

Inequalities



ESSENTIAL QUESTION

How can you use inequalities to solve real-world problems?

MODULE



7

LESSON 7.1

Writing and Solving One-Step Inequalities

COMMON CORE 7.EE.4b

LESSON 7.2

Writing Two-Step Inequalities

COMMON CORE 7.EE.4

LESSON 7.3

Solving Two-Step Inequalities

COMMON CORE 7.EE.4b



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Real-World Video

Many school groups and other organizations hold events to raise money. Members can write and solve inequalities to represent the financial goals they are trying to achieve.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Inverse Operations

EXAMPLE

$$3x = 24$$

$$\frac{3x}{3} = \frac{24}{3}$$

$$x = 8$$

x is multiplied by 3.

Use the inverse operation, division.

Divide both sides by 3.

$$z + 6 = 4$$

$$\underline{-6} = \underline{-6}$$

$$z = -2$$

6 is added to z .

Use the inverse operation, subtraction.

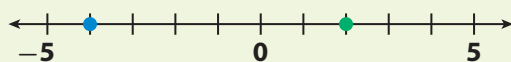
Subtract 6 from both sides.

Solve each equation, using inverse operations.

1. $9w = -54$ _____ 2. $b - 12 = 3$ _____ 3. $\frac{n}{4} = -11$ _____

Locate Points on a Number Line

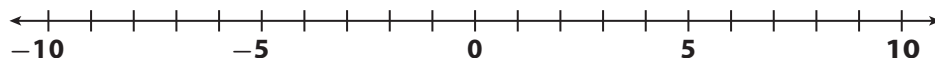
EXAMPLE



Graph $+2$ by starting at 0 and counting 2 units to the right.

Graph -4 by starting at 0 and counting 4 units to the left.

Graph each number on the number line.



4. 3

5. -9

6. 7

7. -3

Integer Operations

EXAMPLE

$$-7 - (-4) = -7 + 4$$

$$= |-7| - |4|$$

$$= 7 - 4, \text{ or } 3$$

$$= -3$$

To subtract an integer, add its opposite.

The signs are different, so find the difference of the absolute values.

Use the sign of the number with the greater absolute value.

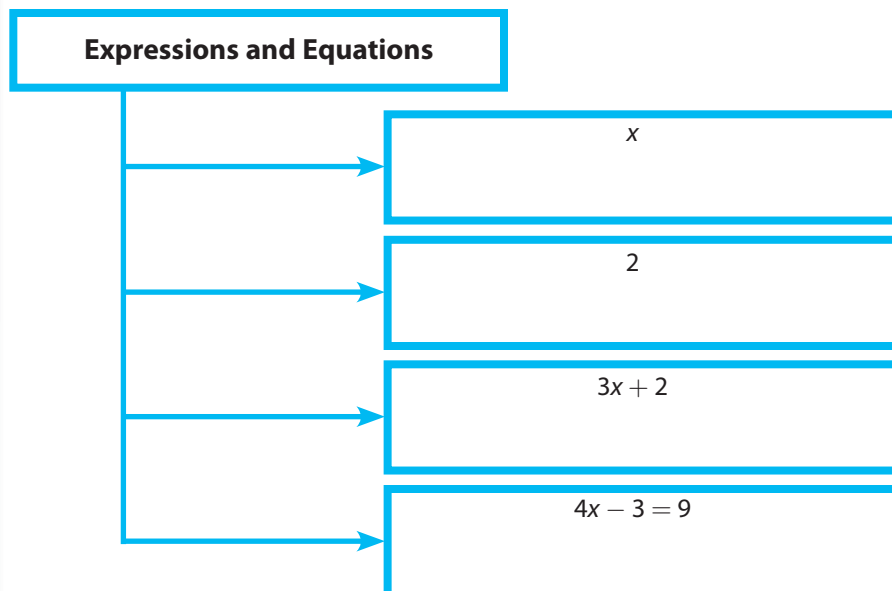
8. $3 - (-5)$ _____ 9. $-4 - 5$ _____ 10. $6 - 10$ _____ 11. $-5 - (-3)$ _____

12. $8 - (-8)$ _____ 13. $9 - 5$ _____ 14. $-3 - 9$ _____ 15. $0 - (-6)$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the graphic. You may put more than one word in each box.



Vocabulary

Review Words

- ✓ algebraic expression (*expresión algebraica*)
- coefficient (*coeficiente*)
- ✓ constant (*constante*)
- ✓ equation (*ecuación*)
- greater than (*mayor que*)
- ✓ inequality (*desigualdad*)
- integers (*enteros*)
- less than (*menor que*)
- operations (*operaciones*)
- solution (*solución*)
- ✓ variable (*variable*)

Understand Vocabulary

Complete each sentence, using the review words.

1. A value of the variable that makes the equation true is a _____.
2. The set of all whole numbers and their opposites are _____.
3. An _____ is an expression that contains at least one variable.

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the module, "Inequalities." Then label each flap with one of the lesson titles in this module. As you study each lesson, write important ideas, such as vocabulary and processes, under the appropriate flap.





MODULE 7

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON
CORE

7.EE.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Key Vocabulary

inequality (*desigualdad*)

A mathematical sentence that shows that two quantities are not equal.

What It Means to You

You will write an inequality to solve a real-world problem.

UNPACKING EXAMPLE 7.EE.4

To rent a certain car for a day costs \$39 plus \$0.29 for every mile the car is driven. Write an inequality to show the maximum number of miles you can drive and keep the rental cost under \$100.

The expression for the cost of the rental is $39 + 0.29m$. The total cost of the rental must be under \$100. So the inequality is as shown.

$$39 + 0.29m < 100$$



COMMON
CORE

7.EE.4b

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Key Vocabulary

solution (*solución*)

The value(s) for the variable that makes the inequality true.

What It Means to You

You will solve inequalities that involve two steps and interpret the solutions.

UNPACKING EXAMPLE 7.EE.4b

Solve and graph the solution of $-3x + 7 > -8$.

$$-3x + 7 > -8$$

$$-3x > -7 - 8 \quad \text{Subtract 7 from both sides.}$$

$$-3x > -15 \quad \text{Simplify.}$$

$$x < 5 \quad \text{Divide both sides by } -5, \text{ and reverse the inequality.}$$



All numbers less than 5 are solutions for this inequality.



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Writing and Solving One-Step Inequalities

Use variables to represent quantities in a real-world or mathematical problem, and construct simple ... inequalities to solve problems by reasoning about the quantities.



ESSENTIAL QUESTION

How do you write and solve one-step inequalities?

EXPLORE ACTIVITY

COMMON
CORE

Prep. for 7.EE.4b

Investigating Inequalities

You know that when you perform any of the four basic operations on both sides of an equation, the resulting equation is still true. What effect does performing these operations on both sides of an *inequality* have?

A Complete the table.

Inequality	Add to both sides:	New Inequality	Is new inequality true or false?
$2 \geq -3$	3		
$-1 \leq 6$	-1		
$-8 > -10$	-8		



Reflect

- Make a Conjecture** When you add the same number to both sides of an inequality, is the inequality still true? Explain how you know that your conjecture holds for *subtracting* the same number.

B Complete the table.

Inequality	Divide both sides by:	New Inequality	Is new inequality true or false?
$4 < 8$	4		
$12 \geq -15$	3		
$-16 \leq 12$	-4		
$15 > 5$	-5		

What do you notice when you divide both sides of an inequality by the same negative number?

Reflect

2. **Make a Conjecture** What could you do to make the inequalities that are not true into true statements?

3. **Communicate Mathematical Ideas** Explain how you know that your conjecture holds for multiplying both sides of an inequality by a negative number.



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Solving Inequalities Involving Addition and Subtraction

You can use properties of inequality to solve inequalities involving addition and subtraction with rational numbers.

Addition and Subtraction Properties of Inequality

Addition Property of Inequality

You can add the same number to both sides of an inequality and the inequality will remain true.

Subtraction Property of Inequality

You can subtract the same number from both sides of an inequality and the inequality will remain true.

EXAMPLE 1

COMMON CORE 7.EE.4b

Solve each inequality. Graph and check the solution.

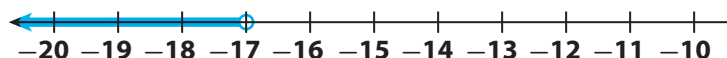
A $x + 5 < -12$

STEP 1 Solve the inequality.

$$x + 5 < -12 \quad \text{Use the Subtraction Property of Inequality.}$$

$$\frac{-5}{x} < \frac{-5}{-17} \quad \text{Subtract 5 from both sides.}$$

STEP 2 Graph the solution.



STEP 3 Check the solution. Substitute a solution from the shaded part of your number line into the original inequality.

$$-18 + 5 \stackrel{?}{<} -12 \quad \text{Substitute } -18 \text{ for } x \text{ into } x + 5 < -12.$$

$$-13 < -12 \quad \text{The inequality is true.}$$



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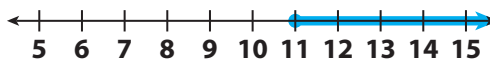
B $8 \leq y - 3$

STEP 1 Solve the inequality.

$$\begin{array}{r} 8 \leq y - 3 \\ + 3 \quad + 3 \\ \hline 11 \leq y \end{array}$$

Use the Addition Property of Inequality.
Add 3 to both sides.
You can rewrite $11 \leq y$ as $y \geq 11$.

STEP 2 Graph the solution.



STEP 3 Check the solution. Substitute a solution from the shaded part of your number line into the original inequality.

$$\begin{array}{l} 8 \stackrel{?}{\leq} 12 - 3 \\ 8 \leq 9 \end{array}$$

Substitute 12 for y in $8 \leq y - 3$.
The inequality is true.

Math Talk

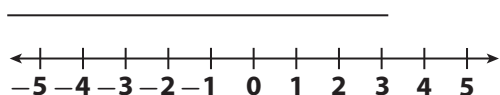
Mathematical Practices

How does the true inequality you found by substituting 12 into the original inequality help you check the solution?

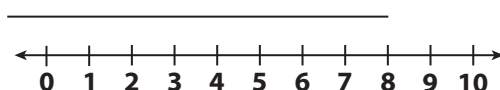
YOUR TURN

Solve each inequality. Graph and check the solution.

4. $y - 5 \geq -7$



5. $21 > 12 + x$



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Solving Inequalities Involving Multiplication and Division

You can use properties of inequality to solve inequalities involving multiplication and division with rational numbers.

Multiplication and Division Properties of Inequality

- You can multiply or divide both sides of an inequality by the same positive number and the inequality will remain true.
- If you multiply or divide both sides of an inequality by the same negative number, you must reverse the inequality symbol for the statement to still be true.



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EXAMPLE 2

COMMON CORE 7.EE.4b

Solve each inequality. Graph and check the solution.

A $\frac{y}{3} \geq 5$

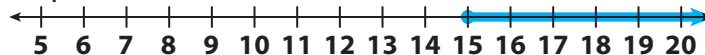
STEP 1 Solve the inequality.

$$3\left(\frac{y}{3}\right) \geq 3(5) \quad \text{Multiply both sides by 3.}$$

$$y \geq 15$$

Use a closed circle to show that 15 is a solution.

STEP 2 Graph the solution.



STEP 3 Check the solution by substituting a solution from the shaded part of the graph into the original inequality. For convenience, choose a multiple of 3.

$$\frac{18}{3} \stackrel{?}{\geq} 5 \quad \text{Substitute 18 for } x \text{ in the original inequality.}$$

$$6 \geq 5 \quad \text{The inequality is true.}$$

B $-4x > 52$

STEP 1 Solve the inequality.

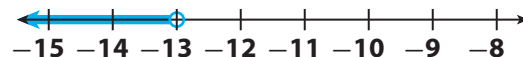
$$-4x > 52$$

$$\frac{-4x}{-4} < \frac{52}{-4}$$

Divide both sides by -4 .
Reverse the inequality symbol.

$$x < -13$$

STEP 2 Graph the solution.



STEP 3 Check your answer using substitution.

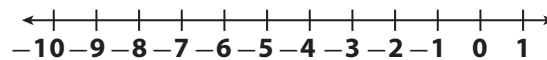
$$-4(-15) \stackrel{?}{>} 52 \quad \text{Substitute } -15 \text{ for } x \text{ in } -4x > 52.$$

$$60 > 52 \quad \text{The statement is true.}$$

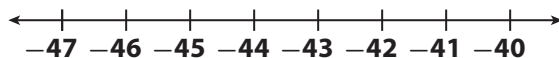
YOUR TURN

Solve each inequality. Graph and check the solution.

6. $-10y < 60$ _____



7. $7 \geq -\frac{t}{6}$ _____



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Solving a Real-World Problem

Although elevations below sea level are represented by negative numbers, we often use absolute values to describe these elevations. For example, -50 feet relative to sea level might be described as 50 feet below sea level.



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EXAMPLE 3

Problem Solving

COMMON CORE

7.EE.4b

A marine submersible descends more than 40 feet below sea level. As it descends from sea level, the change in elevation is -5 feet per second. For how many seconds does it descend?



Analyze Information

Rewrite the question as a statement.

- Find the number of seconds that the submersible descends below sea level.

List the important information:

- Final elevation > 40 feet below sea level or final elevation < -40 feet
- Rate of descent $= -5$ feet per second



Formulate a Plan

Write and solve an inequality. Use this fact:

Rate of change in elevation \times Time in seconds $=$ Final elevation



Solve

$$-5t < -40$$

Rate of change \times Time $<$ Final elevation

$$\frac{-5t}{-5} > \frac{-40}{-5}$$

Divide both sides by -5 . Reverse the inequality symbol.

$$t > 8$$

The submersible descends for more than 8 seconds.



Justify and Evaluate

Check your answer by substituting a value greater than 8 seconds in the original inequality.

$$-5(9) \stackrel{?}{<} -40$$

Substitute 9 for t in the inequality $-5t < -40$.

$$-45 < -40$$

The statement is true.

YOUR TURN

8. Every month, \$35 is withdrawn from Tony's savings account to pay for his gym membership. He has enough savings to withdraw no more than \$315. For how many months can Tony pay for his gym membership?



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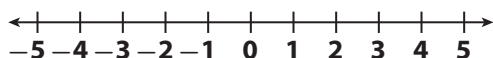
Guided Practice

Write the resulting inequality. (Explore Activity)

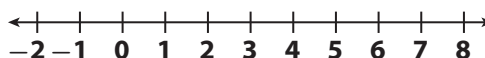
1. $-5 \leq -2$; Add 7 to both sides _____
2. $-6 < -3$; Divide both sides by -3 _____
3. $7 > -4$; Subtract 7 from both sides _____
4. $-1 \geq -8$; Multiply both sides by -2 _____

Solve each inequality. Graph and check the solution. (Examples 1 and 2)

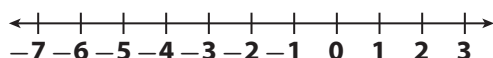
5. $n - 5 \geq -2$ _____



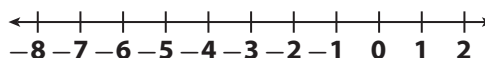
6. $3 + x < 7$ _____



7. $-7y \leq 14$ _____



8. $\frac{b}{5} > -1$ _____



9. For a scientific experiment, a physicist must make sure that the temperature of a metal at 0°C gets no colder than -80°C . The physicist changes the metal's temperature at a steady rate of -4°C per hour. For how long can the physicist change the temperature? (Example 3)

- a. Let t represent temperature in degrees Celsius. Write an inequality. Use the fact that the rate of change in temperature times the number of hours equals the final temperature.

- b. Solve the inequality in part a. How long can the physicist change the temperature of the metal?

- c. The physicist has to repeat the experiment if the metal gets cooler than -80°C . How many hours would the physicist have to cool the metal for this to happen?

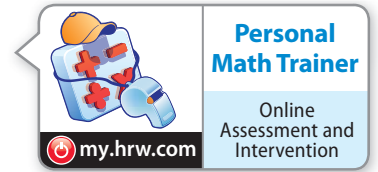


ESSENTIAL QUESTION CHECK-IN

10. Suppose you are solving an inequality. Under what circumstances do you reverse the inequality symbol?

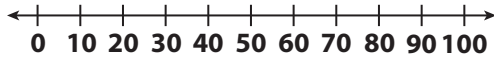
7.1 Independent Practice

COMMON CORE 7.EE.4b

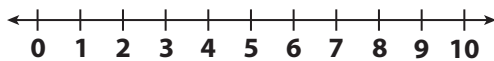


In 11–16, solve each inequality. Graph and check the solution.

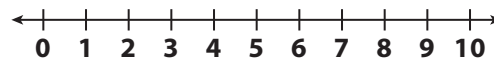
11. $x - 35 > 15$ _____



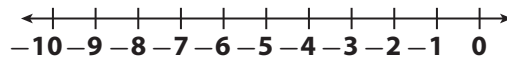
12. $193 + y \geq 201$ _____



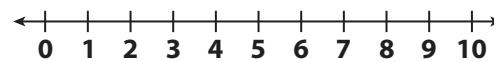
13. $-\frac{q}{7} \geq -1$ _____



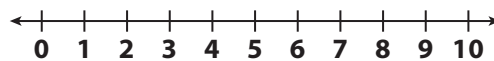
14. $-12x < 60$ _____



15. $5 > z - 3$ _____



16. $0.5 \leq \frac{y}{8}$ _____



17. The vet says that Lena's puppy will grow to be at most 28 inches tall. Lena's puppy is currently 1 foot tall. How many more inches will the puppy grow?

18. In a litter of 7 kittens, each kitten weighs less than 3.5 ounces. Find all the possible values of the combined weights of the kittens.

19. **Geometry** The sides of the hexagon shown are equal in length. The perimeter of the hexagon is at most 42 inches. Find the possible side lengths of the hexagon.



20. To get a free meal at his favorite restaurant, Tom needs to spend \$50 or more at the restaurant. He has already spent \$30.25. How much more does Tom need to spend to get his free meal?

21. To cover a rectangular region of her yard, Penny needs at least 170.5 square feet of sod. The length of the region is 15.5 feet. What are the possible widths of the region?

22. **Draw Conclusions** A submarine descends from sea level to the entrance of an underwater cave. The elevation of the entrance is -120 feet. The rate of change in the submarine's elevation is no greater than -12 feet per second. Can the submarine reach the entrance to the cave in less than 10 seconds? Explain.

The sign shows some prices at a produce stand.

23. Selena has \$10. What is the greatest amount of spinach she can buy?
- _____
24. Gary has enough money to buy at most 5.5 pounds of potatoes. How much money does Gary have?
- _____

Produce	Price per Pound
Onions	\$ 1.25
Yellow Squash	\$ 0.99
Spinach	\$ 3.00
Potatoes	\$ 0.50

25. Florence wants to spend no more than \$3 on onions. Will she be able to buy 2.5 pounds of onions? Explain.
- _____
- _____



FOCUS ON HIGHER ORDER THINKING

26. **Counterexamples** John says that if one side of an inequality is 0, you don't have to reverse the inequality symbol when you multiply or divide both sides by a negative number. Find an inequality that you can use to disprove John's statement. Explain your thinking.
- _____
- _____
- _____

27. **Look for a Pattern** Solve $x + 1 > 10$, $x + 11 > 20$, and $x + 21 > 30$. Describe a pattern. Then use the pattern to predict the solution of $x + 9,991 > 10,000$.
- _____
- _____
- _____

28. **Persevere in Problem Solving** The base of a rectangular prism has a length of 13 inches and a width of $\frac{1}{2}$ inch. The volume of the prism is less than 65 cubic inches. Find all possible heights of the prism. Show your work.
- _____
- _____
- _____

Work Area

Writing Two-Step Inequalities

Use variables to represent quantities in a real-world or mathematical problem, and construct simple... inequalities...



ESSENTIAL QUESTION

How do you write a two-step inequality?

EXPLORE ACTIVITY

COMMON
CORE

Prep for 7.EE.4

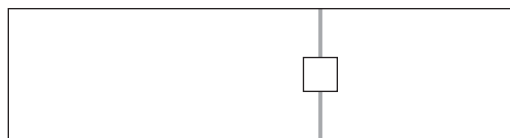
Modeling Two-Step Inequalities

You can use algebra tiles to model two-step inequalities.

Use algebra tiles to model $2k + 5 \geq -3$.

- A** Using the line on the mat, draw in the inequality symbol shown in the inequality.

- B** How can you model the left side of the inequality?



- C** How can you model the right side of the inequality?

- D** Use algebra tiles or draw them to model the inequality on the mat.

Reflect

- 1. Multiple Representations** How does your model differ from the one you would draw to model the equation $2k + 5 = -3$?

- 2.** Why might you need to change the inequality sign when you solve an inequality using algebra tiles?



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Writing Two-Step Inequalities

You can write two-step inequalities to represent real-world problems by translating the words of the problems into numbers, variables, and operations.

EXAMPLE 1



COMMON
CORE

7.EE.4

A mountain climbing team is camped at an altitude of 18,460 feet on Mount Everest. The team wants to reach the 29,029-foot summit within 6 days. Write an inequality to find the average number of feet per day the team must climb to accomplish its objective.



STEP 1

Identify what you are trying to find. This will be the variable in the inequality.

Let d represent the average altitude the team must gain each day.

STEP 2

Identify important information in the problem that you can use to write an inequality.

starting altitude: 18,460 ft

target altitude: 29,029 ft

number of days times altitude gained to reach target altitude: $6 \cdot d$

STEP 3

Use words in the problem to tie the information together and write an inequality.

starting altitude	+	number of days	times	altitude gain	is greater than or equal to	target altitude
↓	↓	↓	↓	↓	↓	↓
18,460	+	6	×	d	\geq	29,029

$$18,460 + 6d \geq 29,029$$

Math Talk

Mathematical Practices

Why is the inequality sign \geq used, rather than an equal sign?

YOUR TURN

- The 45 members of the glee club are trying to raise \$6,000 so they can compete in the state championship. They already have \$1,240. What inequality can you write to find the amount each member must raise, on average, to meet the goal? _____
- Ella has \$40 to spend at the State Fair. Admission is \$6 and each ride costs \$3. Write an inequality to find the greatest number of rides she can go on.



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Writing a Verbal Description of a Two-Step Inequality

You can also write a verbal description to fit a two-step inequality.



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EXAMPLE 2



COMMON CORE 7.EE.4

Write a corresponding real-world problem to represent $2x + 20 \leq 50$.

STEP 1 Analyze what each part of the inequality means mathematically.

x is the solution of the problem, the quantity you are looking for.

$2x$ means that, for a reason given in the problem, the quantity you are looking for is multiplied by 2.

$+ 20$ means that, for a reason given in the problem, 20 is added to $2x$.

≤ 50 means that after multiplying the solution x by 2 and adding 20 to it, the result can be no greater than 50.

STEP 2 Think of some different situations in which a quantity x is multiplied by 2.

You run x miles per day for 2 days.
So, $2x$ is the total distance run.

You buy 2 items each costing x dollars. So, $2x$ is the total cost.

STEP 3 Build on the situation and adjust it to create a verbal description that takes all of the information into account.

- Tomas has run 20 miles so far this week. If he intends to run 50 miles or less, how many miles on average should he run on each of the 2 days remaining in the week?
- Manny buys 2 work shirts that are each the same price. After using a \$20 gift card, he can spend no more than \$50. What is the maximum amount he can spend on each shirt?

YOUR TURN

Write a real-world problem for each inequality.

5. $3x + 10 > 30$

6. $5x - 50 \leq 100$

My Notes



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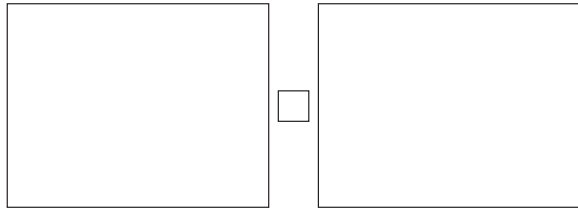
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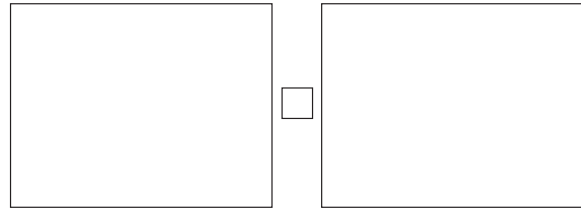
Guided Practice

Draw algebra tiles to model each two-step inequality. (Explore Activity)

1. $4x - 5 < 7$



2. $-3x + 6 > 9$

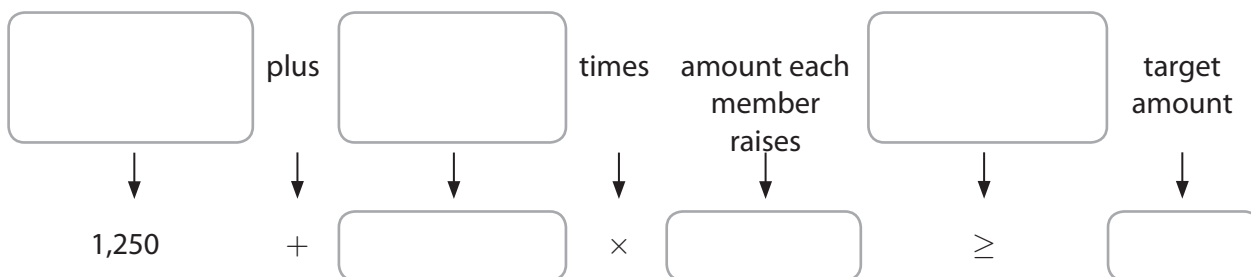


3. The booster club needs to raise at least \$7,000 for new football uniforms. So far, they have raised \$1,250. Write an inequality to find the average amounts each of the 92 members can raise to meet the club's objective. (Example 1)

Let a represent the amount each member must raise.

amount to be raised: amount already raised: number of members:

Use clues in the problem to write an equation.



The inequality that represents the situation is _____.

4. Analyze what each part of $7x - 18 \leq 32$ means mathematically. (Example 2)

x is _____. $7x$ is _____.

-18 means that _____.

≤ 32 means that _____.

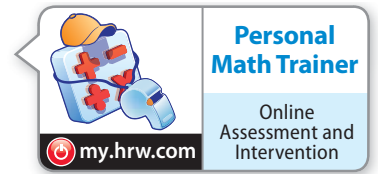
5. Write a real-world problem to represent $7x - 18 \leq 32$.

ESSENTIAL QUESTION CHECK-IN

6. Describe the steps you would follow to write a two-step inequality you can use to solve a real-world problem.

7.2 Independent Practice

COMMON CORE 7.EE.4



- 7.** Three friends earned more than \$200 washing cars. They paid their parents \$28 for supplies and divided the rest of money equally. Write an inequality to find possible amounts each friend earned. Identify what your variable represents.

- 8.** Nick has \$7.00. Bagels cost \$0.75 each, and a small container of cream cheese costs \$1.29. Write an inequality to find the numbers of bagels Nick can buy. Identify what your variable represents.

- 9.** Chet needs to buy 4 work shirts, all costing the same amount. After he uses a \$25 gift certificate, he can spend no more than \$75. Write an inequality to find the possible costs for a shirt. Identify what your variable represents.

- 10.** Due to fire laws, no more than 720 people may attend a performance at Metro Auditorium. The balcony holds 120 people. There are 32 rows on the ground floor, each with the same number of seats. Write an inequality to find the numbers of people that can sit in a ground-floor row if the balcony is full. Identify what your variable represents.

- 11.** Liz earns a salary of \$2,100 per month, plus a commission of 5% of her sales. She wants to earn at least \$2,400 this month. Write an inequality to find amounts of sales that will meet her goal. Identify what your variable represents.

- 12.** Lincoln Middle School plans to collect more than 2,000 cans of food in a food drive. So far, 668 cans have been collected. Write an inequality to find numbers of cans the school can collect on each of the final 7 days of the drive to meet this goal. Identify what your variable represents.

- 13.** Joanna joins a CD club. She pays \$7 per month plus \$10 for each CD that she orders. Write an inequality to find how many CDs she can purchase in a month if she spends no more than \$100. Identify what your variable represents.

- 14.** Lionel wants to buy a belt that costs \$22. He also wants to buy some shirts that are on sale for \$17 each. He has \$80. What inequality can you write to find the number of shirts he can buy? Identify what your variable represents.

15. Write a situation for $15x - 20 \leq 130$ and solve.

Analyze Relationships Write $>$, $<$, \geq , or \leq in the blank to express the given relationship.

16. m is at least 25 m _____ 25 17. k is no greater than 9 k _____ 9
18. p is less than 48 p _____ 48 19. b is no more than -5 b _____ -5
20. h is at most 56 h _____ 56 21. w is no less than 0 w _____ 0

22. **Critical Thinking** Marie scored 95, 86, and 89 on three science tests. She wants her average score for 6 tests to be at least 90. What inequality can you write to find the average scores that she can get on her next three tests to meet this goal? Use s to represent the lowest average score.



FOCUS ON HIGHER ORDER THINKING

23. **Communicate Mathematical Ideas** Write an inequality that expresses the reason the lengths 5 feet, 10 feet, and 20 feet could not be used to make a triangle. Explain how the inequality demonstrates that fact.

24. **Analyze Relationships** The number m satisfies the relationship $m < 0$. Write an inequality expressing the relationship between $-m$ and 0. Explain your reasoning.

25. **Analyze Relationships** The number n satisfies the relationship $n > 0$. Write three inequalities to express the relationship between n and $\frac{1}{n}$.

Work Area

LESSON 7.3 Solving Two-Step Inequalities

COMMON CORE 7.EE.4b

Solve...inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set...and interpret it in the context of the problem.



ESSENTIAL QUESTION

How do you solve a two-step inequality?

Modeling and Solving Two-Step Inequalities

You can solve two-step inequalities using algebra tiles. The method is similar to the one you used to solve two-step equations.



Math On the Spot

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EXAMPLE 1

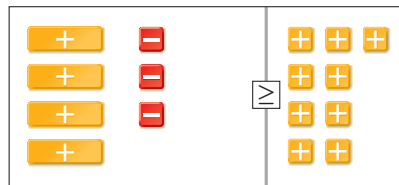
COMMON CORE

Prep for 7.EE.4b

Use algebra tiles to model and solve $4d - 3 \geq 9$.

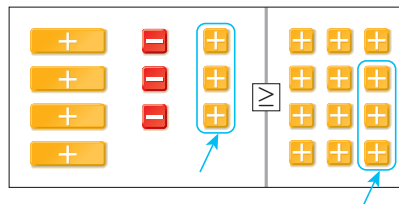
STEP 1

Model the inequality. Use a " \geq " symbol between the mats.



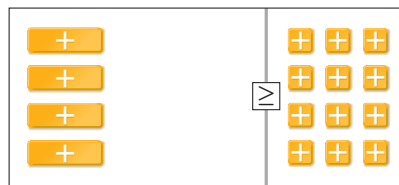
STEP 2

Add three +1 tiles to both sides of the mat.



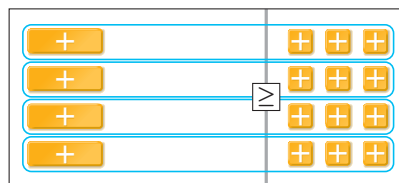
STEP 3

Remove zero pairs from the left side of the mat.



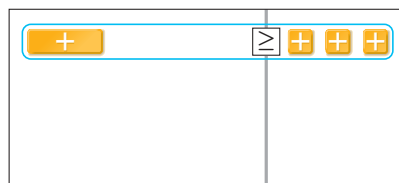
STEP 4

Divide each side into 4 equal groups.



STEP 5

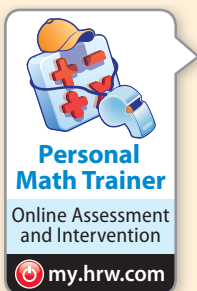
The solution is $d \geq 3$.



Math Talk

Mathematical Practices

Why are three +1-tiles added to both sides of the mat in Step 3?



My Notes

YOUR TURN

Use algebra tiles to model and solve each inequality.

1. $2x + 7 > 11$ _____ 2. $5h - 4 \geq 11$ _____

Solving and Interpreting Solutions

You can apply what you know about solving two-step equations and one-step inequalities to solving two-step inequalities.

EXAMPLE 2



COMMON CORE 7.EE.4b

Serena wants to complete the first 3 miles of a 10-mile run in 45 minutes or less running at a steady pace. The inequality $10 - 0.75p \leq 7$ can be used to find p , the pace, in miles per hour, she can run to reach her goal. Solve the inequality. Then graph and interpret the solution.

STEP 1

Use inverse operations to solve the inequality.

$$10 - 0.75p \leq 7 \quad \text{Subtract 10 from both sides.}$$

$$\underline{-10} \quad \underline{-10}$$

$$-0.75p \leq -3$$

$$\frac{-0.75p}{-0.75} \geq \frac{-3}{-0.75} \quad \begin{array}{l} \text{Divide both sides by } -0.75. \\ \text{Reverse the inequality symbol.} \end{array}$$

$$p \geq 4$$

STEP 2

Graph the inequality and interpret the circle and the arrow.

Serena can meet her goal by running at a pace of 4 miles per hour.

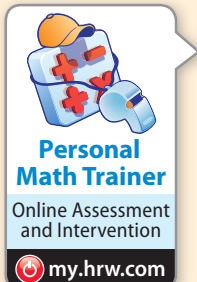
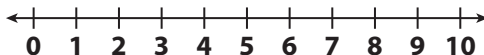


Serena can run at paces faster than 4 miles per hour and reach her goal.

Serena has to run at a steady pace of at least 4 miles per hour.

YOUR TURN

3. Joshua wants to complete the first mile of a 5-mile run in 10 minutes or less running at a steady pace. The inequality $5 - \frac{p}{6} \leq 4$ can be used to find p , the pace, in miles per hour, he can run to reach his goal. Solve the inequality. Then graph and interpret the solution.



Determining if a Given Value Makes the Inequality True

You can use substitution to decide whether a given value is the solution of an inequality.



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EXAMPLE 3



COMMON CORE 7.EE.4b

At Gas 'n' Wash, gasoline sells for \$4.00 a gallon and a car wash costs \$12. Harika wants to have her car washed and keep her total purchase under \$60. The inequality $4g + 12 < 60$ can be used to find g , the number of gallons of gas she can buy. Determine which, if any, of these values is a solution: $g = 10$; $g = 11$; $g = 12$.



STEP 1 Substitute each value for g in the inequality $4g + 12 < 60$.

$$g = 10$$

$$4(10) + 12 < 60$$

$$g = 11$$

$$4(11) + 12 < 60$$

$$g = 12$$

$$4(12) + 12 < 60$$

STEP 2 Evaluate each expression to see if a true inequality results.

$$4(10) + 12 \stackrel{?}{<} 60$$

$$40 + 12 \stackrel{?}{<} 60$$

$$52 \stackrel{?}{<} 60$$

true ✓

$$4(11) + 12 \stackrel{?}{<} 60$$

$$44 + 12 \stackrel{?}{<} 60$$

$$56 \stackrel{?}{<} 60$$

true ✓

$$4(12) + 12 \stackrel{?}{<} 60$$

$$48 + 12 \stackrel{?}{<} 60$$

$$60 \stackrel{?}{<} 60$$

not true ✗

So, Harika can buy 10 or 11 gallons of gas but not 12 gallons.

Check: Solve and graph the inequality.

$$4g + 12 < 60$$

$$4g < 48$$

$$g < 12$$



The closed circle at zero represents the minimum amount she can buy, zero gallons. She cannot buy a negative number of gallons. The open circle at 12 means that she can buy any amount up to but not including 12 gallons.

Math Talk

Mathematical Practices

In Example 2, how will the graph change if Serena's maximum pace is 12 miles per hour?

YOUR TURN

Circle any given values that make the inequality true.

4. $3v - 8 > 22$

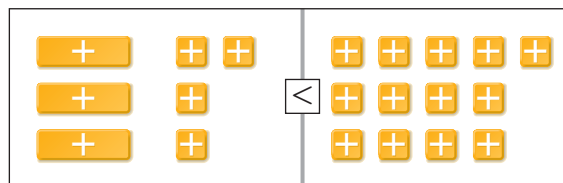
$v = 9; v = 10; v = 11$

5. $5h + 12 \leq -3$

$h = -3; h = -4; h = -5$

Guided Practice

1. Describe how to solve the inequality $3x + 4 < 13$ using algebra tiles. (Example 1)

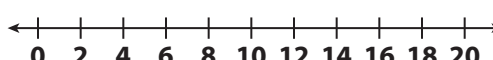


Solve each inequality. Graph and check the solution. (Example 2)

2. $5d - 13 < 32$ _____



3. $-4b + 9 \leq -7$ _____



Circle any given values that make the inequality true. (Example 3)

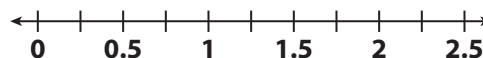
4. $2m + 18 > -4$

$m = -12; m = -11; m = -10$

5. $-6y + 3 \geq 0$

$y = 1; y = \frac{1}{2}; y = 0$

6. Lizzy has 6.5 hours to tutor 4 students and spend 1.5 hours in a lab. She plans to tutor each student the same amount of time. The inequality $6.5 - 4t \geq 1.5$ can be used to find t , the amount of time in hours Lizzy could spend with each student. Solve the inequality. Graph and interpret the solution. Can Lizzy tutor each student for 1.5 hours? Explain. (Examples 2 and 3)

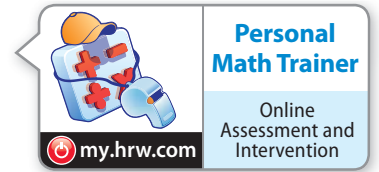


ESSENTIAL QUESTION CHECK-IN

7. How do you solve a two-step inequality?

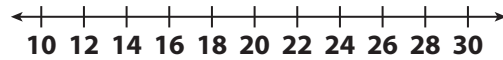
7.3 Independent Practice

COMMON CORE 7.EE.4b

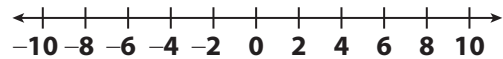


Solve each inequality. Graph and check the solution.

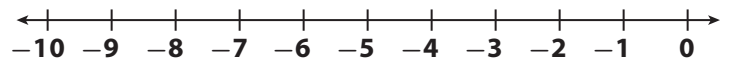
8. $2s + 5 \geq 49$ _____



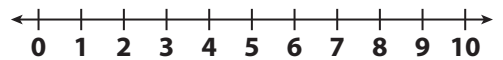
9. $-3t + 9 \geq -21$ _____



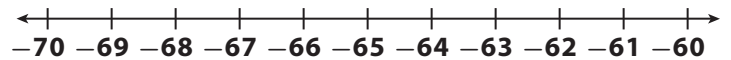
10. $55 > -7v + 6$ _____



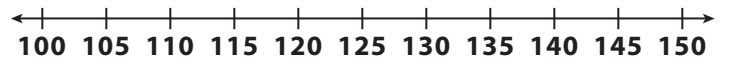
11. $41 > 6m - 7$ _____



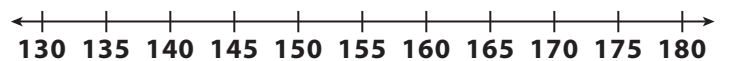
12. $\frac{a}{-8} + 15 > 23$ _____



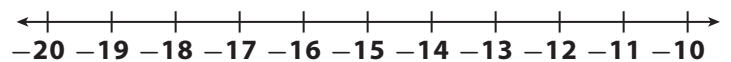
13. $\frac{f}{2} - 22 < 48$ _____



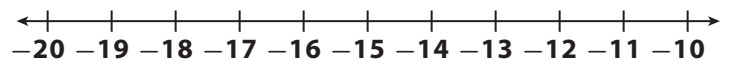
14. $-25 + \frac{t}{2} \geq 50$ _____



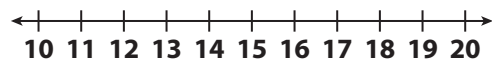
15. $10 + \frac{g}{-9} > 12$ _____



16. $25.2 \leq -1.5y + 1.2$ _____



17. $-3.6 \geq -0.3a + 1.2$ _____



18. **What If?** The perimeter of a rectangle is at most 80 inches. The length of the rectangle is 25 inches. The inequality $80 - 2w \geq 50$ can be used to find w , the width of the rectangle in inches. Solve the inequality and interpret the solution. How will the solution change if the width must be at least 10 inches and a whole number?

- 19. Interpret the Answer** Grace earns \$7 for each car she washes. She always saves \$25 of her weekly earnings. This week, she wants to have at least \$65 in spending money. How many cars must she wash? Write and solve an inequality to represent this situation. Interpret the solution in context.

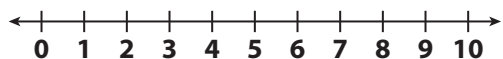


FOCUS ON HIGHER ORDER THINKING

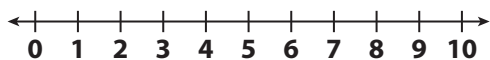
- 20. Critical Thinking** Is there any value of x with the property that $x < x - 1$? Explain your reasoning.

- 21. Analyze Relationships** A *compound inequality* consists of two simple equalities joined by the word “and” or “or.” Graph the solution sets of each of these compound inequalities.

- a.** $x > 2$ and $x < 7$



- b.** $x < 2$ or $x > 7$



- c.** Describe the solution set of the compound inequality $x < 2$ and $x > 7$.

- d.** Describe the solution set of the compound inequality $x > 2$ or $x < 7$.

- 22. Communicate Mathematical Ideas** Joseph used the problem-solving strategy Work Backward to solve the inequality $2n + 5 < 13$. Shawnee solved the inequality using the algebraic method you used in this lesson. Compare the two methods.


Work Area

Ready to Go On?



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7.1 Writing and Solving One-Step Inequalities

Solve each inequality.

1. $n + 7 < -3$ _____

2. $5p \geq -30$ _____

3. $14 < k + 11$ _____

4. $\frac{d}{-3} \leq -6$ _____

5. $c - 2.5 \leq 2.5$ _____

6. $12 \geq -3b$ _____

7. Jose has scored 562 points on his math tests so far this semester. To get an A for the semester, he must score at least 650 points. Write and solve an inequality to find the minimum number of points he must score on the remaining tests in order to get an A.

7.2 Writing Two-Step Inequalities

8. During a scuba dive, Lainey descended to a point 20 feet below the ocean surface. She continued her descent at a rate of 20 feet per minute. Write an inequality you could solve to find the number of minutes she can continue to descend if she does not want to reach a point more than 100 feet below the ocean surface.

7.3 Solving Two-Step Inequalities

Solve.

9. $2s + 3 > 15$ _____

10. $-\frac{d}{12} - 6 < 1$ _____

11. $-6w - 18 \geq 36$ _____

12. $\frac{z}{4} + 22 \leq 38$ _____

13. $\frac{b}{9} - 34 < -36$ _____

14. $-2p + 12 > 8$ _____



ESSENTIAL QUESTION

15. How can you recognize whether a real-world situation should be represented by an equation or an inequality?

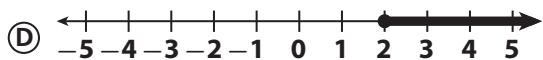
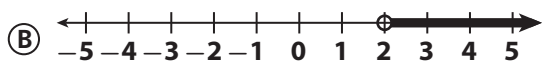
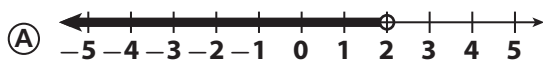


Assessment Readiness



Selected Response

1. Which graph models the solution of the inequality $-6 \leq -3x$?



2. A taxi cab costs \$1.75 for the first mile and \$0.75 for each additional mile. You have \$20 to spend on your ride. Which inequality could be solved to find how many miles you can travel, if n is the number of additional miles?

(A) $1.75n + 0.75 \geq 20$

(B) $1.75n + 0.75 \leq 20$

(C) $0.75n + 1.75 \geq 20$

(D) $0.75n + 1.75 \leq 20$

3. The inequality $\frac{9}{5}C + 32 < -40$ can be used to find Celsius temperatures that are less than -40° Fahrenheit. What is the solution of the inequality?

(A) $C < 40$

(C) $C < -40$

(B) $C < -\frac{40}{9}$

(D) $C < -\frac{72}{5}$

4. The 30 members of a choir are trying to raise at least \$1,500 to cover travel costs to a singing camp. They have already raised \$600. Which inequality could you solve to find the average amounts each member can raise that will at least meet the goal?

(A) $30x + 600 > 1,500$

(B) $30x + 600 \geq 1,500$

(C) $30x + 600 < 1,500$

(D) $30x + 600 \leq 1,500$

5. Which represents the solution for the inequality $3x - 7 > 5$?

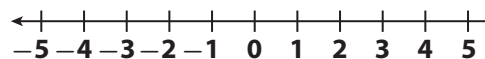
(A) $x < 4$

(C) $x > 4$

(B) $x \leq 4$

(D) $x \geq 4$

6. Which inequality has the following graphed solution?



(A) $3x + 8 \leq 2$

(C) $2x + 5 \leq 1$

(B) $4x + 12 < 4$

(D) $3x + 6 < 3$

7. Divide: $-36 \div 6$.

(A) 30

(C) -6

(B) 6

(D) -30

8. Eleni bought 2 pounds of grapes at a cost of \$3.49 per pound. She paid with a \$10 bill. How much change did she get back?

(A) \$3.02

(C) \$6.51

(B) \$4.51

(D) \$6.98

Mini-Task

9. In golf, the lower your score, the better. Negative scores are best of all. Teri scored +1 on each of the first three holes at a nine-hole miniature golf course. Her goal is a total score of -9 or better after she has completed the final six holes.

- a. Let h represent the score Teri must average on each of the last six holes in order to meet her goal. Write a two-step inequality you can solve to find h .

- b. Solve the inequality.
