students may do just as well looking at the difference between the two sets of data points, if that is how they chose to represent the data.
- They may also struggle with finding a one-time setup fee, then a per-shirt price. A suggestion is selecting a one-time setup between $0 and $30. Find the difference of this fee and the $100 that 10 shirts should cost. Divide the difference by 10. This will give the per-shirt price.

After the Task:

This can be related to clubs that students are a part of at school or athletic teams. There are times when they need to raise money and need to shop for the best deal in order to make the most profit.

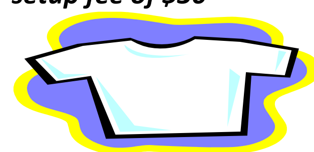
Student Instructional Task

The Math Club has chosen to make T-shirts with the school's logo and sell them for a fundraiser. It is your responsibility to help decide where to purchase the shirts from. There are two local competitors. Use the information from the ads to answer the questions.

Printin' T's

Just $10 for each T-shirt!!!

$7 per shirt + one-time setup fee of $30



Bright Stitches

1. When is it better to order from *Printin' T's*? When is it better to order from *Bright Stitches*? Explain your choices using a graph, algebra, and/or a table.

2. Choose a company from the two local competitors. Using the company you have chosen, how much must the Math Club sell each shirt for in order to make a profit? Explain your reasons for choosing the company and the price.

3. A third company is going to sell T-shirts in your area. This company doesn't want to ever be the cheapest or most expensive company. They want their T-shirts to be priced between their competitors.
   How could the poster for *Your Way T's* be completed to meet these criteria? Explain your reasoning.

Your Way T's

One-time setup fee: ___
Price per shirt printed: ___

Instructional Task Exemplar Response

The Math Club has chosen to make T-shirts with the school's logo and sell them for a fundraiser. It is your responsibility to help decide where to purchase the shirts from. There are two local competitors. Use the information from the ads to answer the questions.

1. When is it better to order from *Printin' T's*? When is it better to order from *Bright Stitches*? Explain your choices using a graph, algebra, and/or a table.

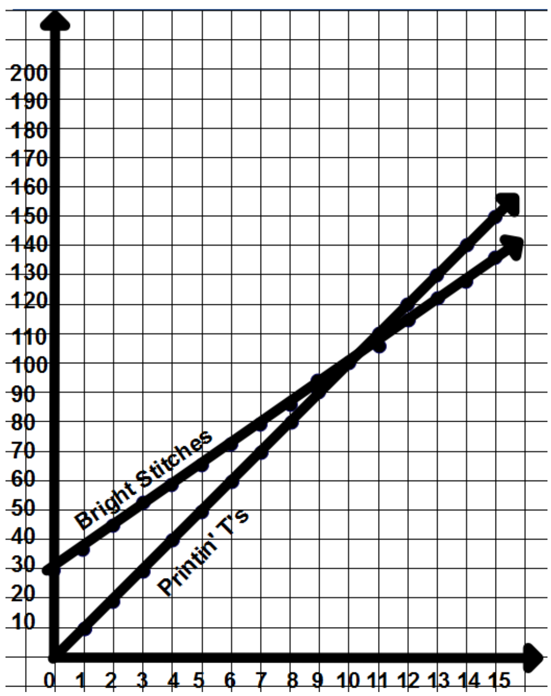
An equation for Printin' T's would be: 10t = c where t is the number of T-shirts and c is the total cost.

An equation for Bright Stitches would be: 30 + 7t = c where t is the number of T-shirts and c is the total cost.

Data points created using the equations:

10t = c		*30 + 7t = c*	
(0, 0)	*(10, 100)*	*(0, 30)*	*(10, 100)*
(1, 10)	*(11, 110)*	*(1, 37)*	*(11, 107)*
(2, 20)	*(12, 120)*	*(2, 44)*	*(12, 114)*
(3, 30)		*(3, 51)*	
(4, 40)		*(4, 58)*	
(5, 50)		*(5, 65)*	
(6, 60)		*(6, 72)*	
(7, 70)		*(7, 79)*	
(8, 80)		*(8, 86)*	
(9, 90)		*(9, 93)*	

When the two equations are graphed, the better value at each point is visible. The better value will be the company represented by the lower line at a given x value.



The club should choose Printin' T's when the T-shirt order is less than 10 shirts, because up until that point, it is the better price. The club should choose Bright Stitches when the T-shirt order is more than 10 shirts, because after that point, it is the better price. If the order is exactly 10 shirts, the price will be $100 at both places.

2. Choose a company from the two local competitors. Using the company you have chosen, how much must the Math Club sell each shirt for in order to make a profit? Explain your reasons for choosing the company and the price.
   Sample response: Since the club will most likely sell more than 10 shirts, the club would use Bright Stitches. If 10 shirts were ordered, each shirt would cost $10 to make. After this point, each shirt becomes cheaper. If the shirts are sold for $10, the club would make a profit.

3. A third company is going to sell T-shirts in your area. This company doesn't want to ever be the cheapest or most expensive company. They want their T-shirts to be priced between their competitors.
How could the poster for *Your Way T's* be completed to meet these criteria? Explain your reasoning.

Your Way T's

One-time setup fee: ___
Price per shirt printed: ___

This answer can be different for each student. However, it must meet the criteria of being between the two competitors at all times. There should be an adequate explanation of how the price fits the parameters.

For example: One-time setup fee: $15. Price per shirt printed: $8.50

This would make the shirts equal at $100 and between the other two at all points.

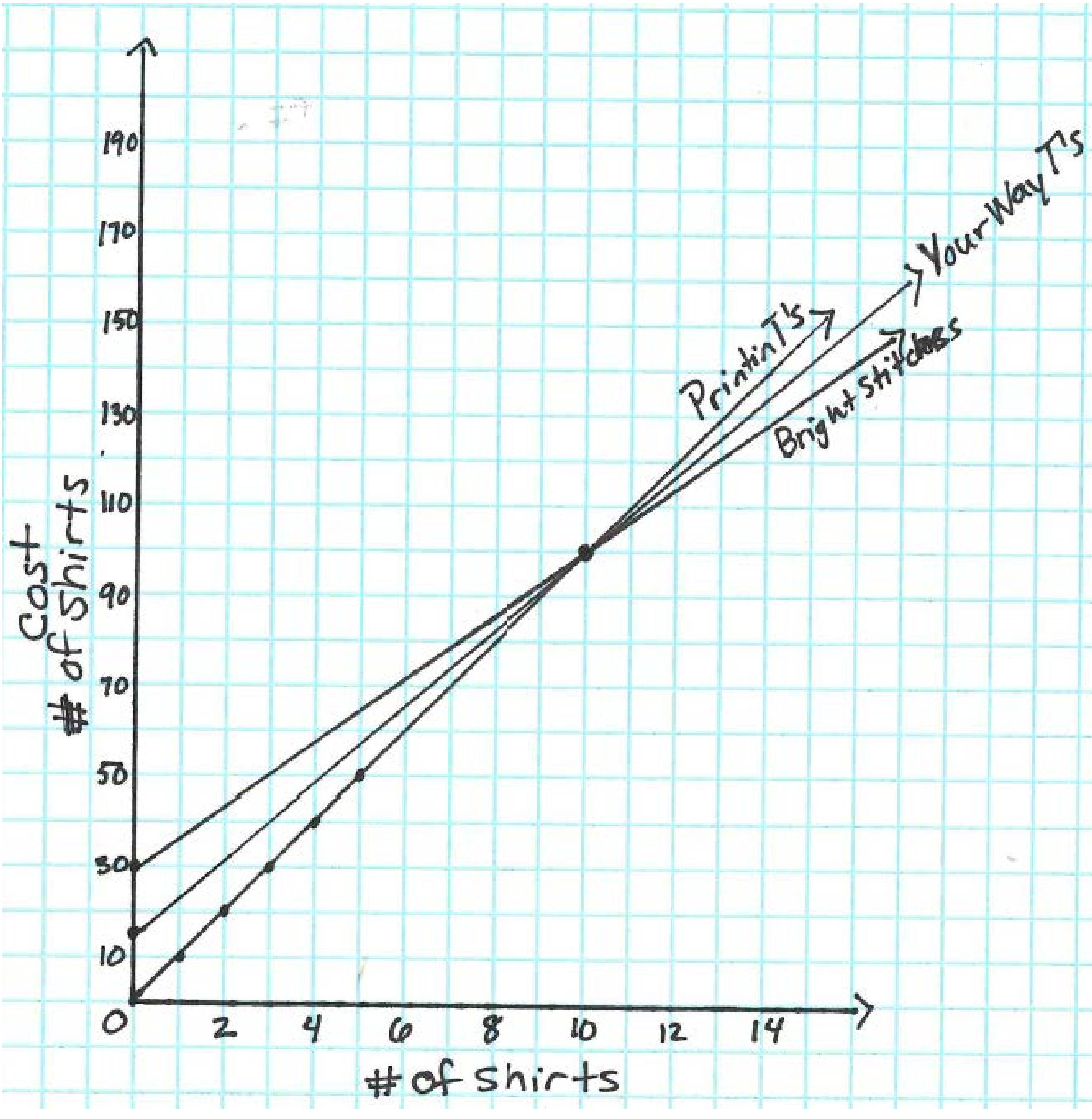
A graph or data points table could be used.

Data points:

15 + 8.5t = c

(0, 15)	*(7, 74.50)*	
(1, 23.50)	*(8, 83)*	
(2, 32)	*(9, 91.50)*	
(3, 41.50)	*(10, 100)*	****Notice the point (10, 100). That is a must.*
(4, 49)	*(11, 108.50)*	
(5, 57.50)	*(12, 117)*	
(6, 66)		

Graph for these points proving that it fits the parameters:



Game Design (IT)

Overview

Students will use congruence transformations to create a game on a grid.

Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments, of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.G.A.1	• 7.G.A.2 • 7.G.B.5	1. $\overline{AB}$ is reflected over line *l*. What must be true about the resulting image $\overline{A'B'}$? *a. The resulting image, $\overline{A'B'}$, is the same length as $\overline{AB}$.* 2. $\Delta\, ABC$ is rotated around point *A*. What is true about angle *B'C'A'*? *a. Angle B'C'A' is congruent to angle BCA.*	• http://learnzillion.com/lessonsets/473-verify-properties-of-rotations-reflections-and-translations
8.G.A.2	• 8.G.A.1	1. Two triangles are drawn on a coordinate plane. How can you tell if they are congruent? a. The two triangles are congruent if one triangle can be obtained from the other through a sequence of rotations, reflections, and/or translations. 2. http://www.illustrativemathematics.org/illustrations/646	• http://learnzillion.com/lessonsets/528-understand-congruency-in-twodimensional-figures • http://learnzillion.com/lessonsets/466-assess-congruence-using-rotations-reflections-and-translations

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		3. http://www.illustrativemathematics.org/illustrations/1228 4. http://www.illustrativemathematics.org/illustrations/1231	
8.G.A.3	• 6.G.A.3 • 8.G.A.1	1. Graph rectangle *RSTU* with vertices $R(2,3), S(3,1), T(-1,-1)$, and $U(-2,1)$. Rotate rectangle *RSTU* 90° clockwise around the origin. What are the coordinates of *R'S'T'U'*? a. *R'(3, -2), S'(1, -3), T'(-1, 1), and U'(1, 2)* 2. http://www.illustrativemathematics.org/illustrations/1243	• http://www.illustrativemathematics.org/illustrations/1188 • http://learnzillion.com/lessonsets/534-describe-the-effect-of-dilations-translations-rotations-and-reflections-on-twodimensional-figures-using-coordinates • http://learnzillion.com/lessonsets/476-describe-the-effects-of-dilations-translations-rotations-and-reflections-using-coordinates

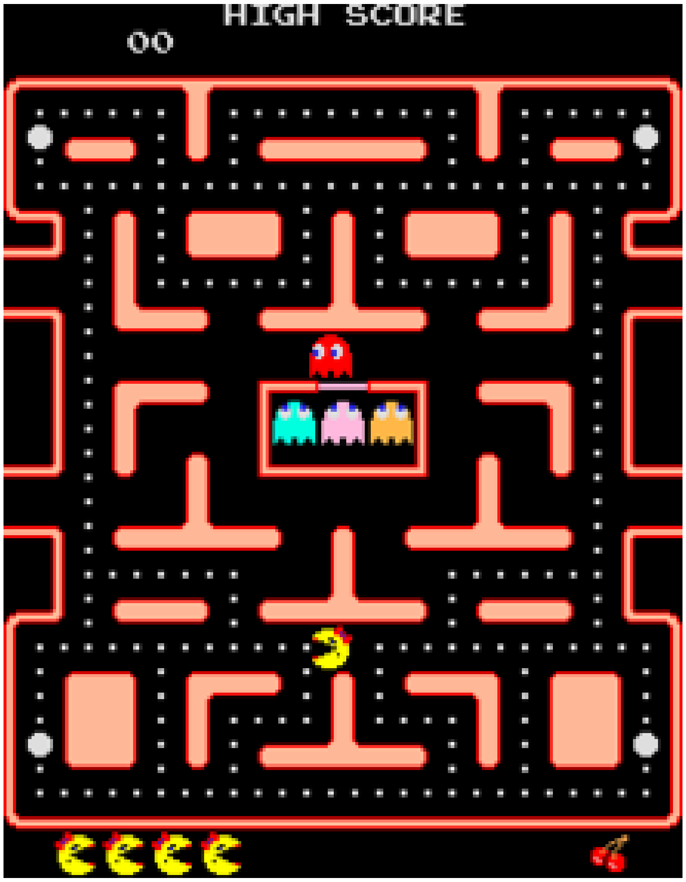
During the Task:

- Students will find it easier to use and describe reflections and translations. They may need some assistance in using and describing rotations to create the game.
- To help students determine whether they have completed all of the requirements, teachers may wish to create a checklist for students to follow.
- Watch for students who may change the shapes, lengths, or measures of angles as they create the game. Discuss with students the meaning of congruence transformations and how they apply to this task.

After the Task:

Have students identify transformations in other areas like classrooms, the gym, cafeteria, school yard, etc. Teachers can also have students find examples of transformations at home. When identifying examples of transformations, students would need to explain how they know the example represents the transformation.

Student Instructional Task



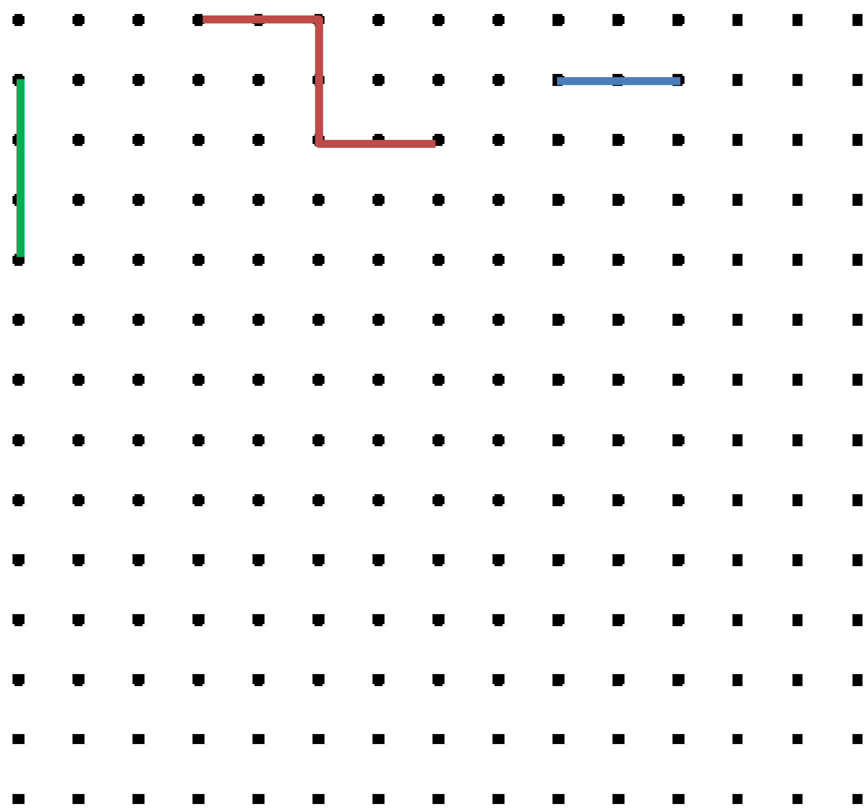
Source for picture: http://en.wikipedia.org/wiki/Ms._Pac-Man

Above is the game screen for Ms. Pac-Man. Reflections, translations, and rotations were used to place the "walls" between which Ms. Pan-Man can travel. Take a minute to locate reflections, translations, and rotations of the "walls" in the screen above.

Design a game of your own below. Create your own game and game characters. Be sure your game meets the following requirements:

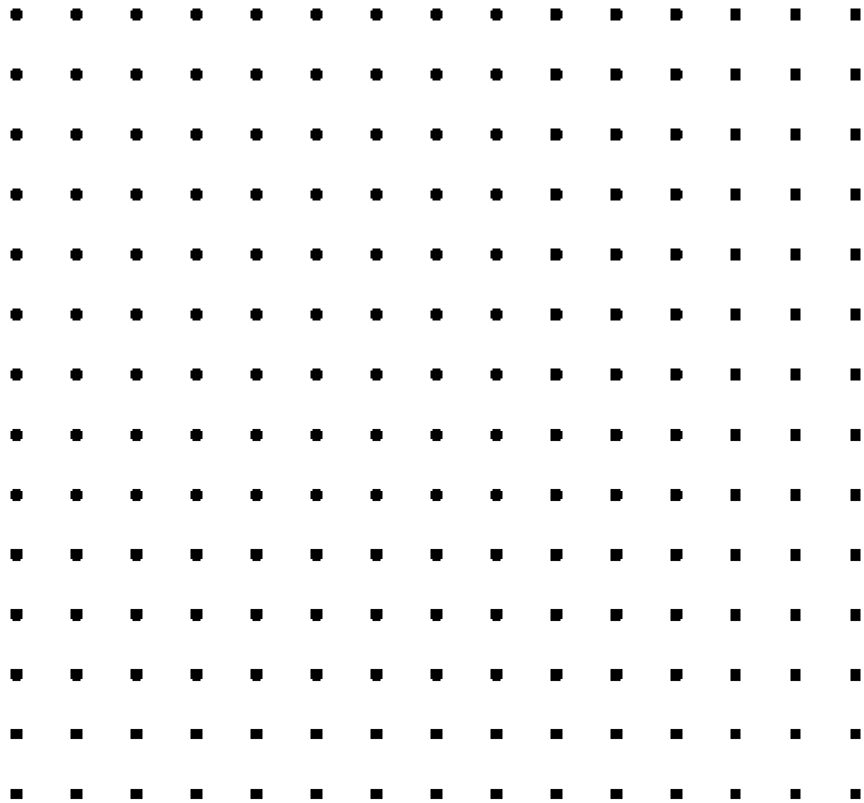
- Identify the goal of your game. The goal may be for your game character to eat all of the dots, as in Ms. Pac-Man, or you may have a different goal altogether.
- Use at least two of the three figures below. You may also use other figures you create. Draw any figures you create below.
- Use at least three different figures in the game.
- Each different figure used must be a different color. When each figure is transformed, the new image must be the same color as the original figure.
- The interior walls of your game must be the figures you have created or chosen and the images of those figures that have been reflected, translated, and rotated.
- There must be at least one use of each type of transformation.
- There must be at least two series of transformations. (You will likely have many more.)
- Exterior walls and other objects may be drawn, if needed, to complete your game.

Use the grid below to draw any additional figures you plan to use in your game that will be transformed using reflections, rotations, and/or translations. Be sure to use a different color for each different figure.



Create your game using the grid on the next page.

Use the blank grid below to create your game.



Complete parts A through H on the following page.

After you have created your game, complete the following about your game:

A. Identify a reflection you used in the game. Explain how you know this is a reflection.

B. Identify a translation you used in the game. Explain how you know this is a translation.

C. Identify a rotation you used in the game. Explain how you know this is a rotation.

D. Identify and list the steps of two transformations that involved a series of reflections, translations, and/or rotations.

Next, exchange games with your partner. Do not show them your answers to parts A through D above. Using your partner's game, complete the following. After you identify the transformations, discuss your findings with your partner.

E. Identify a reflection used in your partner's game. Explain how you know this is a reflection.

F. Identify a translation used in your partner's game. Explain how you know this is a translation.

G. Identify a rotation used in your partner's game. Explain how you know this is a rotation.

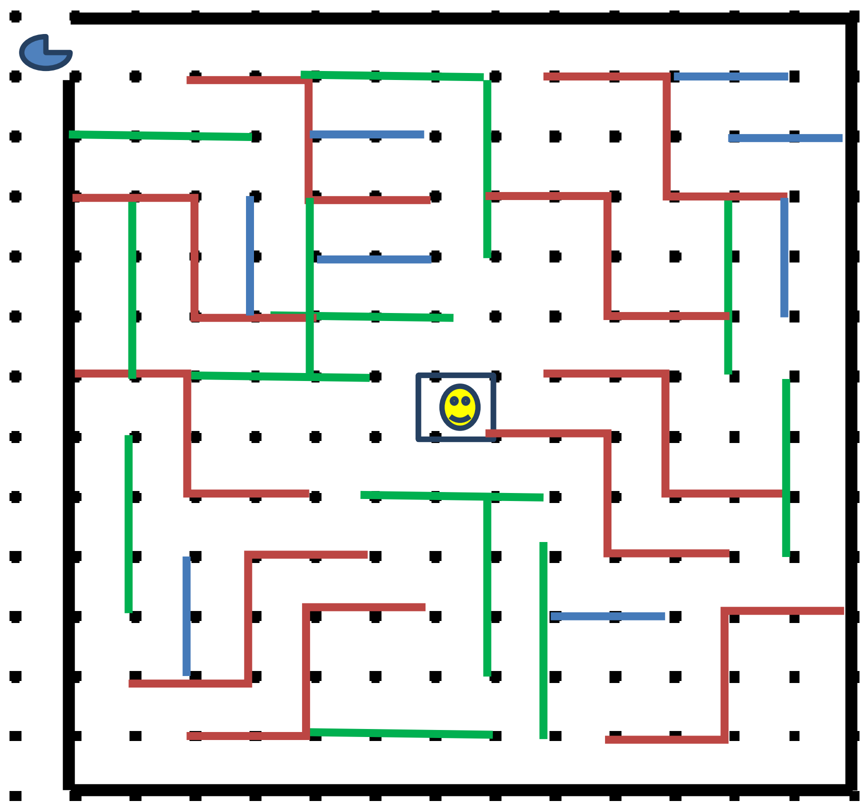
H. Identify and list the steps of one transformation that involves a series of reflections, translations, and/or rotations.

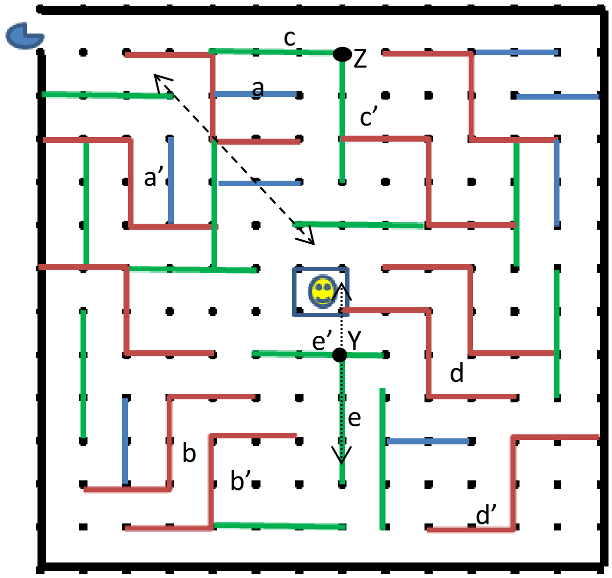
Now, play your partner's game. ☺

Instructional Task Exemplar Response

This is a sample response. This is an open-ended task, and students will create very different games that meet the criteria.

Pac-Man Finds Smiley





After you have created your game, complete the following about your game:

A. Identify a reflection you used in the game. Explain how you know this is a reflection.

$$a \rightarrow a' \, reflected\ over\ the\ dashed\ line.$$

Check student's explanation.

B. Identify a translation you used in the game. Explain how you know this is a translation.

$$b \rightarrow b' \, Translated\ down\ one, right\ one.$$

Check student's explanation.

C. Identify a rotation you used in the game. Explain how you know this is a rotation.

$$c \rightarrow c' \, Rotated\ 90^{\circ}\ counterclockwise\ about\ point\ Z.$$

Check student's explanation.

D. Identify and list the steps of two transformations that involved a series of reflections, translations, and/or rotations.

$$d \rightarrow d' \, Reflected\ over\ dotted\ line, then\ translated\ down\ 3\ and\ right\ 6.$$

$$e \rightarrow e' \, Rotated\ 90^{\circ} clockwise\ about\ point\ Y, then\ translated\ right\ one.$$

Next, exchange games with your partner. Do not show them your answers to parts A through D above. Using your partner's game, complete the following. After you identify the transformations, discuss your findings with your partner. *Solutions for this section will vary based on student observations.*

Tank Volume (IT)

Overview

This task provides students an opportunity to use their skill and fluency working with volume formulas for cones, cylinders, and spheres in a real-world context. Some of the questions in this task have specific, correct answers (#1, 2, 3, and 5), while other questions (#4 and 6) require students to make assumptions and design tanks to meet given specifications. The dimensions and volumes involve numbers that may be overwhelming to students at first, but as they persevere through the problems they should be able to get answers.

Standards

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Work with radicals and integer exponents.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.NS.A.2	• 7.NS.A.2d	1. Approximate the value of each expression: (sample answers are provided; other approximations are dependent upon the rational number used for the irrational value) a. $\pi + \pi$ i. Approximately 6.28 b. 10π i. Approximately 31.4 c. $\frac{4}{3}\pi$ i. Approximately 4 d. $\frac{1}{3}\pi(4)^2$ i. Approximately 16	• http://www.illustrativemathematics.org/illustrations/604 • http://www.illustrativemathematics.org/illustrations/593 • http://learnzillion.com/lessonsets/41-understand-rational-and-irrational-numbers

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		2. http://www.illustrativemathematics.org/illustrations/335 3. http://www.illustrativemathematics.org/illustrations/334 4. http://www.illustrativemathematics.org/illustrations/1538	
8.EE.A.2	• 6.EE.B.5 • 7.NS.A.3	1. Solve for x. For irrational answers provide both an exact and approximate answer. a. $x^2 = 16$ i. $x = 4$ b. $x^2 = 200$ i. $x = \sqrt{200} \approx 14.14$ c. $x^2 = \frac{25}{16}$ i. $x = \frac{5}{4}$ 2. Solve for x. For irrational answers provide both an exact and approximate answer. a. $x^3 = 27$ i. $x = 3$ b. $x^3 = 40$ i. $x = \sqrt[3]{40} \approx 3.42$ c. $x^3 = \frac{64}{125}$ i. $x = \frac{4}{5}$	• http://www.illustrativemathematics.org/illustrations/673 • http://www.illustrativemathematics.org/illustrations/298 • http://learnzillion.com/lessonsets/351-understand-and-evaluate-square-roots-and-cube-roots • http://learnzillion.com/lessonsets/45-understand-perfect-cubes-and-cube-roots • http://learnzillion.com/lessonsets/44-understand-perfect-squares-and-square-roots
8.G.C.9	• 8.EE.A.2	1. Write the formula that could be used to calculate the volume of a: a. Cylinder i. $V = \pi r^2 h$ b. Cone i. $V = \frac{1}{3}\pi r^2 h$ c. Sphere i. $V = \frac{4}{3}\pi r^2$ 2. http://www.illustrativemathematics.org/illustrations/520 3. http://www.illustrativemathematics.org/illustrations/517 4. http://www.illustrativemathematics.org/illustrations/521	• http://learnzillion.com/lessonsets/704-find-volumes-of-cones-cylinders-and-spheres • http://learnzillion.com/lessonsets/286-know-and-use-the-formulas-for-volumes-of-cones-cylinders-and-spheres

During the Task:

- Several of the questions in this task are multistep. For example, in #1 students must (a) find the volume of Tank 2 and (b) use this volume along with the provided volume of Tank 1 to determine how many times the volume of Tank 2 is the volume of Tank 1. On any of these questions, students may need help getting started. Questions like, “What is the problem asking you to find?”, “What information do you have?”, “What additional information do you need?”, etc., may help students get started.

- Some students may be intimidated by the arithmetic involved in this task. If students get stuck working with the arithmetic, remind students that they can reduce fractions before multiplying, encourage students to take the arithmetic one step at a time, or provide a calculator to weaker students.

After the Task:

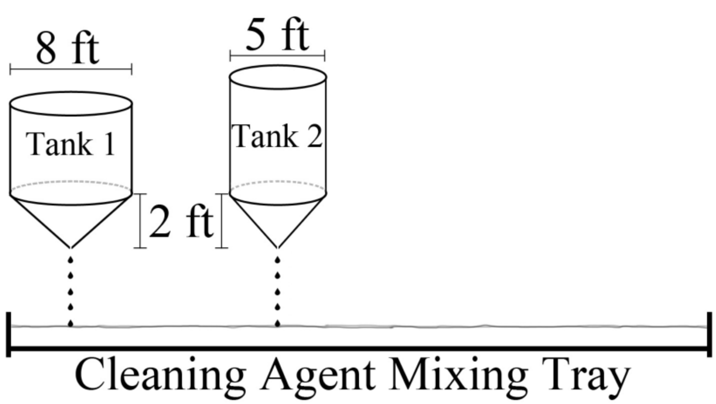
Ultimately, this is a design task. Students need to use what they know about the volume of cylinders, cones, and spheres to perform calculations and then to design tanks to meet specifications. This could be applied to any situation where students need to calculate or approximate volumes of known figures. For example, students may use some form of this in cooking to determine appropriate containers for drinks, soups, gravy, etc., when preparing food for a group of people.

Student Instructional Task

In response to an increase in demand for glass cleaner, the CEO of Shining Glass made the decision to open a new factory to manufacture more glass cleaner. You have been asked to help make the process more efficient. Four of the ingredients are cleaning agents, and these are mixed together before being added to the remaining ingredients.

Your first job is to put together the section of the factory where the four cleaning agents are mixed. When you started working on the project, two of the four tanks of cleaning agents had already been installed. Before deciding what to do with the other two tanks, you need to gather information about Tank 1 and Tank 2. Both tanks are comprised of a cylinder section and a cone section. Both cone sections have a height of 2 ft. The diameter of Tank 1 is 8 ft. and the diameter of Tank 2 is 5 ft. On the side of Tank 1 you found a label stating the volume of the <u>cylinder section only</u>: $V = 96\pi$ ft.^{3}. On the side of Tank 2 you found a label stating the volume of the <u>entire tank</u>: $V = \frac{325}{6}\pi$ ft.^{3}. (When approximating answers use $\pi \approx 3$).

(not necessarily drawn to scale)



1. How many times the total volume of Tank 2 is the total volume of Tank 1? Justify your answer.

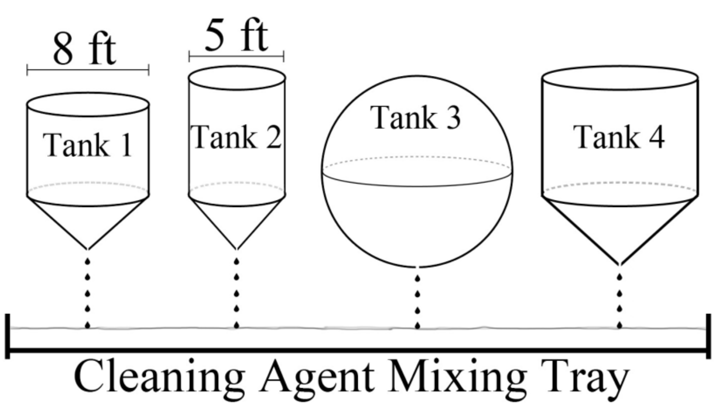
2. The volumes of Tank 1 and Tank 2 were chosen such that using a full tank of each will produce the correct ratio of the two liquids. Both Tank 1 and Tank 2 are set to constantly add liquid to the mixture. If Tank 1 is set to add 16 ft^3 of liquid to the mixture every hour, at what rate should Tank 2 add liquid in order to keep the correct ratio of the two liquids? Show how you found your answer.

The volume of Tank 2 is $\frac{325}{6}\pi$ ft.^{3} and the combined volume of Tank 1 and Tank 2 is $\frac{965}{6}\pi$ ft.^{3} . You now need to add two new tanks for the other two cleaning agents. The third liquid will be dispensed from a spherical tank, and the fourth liquid will be dispensed from a tank with the same basic shape as Tanks 1 and 2. Mixing full tanks of Tanks 1, 2, 3, and 4 will produce the correct ratio of the four liquids.

Tank 3: The cleaning agent mixture will contain more liquid from Tank 3 than from Tanks 1 and 2 combined. To maintain the correct ratio of liquids, the liquid used from Tank 3 should represent $\frac{2000}{965}$ times as much liquid as from Tanks 1 and 2 combined.

Tank 4: The volume of the <u>cone</u> section of Tank 4 is $\frac{10}{13}$ of the entire volume of Tank 2. The <u>cylinder</u> section of Tank 4 needs to hold six times as much liquid as the cone section of Tank 4.

(not necessarily drawn to scale)



3. What would the diameter of Tank 3 need to be? Show all work. *Include the exact diameter and an approximation of the diameter.*

4. What dimensions for Tank 4 would produce the required volume? Justify your answer.

5. Together Tanks 1 and 2 add $\frac{193}{8}\ ft^3$ of liquid to the mixing tray each hour. At what rate should Tank 3 and Tank 4 add liquid in order to keep the correct ratio of liquids? Justify your answer.

6. Two large cylinder-shaped holding tanks (Tank 5 and Tank 6) are used to store the cleaning agent mixture before it is added to the rest of the ingredients. These tanks need to be designed to hold approximately2500 ft^3 of liquid. Tank 6 will be turned sideways on top of Tank 5, so the height of Tank 6 should equal the diameter of Tank 5. Design these two tanks to meet the required specifications. *Show your work.*

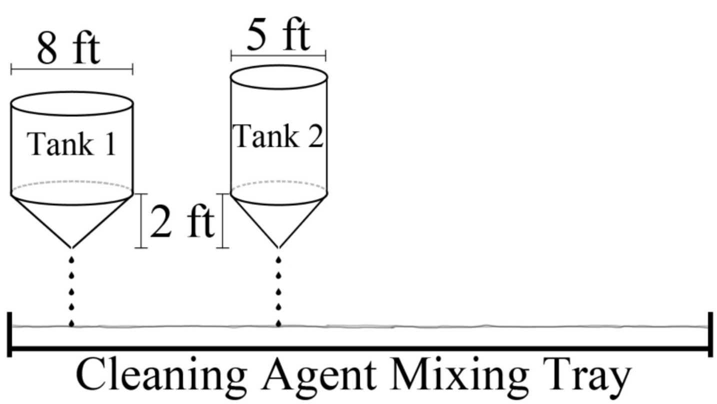
Task adapted with permission from Universal Achievement, LLC

Instructional Task Exemplar Response

In response to an increase in demand for glass cleaner, the CEO of Shining Glass made the decision to open a new factory to manufacture more glass cleaner. You have been asked to help make the process more efficient. Four of the ingredients are cleaning agents, and these are mixed together before being added to the remaining ingredients.

Your first job is to put together the section of the factory where the four cleaning agents are mixed. When you started working on the project, two of the four tanks of cleaning agents had already been installed. Before deciding what to do with the other two tanks, you need to gather information about Tank 1 and Tank 2. Both tanks are comprised of a cylinder section and a cone section. Both cone sections have a height of 2 ft. The diameter of Tank 1 is 8 ft. and the diameter of Tank 2 is 5 ft. On the side of Tank 1 you found a label stating the volume of the cylinder section only: $V = 96\pi$ ft.^{3}. On the side of Tank 2 you found a label stating the volume of the entire tank: $V = \frac{325}{6}\pi$ ft.^{3}. (When approximating answers use $\pi \approx 3$).

(not necessarily drawn to scale)



1. How many times the total volume of Tank 2 is the total volume of Tank 1? Justify your answer.

Volume of Tank 2: $\frac{325}{6}\pi \text{ ft}^3 \approx \frac{325}{6}(3) \text{ ft}^3 \approx \frac{325}{2} \text{ ft}^3$

Tank 1 (diameter = 8 ft, radius = 4 ft)

Cylinder section: $V = 96\pi \text{ ft}^3$

Cone section: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4 \text{ ft})^2(2 \text{ ft})$

$V = \frac{1}{3}\pi(16 \text{ ft}^2)(2 \text{ ft})$

$V = \frac{32}{3}\pi \text{ ft}^3$

Total Volume $= 96\pi \text{ ft}^3 + \frac{32}{3}\pi \text{ ft}^3$

$= \frac{288}{3}\pi \text{ ft}^3 + \frac{32}{3}\pi \text{ ft}^3$

$= \frac{320}{3}\pi \text{ ft}^3$

$\approx \frac{320}{3}(3) \text{ ft}^3 \approx 320 \text{ ft}^3$

Tank 1 has a volume of $\frac{320}{3}\pi \text{ ft}^3$ (approximately 320 ft^3) and Tank 2 has a volume of $\frac{325}{6}\pi \text{ ft}^3$ (approximately 162.5 ft^3).

(Volume of Tank 2)(x) = (Volume of Tank 1)

$\left(\frac{325}{6}\pi \text{ ft}^3\right)x = \frac{320}{3}\pi \text{ ft}^3$

$x = \dfrac{\frac{320}{3}\cancel{\pi}\,\cancel{\text{ft}^3}}{\frac{325}{6}\cancel{\pi}\,\cancel{\text{ft}^3}} = \frac{320}{3} \div \frac{325}{6} = \frac{\cancel{320}^{64}}{\cancel{3}_{1}} \cdot \frac{\cancel{6}^{2}}{\cancel{325}_{65}} = \frac{128}{65}$

Tank 1 has a volume $\frac{128}{65}$ times the volume of Tank 2.

2. The volumes of Tank 1 and Tank 2 were chosen such that using a full tank of each will produce the correct ratio of the two liquids. Both Tank 1 and Tank 2 are set to constantly add liquid to the mixture. If Tank 1 is set to add 16 $ft.^3$ of liquid to the mixture every hour, at what rate should Tank 2 add liquid in order to keep the correct ratio of the two liquids? Show how you found your answer.

$$\left(\text{Volume of Tank 2}\right)\left(\frac{128}{65}\right) = \left(\text{Volume of Tank 1}\right)$$

$$\left(\text{Rate for Tank 2}\right)\left(\frac{128}{65}\right) = \left(\text{Rate for Tank 1}\right)$$

$$\text{Rate for Tank 2} = \left(16\ \frac{ft^3}{hr}\right)\cdot\left(\frac{65}{128}\right)$$

$$\text{Rate for Tank 2} = \frac{(\overset{1}{\cancel{16}})(65)}{\underset{8}{\cancel{128}}}\ \frac{ft^3}{hr}$$

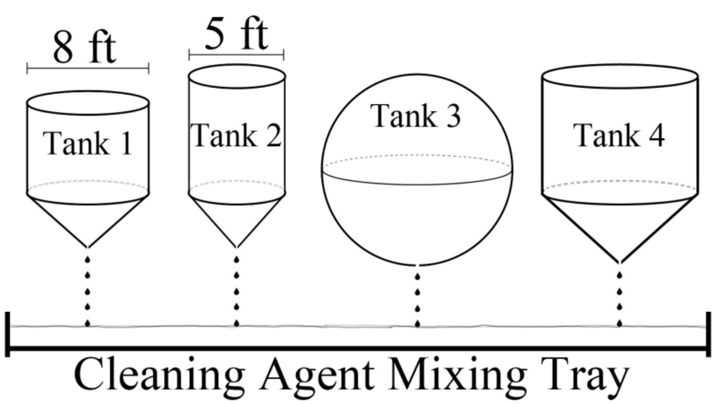
$$\text{Rate for Tank 2} = \frac{65}{8}\ \frac{ft^3}{hr}$$

The volume of Tank 2 is $\frac{325}{6}\pi$ ft.^{3} and the combined volume of Tank 1 and Tank 2 is $\frac{965}{6}\pi$ ft.^{3} . You now need to add two new tanks for the other two cleaning agents. The third liquid will be dispensed from a spherical tank, and the fourth liquid will be dispensed from a tank with the same basic shape as Tanks 1 and 2. Mixing full tanks of Tanks 1, 2, 3, and 4 will produce the correct ratio of the four liquids.

Tank 3: The cleaning agent mixture will contain more liquid from Tank 3 than from Tanks 1 and 2 combined. To maintain the correct ratio of liquids, the liquid used from Tank 3 should represent $\frac{2000}{965}$ times as much liquid as from Tanks 1 and 2 combined.

Tank 4: The volume of the <u>cone</u> section of Tank 4 is $\frac{10}{13}$ of the entire volume of Tank 2. The <u>cylinder</u> section of Tank 4 needs to hold six times as much liquid as the cone section of Tank 4.

(not necessarily drawn to scale)



3. What would the diameter of Tank 3 need to be? Show all work. *Include the exact diameter and an approximation of the diameter.*

Tank 3 Volume $= \left(\frac{\overset{1000}{\cancel{2000}}}{\underset{1}{\cancel{965}}}\right)\left(\frac{\overset{1}{\cancel{965}}}{\underset{3}{\cancel{6}}}\pi \text{ ft}^3\right) = \frac{1000}{3}\pi \text{ ft}^3$

For a sphere: $V = \frac{4}{3}\pi r^3$

$\frac{1000}{3}\pi \text{ ft}^3 = \frac{4}{3}\pi r^3$

$\frac{\overset{250}{\cancel{1000}}\overset{1}{\cancel{\pi}}}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1\ 1}{\cancel{4\pi}}} \text{ ft}^3 = r^3$

$250 \text{ ft}^3 = r^3$

Diameter

$r = \sqrt[3]{250} \text{ ft} \approx 6.3 \text{ ft} \longrightarrow 2\sqrt[3]{250} \text{ ft} \approx 12.6 \text{ ft}$

4. What dimensions for Tank 4 would produce the required volume? Justify your answer.

***Note: The work shown here is a sample answer. Students may find other dimensions that work, as long as they can justify their answer.*

Cone section

$$\frac{\cancel{10}^{5}}{\cancel{13}_{1}}\left(\frac{\cancel{325}^{25}}{\cancel{6}_{3}}\pi\ ft^3\right) = \frac{125}{3}\pi\ ft^3$$

Cylinder Section

$$\cancel{6}^{2}\left(\frac{125}{\cancel{3}_{1}}\pi\ ft^3\right) = 250\pi\ ft^3$$

Different combinations of heights & radii could work. I will use a radius of 5 ft.

Cone section: $V = \frac{125}{3}\pi\ ft^3 = \frac{1}{3}\pi(5\ ft)^2 h$

$$h = \frac{\frac{125}{3}\pi\ ft^3}{\frac{25}{3}\pi\ ft^2} = \underline{5\ ft}$$

Cylinder Section: $V = 250\pi\ ft^3 = \pi(5\ ft)^2 h$

$$h = \frac{250\pi\ ft^3}{25\pi\ ft^2} = \underline{10\ ft}$$

The radius could be 5 feet, and then the height of the cone section will be 5 feet and the height of the cylinder section would be 10 ft.

5. Together Tanks 1 and 2 add $\frac{193}{8}\ ft.^3$ of liquid to the mixing tray each hour. At what rate should Tank 3 and Tank 4 add liquid in order to keep the correct ratio of liquids? Justify your answer.

Tank 3

$$\frac{\text{Volume of Tank 3}}{\text{Volume of Tanks 1+2}} = \frac{\text{Rate for Tank 3}}{\text{Rate for Tanks 1+2}} \rightarrow \frac{\frac{1000}{3}\cancel{\pi\ ft^3}}{\frac{965}{6}\cancel{\pi\ ft^3}} = \frac{x}{\frac{193}{8}\ \frac{ft^3}{hr}}$$

$$x = \frac{1000}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{965}}} \cdot \frac{\overset{1}{\cancel{193}}}{8}\ \frac{ft^3}{hr} = \frac{(\overset{125}{\cancel{1000}})(2)}{(5)(\underset{1}{\cancel{8}})}\ \frac{ft^3}{hr} = \frac{(\overset{25}{\cancel{125}})(2)}{\underset{1}{\cancel{5}}}\ \frac{ft^3}{hr} = \boxed{50\ \frac{ft^3}{hr}}$$

Tank 4

$$\text{Volume} = \frac{125}{3}\pi\ ft^3 + 250\ \pi\ ft^3 = \frac{875}{3}\pi\ ft^3$$

$$\frac{\text{Volume of Tank 4}}{\text{Volume of Tanks 1+2}} = \frac{\text{Rate for Tank 4}}{\text{Rate for Tanks 1+2}} \rightarrow \frac{\frac{875}{3}\cancel{\pi\ ft^3}}{\frac{965}{6}\cancel{\pi\ ft^3}} = \frac{x}{\frac{193}{8}\ \frac{ft^3}{hr}}$$

$$x = \frac{875}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{965}}} \cdot \frac{\overset{1}{\cancel{193}}}{8}\ \frac{ft^3}{hr} = \frac{(\overset{175}{\cancel{875}})(\overset{1}{\cancel{2}})}{(\underset{1}{\cancel{5}})(\underset{4}{\cancel{8}})}\ \frac{ft^3}{hr} = \boxed{\frac{175}{4}\ \frac{ft^3}{hr}}$$

6. Two large cylinder-shaped holding tanks (Tank 5 and Tank 6) are used to store the cleaning agent mixture before it is added to the rest of the ingredients. These tanks need to be designed to hold approximately2500 $ft.^3$ of liquid. Tank 6 will be turned sideways on top of Tank 5, so the height of Tank 6 should equal the diameter of Tank 5. Design these two tanks to meet the required specifications. *Show your work.*

Tank 5

Assume a radius of 8 feet and a height of 10 feet.

$$V = \pi(8\text{ ft})^2(10\text{ ft}) = \underline{640\pi\text{ ft}^3}$$

This is an approximate volume of $640(3)\text{ ft}^3 = 1920\text{ ft}^3$

Tank 6 needs to hold approximately $(2500-1920)\text{ ft}^3$ of liquid and have a height of 16 ft.

$$580\text{ ft}^3 = \pi(r)^2(16\text{ ft})$$

$$\frac{580\text{ ft}^3}{16\pi\text{ ft}} = r^2$$

$$r^2 \approx \frac{\overset{145}{\cancel{580}}}{(\underset{4}{\cancel{16}})(3)}\text{ ft}^2 = \frac{145}{12}\text{ ft}^2$$

$$r \approx \sqrt{\frac{145}{12}\text{ ft}^2} \approx \sqrt{12.1}\text{ ft} \approx 3.5\text{ ft}$$

These dimensions should result in a combined volume approximately equal to 2500 ft^3.

Tank 5

Radius: 8 feet

Height: 10 feet

Tank 6

Radius: 3.5 feet

Height: 16 feet

Pythagorean Theorem Proof (IT)

Overview

Students will demonstrate their ability to work through and explain a proof of the Pythagorean Theorem. This task does not address the converse of the Pythagorean Theorem. The task walks the student step-by-step through a proof of the Pythagorean Theorem. To end the task, students pull all of the steps together to explain the proof using the provided diagram.

Students will cut out figures to complete this proof. The pieces will either need to be pre-cut, or students will need to cut them out. Keeping everything neat and together is easier if the students glue their figures to a separate sheet of paper. Provide students with additional paper and glue sticks as needed.

Standards

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.G.B.6	• 7.G.B.6	1. You find a triangle on your desk. Answer the following questions: a. How can you tell if it is a right triangle using only a protractor? i. Measure the angles. If the triangle has a 90° angle, then the triangle is a right triangle. b. If it is a right triangle, which side will be the longest? i. The side across from the biggest angle (the hypotenuse, because it is across from the right angle.) 2. How can you find the area of a rectangle? a. Multiply the base by the height 3. How can you find the area of a square? a. Square the length of one side OR multiply the base by the height 4. Given the area of a square, how can you find the length of each side? a. Find the square root of the area 5. http://www.illustrativemathematics.org/illustrations/724	• http://learnzillion.com/lessonsets/450-explain-a-proof-of-the-pythagorean-theorem-and-its-converse • http://learnzillion.com/lessonsets/279-prove-and-apply-the-pythagorean-theorem-to-determine-unknown-side-lengths-in-righttriangles

During the Task:

The teacher should monitor the progress of students/groups to make sure:

1. Students are using the correct pieces and arranging them correctly as they go through the proof.
2. Students are correctly labeling and determining lengths in terms of a, b, and/or c.
3. Students are correctly calculating areas and relating areas of Figures 1 and 3.

Toward the end of the task, students are asked if this proof could be generalized to other types of triangles. The anticipated response is not a formal proof of why this could or could not work. Students can informally answer this question using what they know about types of triangles. For example, the same sets of figures could not be formed with acute or obtuse triangles, as this proof depends on making rectangles and squares using repeated copies of the same triangle. Putting together multiple copies of the same acute or obtuse triangle will form parallelograms, not rectangles. This connection between parallelograms and triangles is first developed in 6^{th} grade as students start to investigate the area formulas of parallelograms and triangles.

After the Task:

If this task is used when students have had limited exposure to the Pythagorean Theorem, students may later be asked to use the Pythagorean Theorem to find missing side lengths. For then they will have established why the Pythagorean Theorem is true.

For students who are having trouble really understanding the proof with the variables a, b, and c, the teacher could ask them to walk through this proof again with known side lengths that are Pythagorean triples like 3, 4, and 5 or 7, 24, and 25.

The teacher should relate the usefulness of the Pythagorean Theorem to any real-world situation involving right triangles, for example, building a truss, constructing a frame (with a diagonal support) as the foundation of a stage or box, calculating the required length of a cable to support a cell tower, etc.

Student Instructional Task

As you work through this lesson, use the Pythagorean Theorem Proof page at the end of this lesson.

1. Cut out all of the shapes (including the Area and Figure boxes) on the Pythagorean Theorem Proof page at the end of the lesson.
2. You should now have eight congruent triangles cut out. These triangles are all right triangles. Draw a right angle on each triangle to indicate which of the three angles is the right angle.
3. Label the hypotenuse of each triangle as length “c,” label the longer leg of each triangle “b,” and label the shorter leg of each triangle “a.”
4. Determine and label the side lengths of the two largest squares in terms of “a,” “b,” and/or “c.”
5. Arrange four triangles and one square in the configuration below, and place the “Figure 1” label under the shapes. Tape or glue these to a separate sheet of paper.

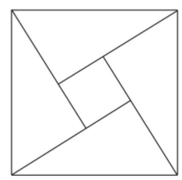


Figure 1

Is Figure 1 a square? *Support your answer with evidence about both the side lengths and angle measures.*

6. Place one of the “Area:” rectangles under Figure 1, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 1.

7. Leave Figure 1 arranged as it is. There should be four more triangles and one more square that are the same sizes as the five shapes used in Figure 1. Arrange these in the configuration below, and place the "Figure 2" label under the shapes. Tape or glue this next to Figure 1.

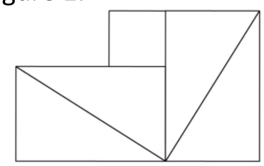
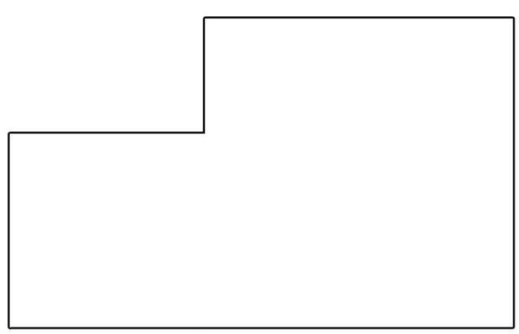


Figure 2

8. Place one of the "Area:" rectangles under Figure 2, and write the area in terms of "a," "b," and/or "c" on the "Area:" rectangle. Explain here how you determined the area of Figure 2.

9. Is the total area of Figure 2 larger, smaller, or the same as the total area of Figure 1? Provide evidence to support your answer.

10. The perimeter of Figure 2 should be shaped like the diagram below.



Determine the lengths of all 6 sides of this diagram in terms of “a,” “b,” and/or “c.” Label all of these lengths on the diagram above.

11. Arrange the two remaining squares in the configuration below, and place the “Figure 3” label under the shapes. Tape or glue these next to Figure 2.

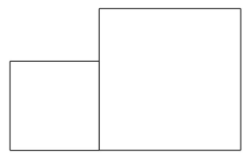


Figure 3

12. Place one of the “Area:” rectangles under Figure 3, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 3.

13. Compare the perimeters of Figure 2 and Figure 3.

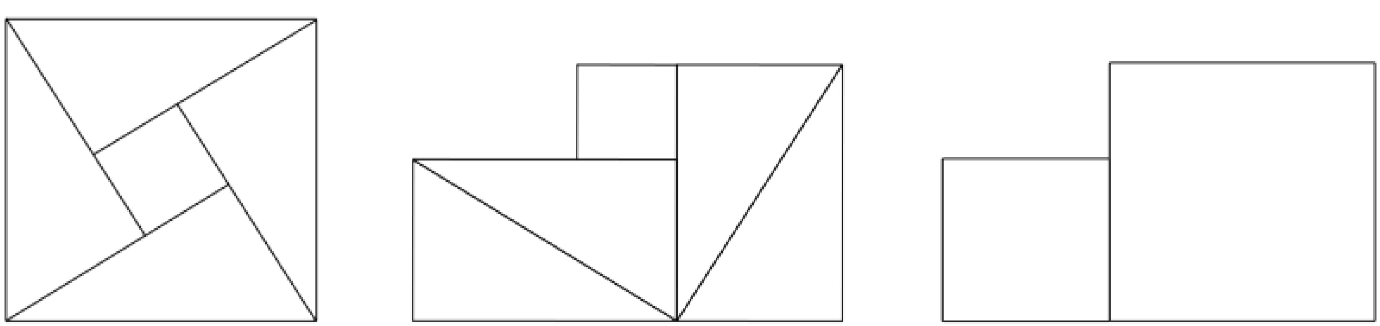
14. Compare the total areas of Figures 1, 2, and 3.

15. Write an equation relating your expression for the area of Figure 1 and your expression for the area of Figure 3. How do these areas and this equation support what you know about the Pythagorean Theorem?

16. Make a conjecture about why this same process could not be repeated with acute or obtuse triangles.

17. Would your equation from Problem #15 be true for acute or obtuse triangles? *Explain.*

18. The basic shapes from this proof are shown below. Use these figures to explain a proof of the Pythagorean Theorem. Label any side lengths needed for the proof, and write explanations below the figures.

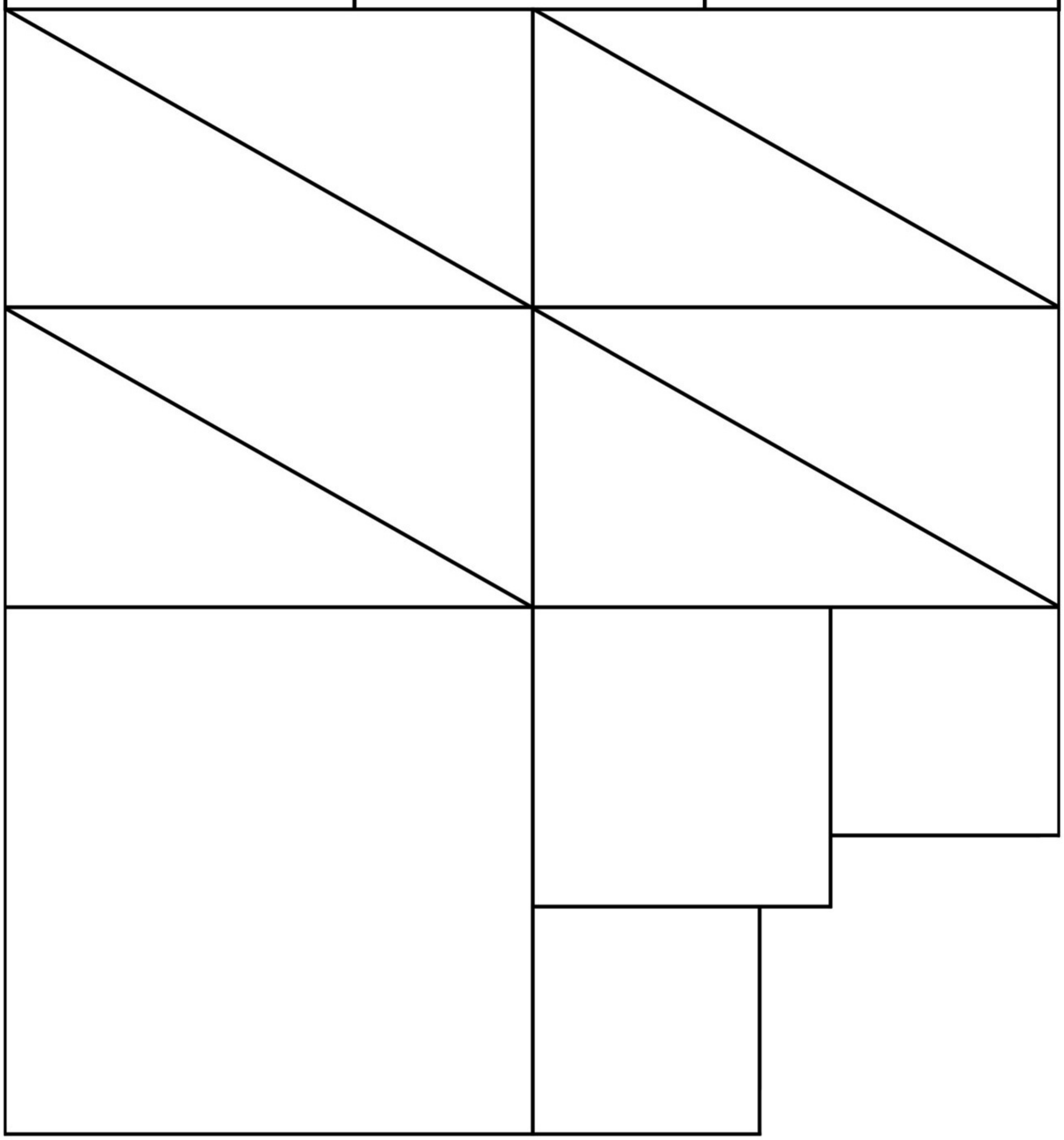


Task adapted with permission from Universal Achievement, LLC

Pythagorean Theorem Proof

(cut out all of the shapes below)

Figure 1	Figure 2	Figure 3
Area:	Area:	Area:



Instructional Task Exemplar Response

As you work through this lesson, use the Pythagorean Theorem Proof page at the end of this lesson.

1. Cut out all of the shapes (including the "Area" and "Figure" boxes) on the Pythagorean Theorem Proof page at the end of the lesson.
2. You should now have eight congruent triangles cut out. These triangles are all right triangles. Draw a right angle on each triangle to indicate which of the three angles is the right angle.
3. Label the hypotenuse of each triangle as length "c," label the longer leg of each triangle "b," and label the shorter leg of each triangle "a."
4. Determine and label the side lengths of the two largest squares in terms of "a," "b," and/or "c."
5. Arrange four triangles and one square in the configuration below, and place the "Figure 1" label under the shapes. Tape or glue these to a separate sheet of paper.

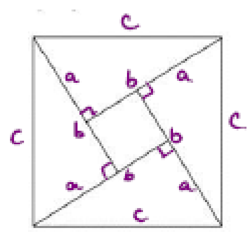
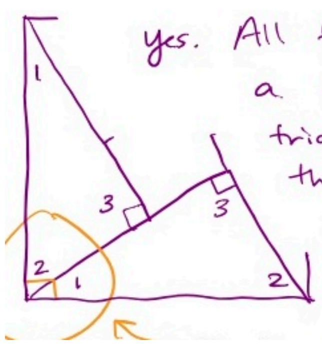


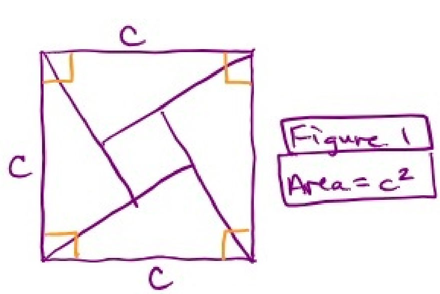
Figure 1

Is Figure 1 a square? *Support your answer with evidence about both the side lengths and angle measures.*



yes. All four sides have a side length of "c." Also, to be a square, the corners must be right angles. Since the triangles are congruent, the angles in each triangle must have the same measure. If ∠3 is a right angle m∠1 + m∠2 must add to 90° (to get 180° total degrees). If m∠1+m∠2 = 90°, then each corner of the square must be a right angle.

6. Place one of the "Area:" rectangles under Figure 1, and write the area in terms of "a," "b," and/or "c" on the "Area:" rectangle. Explain here how you determined the area of Figure 1.



Since Figure 1 is a square, and all sides have a length of "c" the area is:

Area = $c \cdot c = c^2$

7. Leave Figure 1 arranged as it is. There should be four more triangles and one more square that are the same sizes as the five shapes used in Figure 1. Arrange these in the configuration below, and place the “Figure 2” label under the shapes. Tape or glue this next to Figure 1.

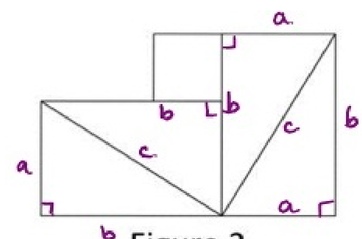
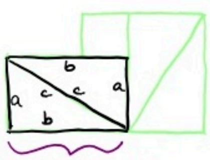
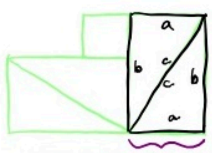


Figure 2

8. Place one of the “Area:” rectangles under Figure 2, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 2.

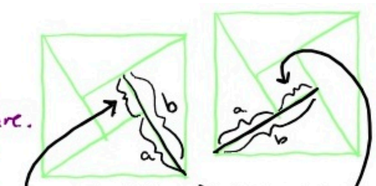


Rectangle with side lengths of b + a.
Area = (a)(b) = ab



Also a rectangle with side lengths of a + b
Area = (a)(b) = ab

Now, I just need the area of the little square. Looking back to Figure 1 (since the little square is the same size)...



this little piece must be the difference between b + a, so "b − a". The area must be (b − a)(b − a)

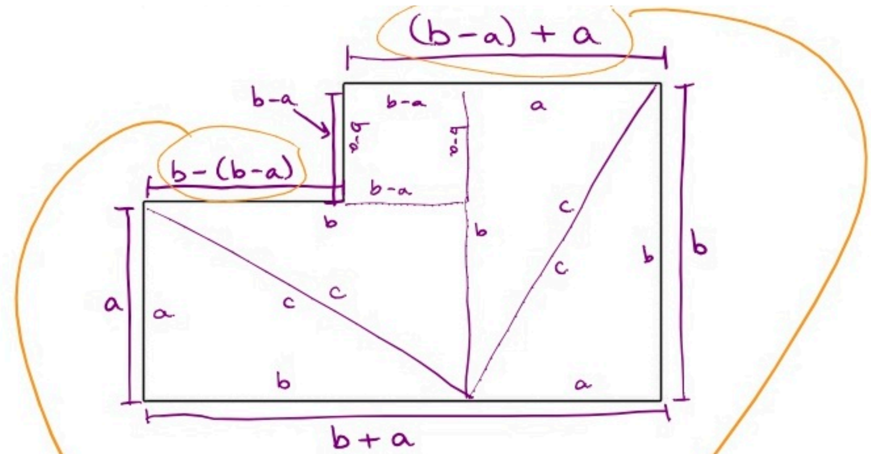
Total Area

$ab + ab + (b-a)(b-a)$

9. Is the total area of Figure 2 larger, smaller, or the same as the total area of Figure 1? Provide evidence to support your answer.

The same. Even though the expressions don't look the same (c^2 vs. $2ab + (b-a)(b-a)$), the areas must be the same because the two figures are made up of the same 5 shapes.

10. The perimeter of Figure 2 should be shaped like the diagram below.



Determine the lengths of all 6 sides of this diagram in terms of “a”, “b”, and/or “c”. Label all of these lengths on the diagram above.

$b-(b-a)=$
$=b-b+a$
$=a$

$(b-a)+a=$
$=b-a+a$
$=b$

11. Arrange the two remaining squares in the configuration below, and place the “Figure 3” label under the shapes. Tape or glue these next to Figure 2.

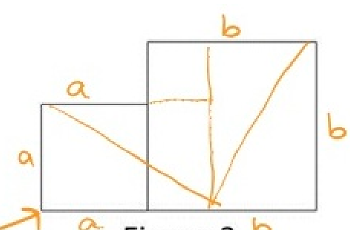
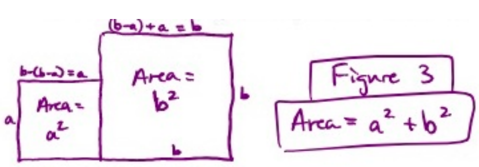


Figure 3

Same size shape as figure 2

12. Place one of the “Area:” rectangles under Figure 3, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 3.

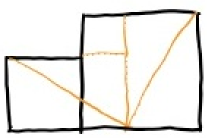


I used the perimeters from #10 to figure out the side lengths of these two squares. Then, I held the squares next to the sides of one triangle to verify that the side lengths were indeed "a" + "b"

13. Compare the perimeters of Figure 2 and Figure 3.

The perimeters are the same. Each side on the outer edge of Figure 1 has the same length as the corresponding side on figure 2.

14. Compare the total areas of Figures 1, 2, and 3.



All three areas must be exactly the same. Figure 3 could be divided like Figure 2 and all of the lengths are what they are supposed to be.

15. Write an equation relating your expression for the area of Figure 1 and your expression for the area of Figure 3. How do these areas and this equation support what you know about the Pythagorean Theorem?

$c^2 = a^2 + b^2$

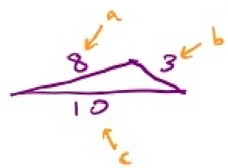
This is the Pythagorean theorem. With any right triangles I should be able to repeat this procedure, so $c^2 = a^2 + b^2$ should be true for any right triangles.

16. Make a conjecture about why this same process could not be repeated with acute or obtuse triangles.
***Note: This is a sample response. Other explanations are acceptable if they are grounded in solid mathematical reasoning.*

With acute or obtuse triangles these configurations could not be made. Right angles could not be formed to create rectangles + squares like we did above.

17. Would your equation from Problem #15 be true for acute or obtuse triangles? *Explain.*
***Note: This is a sample response. Other explanations are acceptable if they are grounded in solid mathematical reasoning.*

Not necessarily. These same figures could not be made, so the equation may not be true. But, looking at a non-right triangle,



$8^2 + 3^2 \stackrel{?}{=} 10^2$

$64 + 9 \stackrel{?}{=} 100$

$73 \neq 100$

Does not work

18. The basic shapes from this proof are shown below. Use these figures to explain a proof of the Pythagorean Theorem. Label any side lengths needed for the proof, and write explanations below the figures.

***Note: This is a sample explanation of this proof. Other explanations should be reviewed for accuracy and conceptual understanding of the Pythagorean Theorem.*

Let all triangles be congruent right triangles with side lengths a, b, + c.

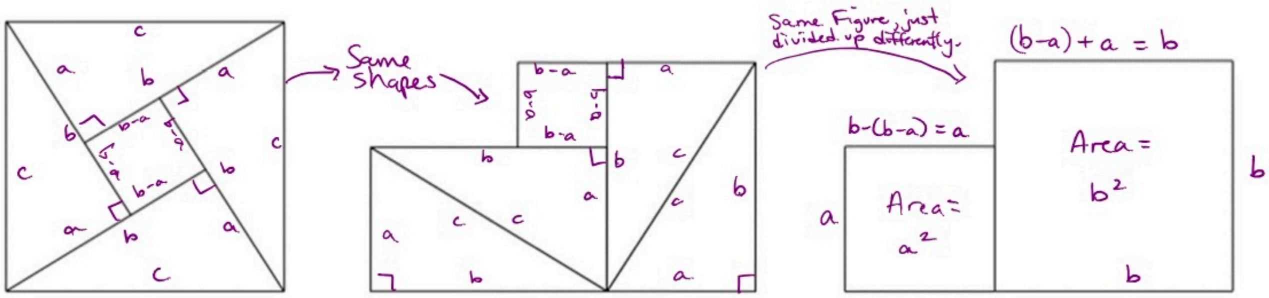


Figure 1 $\boxed{\text{Area} = c^2}$

Figure 2 $\boxed{\text{Area} = 2ab + (b-a)(b-a)}$

Figure 3 Area $= a^2 + b^2$

All three figures must have the same area, therefore

$$c^2 = 2ab + (b-a)(b-a) = a^2 + b^2$$

and

$$\underline{c^2 = a^2 + b^2}$$

APPENDIX

Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a+b)(x+y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a+b+c)(x+y). Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

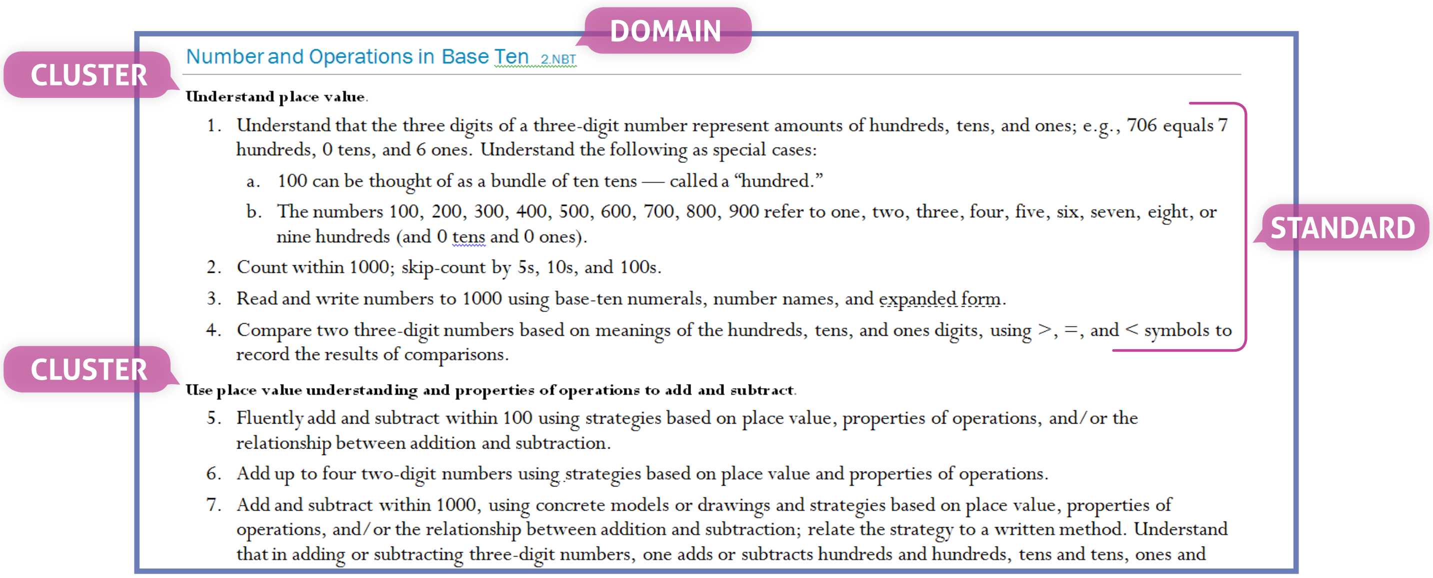
The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y–2)/(x–1)=3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

In Grade 6, instructional time should focus on four critical areas:

1) Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
2) Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;
3) Writing, interpreting, and using expressions and equations; and
4) Developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practices</th></tr>
<tr><td>Ratios and Proportional Relationships</td><td>• Understand ratio concepts and use ratio reasoning to solve problems.</td><td rowspan="5">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>The Number System</td><td>• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
• Compute fluently with multi-digit numbers and find common factors and multiples.
• Apply and extend previous understandings of numbers to the system of rational numbers.</td></tr>
<tr><td>Expressions and Equations</td><td>• Apply and extend previous understandings of arithmetic to algebraic expressions.
• Reason about and solve one-variable equations and inequalities.
• Represent and analyze quantitative relationships between dependent and independent variables.</td></tr>
<tr><td>Geometry</td><td>• Solve real-world and mathematical problems involving area, surface area, and volume.</td></tr>
<tr><td>Statistics and Probability</td><td>• Develop understanding of statistical variability.
• Summarize and describe distributions.</td></tr>
</table>

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[8]

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (⅔) ÷ (¾) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (⅔) ÷ (¾) = 8/9 because ¾ of 8/9 is ⅔. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many ¾-cup servings are in ⅔ of a cup of yogurt? How wide is a rectangular strip of land with length ¾ mi and area ½ square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9+2).*

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

[8] Expectations for unit rates in this grade are limited to non-complex fractions.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.*
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.*
   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.*
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8+7)$ as a product of two factors; view $(8+7)$ as both a single entity and a sum of two terms.*
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V=s^3$ and $A=6\,s^2$ to find the volume and surface area of a cube with sides of length $s=½$.*
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3\,(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6\,(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

In Grade 7, instructional time should focus on four critical areas:

1) Developing understanding of and applying proportional relationships;
2) Developing understanding of operations with rational numbers and working with expressions and linear equations;
3) Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
4) Drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7 Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practices</th></tr>
<tr><td>Ratios and Proportional Relationships</td><td>• Analyze proportional relationships and use them to solve real-world and mathematical problems.</td><td rowspan="5">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>The Number System</td><td>• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</td></tr>
<tr><td>Expressions and Equations</td><td>• Use properties of operations to generate equivalent expressions.
• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</td></tr>
<tr><td>Geometry</td><td>• Draw, construct and describe geometrical figures and describe the relationships between them.
• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</td></tr>
<tr><td>Statistics and Probability</td><td>• Use random sampling to draw inferences about a population.
• Draw informal comparative inferences about two populations.
• Investigate chance processes and develop, use, and evaluate probability models.</td></tr>
</table>

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
e. Solve real-world and mathematical problems involving the four operations with rational numbers.[9]

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

    a. Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
    b. Solve word problems leading to inequalities of the form $px+q>r$ or $px+q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

[9] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

   a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

   b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Grade 8 Mathematics Standards

In Grade 8, instructional time should focus on three critical areas:

1) Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
2) Grasping the concept of a function and using functions to describe quantitative relationships;
3) Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount mA. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practices</th></tr>
<tr><td>The Number System</td><td>• Know that there are numbers that are not rational, and approximate them by rational numbers.</td><td rowspan="5">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Expressions and Equations</td><td>• Work with radicals and integer exponents.
• Understand the connections between proportional relationships, lines, and linear equations.
• Analyze and solve linear equations and pairs of simultaneous linear equations.</td></tr>
<tr><td>Functions</td><td>• Define, evaluate, and compare functions.
• Use functions to model relationships between quantities.</td></tr>
<tr><td>Geometry</td><td>• Understand congruence and similarity using physical models, transparencies, or geometry software.
• Understand and apply the Pythagorean Theorem.
• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.</td></tr>
<tr><td>Statistics and Probability</td><td>• Investigate patterns of association in bivariate data.</td></tr>
</table>

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3–5 = 3–3 = \frac{1}{3}3 = \frac{1}{2}7$.

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
    c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[10]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two *(x, y)* values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[10] Function notation is not required in Grade 8.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*