Englewood Public School District
Precalculus – Graphical, Numerical, Algebraic
Fourth Marking Period

Unit 4: Probability, Statistics and an introduction to Calculus

Overview: During this unit, students will review the concepts found in probability and statistics and further their knowledge by looking at different models and distributions. Students will also begin to study limits and integrals as a way to get them prepared for Calculus.

Time Frame: 43 to 47 Days

Enduring Understandings:

- *Experimental probability is based on the results of an experiment. Theoretical probability is the mathematical chance it will happen.*
- *The probability of an event is the value that its relative frequency of occurrence approaches in the long run.*
- *Probability calculates the likelihood that something will happen and assists us in making predictions and informed decisions.*
- *We can use decimals, fractions or percentages to describe the chance of an event happening.*
- *We can use diagrams, models or algebraic equations and expressions to illustrate different probabilities and outcomes.*
- *Statistics describes properties of data, such as counts, percentages or averages.*
- *We can use charts, graphs, plots, histograms and frequency tables to illustrate different statistics.*
- *Sometimes, one representation will be better than another to describe certain statistics.*
- *To effectively compare sets of data, we need to think about how the data is distributed by looking at the shape, the center and the spread of distribution.*
- *A boxplot (box-and-whisker plot) includes five pieces of data: the three quartiles, the maximum and the minimum.*
- *Standard deviation describes how data is spread out from a particular middle value.*
- *Binomial distribution is a frequency distribution of the possible number of successful outcomes in a given number of trials in each of which there is the same probability of success.*
- *Correlation describes the strength and direction of the relationship between two variables.*
- *Average velocity is the change in position divided by the change in time.*
- *Instantaneous velocity is the change in position at a specific point in time.*
- *The notation* $\lim_{x \to a} f(x) = L$ *means that f(x) gets arbitrarily close to L as x gets arbitrarily close (but not equal to) a.*
- *The derivative of a function at a given point is given by the notation* $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$. *It is also the slope of the tangent line to the graph of f at (a, f(a)).*
- *The derivative of a function with respect to x is given by the notation* $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.
- *An integral can be described as the area under a curve.*
- *A limit does not exist if the left-hand limit does not equal the right-hand limit.*
- *We can find derivatives when we are given a set of data.*

Essential Questions:

- *What is the difference between experimental and theoretical probability?*
- *What effects the probability that a specific event will occur?*
- *How do we know whether an event is independent or dependent?*
- *How can we use modeling to form a prediction of an event occurring?*
- *How do we illustrate the probability of certain events?*
- *How can we illustrate different sets of statistics?*
- *What is the best illustration to use to describe different statistics?*
- *When comparing sets of data, what do we need to look at?*
- *What are the five points illustrated in a box-and-whisker plot?*
- *How are measures of central tendency different from standard deviation?*
- *What is binomial distribution and why is it important?*
- *What does the correlation between sets of data describe?*
- *What is the difference between average velocity and instantaneous velocity?*
- *When we find the derivative of a function at a certain point, what does that really mean?*
- *How do we find the derivative of a function algebraically?*
- *What does it mean to say that a function is discontinuous at a certain point?*
- *When does a limit not exist?*
- *What is an integral?*
- *How can we find derivatives given a set of data?*

Standards	Topics and Objectives	Activities	Resources	Assessments
MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8 **N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**Topics** Sample space, tree diagrams, Venn diagrams, conditional probability, quantitative data, stem plots, frequency tables, histograms, time plots, box-and-whisker plots, quartiles, mean, median, mode, interquartile range, range, variance, standard deviation, normal distributions, correlations, samples, surveys, randomness,	Guess the probability https://www.illustrativemathematics.org/content-standards/tasks/2120 Last person standing https://www.illustrativemathematics.org/content-standards/tasks/2119 Two-way tables and probability	Pearson Chapters 10, 11 Illustrative Mathematics https://www.illustrativemathematics.org/ Alabama Learning Exchange http://alex.state.al.us/search.php?fa_submit=ALLPLANS Texas Instruments http://education.ti.com	Text book pages: 747 – 752, 793 - 795

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

S-CP. A. . Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP. A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP. A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

average velocity, instantaneous velocity, limits, derivatives, integrals, one and two-sided limits, continuous functions, discontinuity.

Twenty-First Century Themes and Skills include:

- The Four C's
- Global awareness
- Financial, economic, business and entrepreneurial literacy

Objectives

Students will

- Identify a sample space
- Calculate probabilities and conditional probabilities with equally likely or unlikely outcomes
- Distinguish between categorical and quantitative variables
- Use various kinds of graphs to display data
- Write outcomes of different events
- Read tables and graphs illustrating different events and be able to answer questions based on those representations

https://www.illustrativemathematics.org/content-standards/tasks/2045

The addition rule
https://www.illustrativemathematics.org/content-standards/tasks/1885

Describing events
https://www.illustrativemathematics.org/content-standards/tasks/1884

Finding probabilities of compound events
https://www.illustrativemathematics.org/content-standards/tasks/1831

False positive test results
https://www.illustrativemathematics.org/content-standards/tasks/1601

But mango is my favorite
https://www.illustrativemathematics.org/content-standards/tasks/1333

Alex, Mel and Chelsea play a game
https://www.illustrativemathematics.org/content-standards/tasks/1035

Coffee at Mom's diner

Mathematics Assessment Project
http://map.mathshell.org/

E-math instruction workbook
https://emathinstruction.com/

S-MD.B.5a Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant. **S-CP. A.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. **S-CP. B.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **S-CP. B.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. **S-CP. B.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. **S-CP. B.9** Use permutations and combinations to compute	• Find and use measures of central tendency • Find and use the five-number summary of data • Use a boxplot to describe data • Use standard deviation to describe data • Use normal distribution to describe data • Create a box plot given data • Create a histogram given data • Create a probability model for a random variable • Work with binomial and Normal models • Determine statistical significance • Understand important aspects of study design • Understand the role of randomness in sampling, experimentation and simulation • Calculate instantaneous velocities and derivatives using limits • Calculate definite integrals using areas • Use the properties of limits • Evaluate one sided and two-sided limits	https://www.illustrativemathematics.org/content-standards/tasks/1024 Do you fit in this car? https://www.illustrativemathematics.org/content-standards/tasks/1020 Breakfast before school https://www.illustrativemathematics.org/content-standards/tasks/1019 Bob's bagel shop https://www.illustrativemathematics.org/content-standards/tasks/1023 Sounds really good! (sort of….) https://www.illustrativemathematics.org/content-standards/tasks/1217 Margin of error for estimating a population mean https://www.illustrativemathematics.org/content-standards/tasks/1956 Identifying outliers https://www.illustrativemathematics.org/content-standards/tasks/1888

probabilities of compound events and solve problems. **S-ID. A.1** Represent data with plots on the real number line (dot plots, histograms, and box plots). **S-ID. B.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **S-CP. A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **S-ID. A.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range,	• Evaluate limits involving infinity • Estimate derivatives and integrals using numerical techniques	Measuring variability in a data set https://www.illustrativemathematics.org/content-standards/tasks/1887 Laptop battery charge 2 https://www.illustrativemathematics.org/content-standards/tasks/1559 Olympic Men's 100-meter dash https://www.illustrativemathematics.org/content-standards/tasks/1554 Words and music II https://www.illustrativemathematics.org/content-standards/tasks/1029 The story of a flight https://www.illustrativemathematics.org/content-standards/tasks/2095 Mathemafish population https://www.illustrativemathematics.org/content-standards/tasks/686 Know your limits https://alex.state.al.us/lesson_view.php?id=27492 Basic limits

standard deviation) of two or more different data sets. **S-ID. A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **S-ID. A.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **S-IC.A.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S-MD.A.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	https://education.ti.com/en/timathnspired/us/detail?id=CC9F89F49B89439993768B40FC9CFEBF&t=5E2A88F117944527ACAEC97F6BF4FEB3 Limits at infinity https://education.ti.com/en/timathnspired/us/detail?id=91CD60911DCE4671A22FE6620D441673&t=5E2A88F117944527ACAEC97F6BF4FEB3 Limits of functions https://education.ti.com/en/timathnspired/us/detail?id=62B8E853318D482CA6BBF2C8C729D28F&t=5E2A88F117944527ACAEC97F6BF4FEB3 Derivative grapher https://education.ti.com/en/timathnspired/us/detail?id=72767CA0F16B4C57AC4C7DB824CAC6B9&t=E990A0CFBE4A4B2CAF83A1416EA537B8 Graphical derivatives https://education.ti.com/en/timathnspired/us/detail?id=CA00EC1A39194984980F9BCD8E229409&t=E990A0CFBE4A4B2CAF83A1416EA537B8

S-MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **S-MD.A.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. **S-MD.A.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. **S-MD.B.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values **S-MD.B.5b** Evaluate and compare strategies on the basis of expected values. **S-MD.B.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Tangent line demonstration https://education.ti.com/en/timathnspired/us/detail?id=604E5EB89D5D407B9E6B745B152B775D&t=E990A0CFBE4A4B2CAF83A1416EA537B8 Slopes of secant lines https://education.ti.com/en/timathnspired/us/detail?id=E4E0E85D5B564B35A6D69751C1BF4D39&t=E990A0CFBE4A4B2CAF83A1416EA537B8 Exploring the normal curve family https://education.ti.com/en/timathnspired/us/detail?id=ABD000AED86A4DE89E4EFF8696ADECC9&t=2B7EAC4F5F364F1797BCA87943CDEF11 Looking normal https://education.ti.com/en/timathnspired/us/detail?id=4B94B00D6ED2445EB00084301D4293B7&t=2B7EAC4F5F364F1797BCA87943CDEF11 Conditional probability https://education.ti.com/en/timathnspired/us/detail?id

S-MD.B.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) **S-IC.A.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **S-IC.B.3**. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S-IC.B.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S-IC.B.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if	=BC6AFC94F54A4AAB 9C5284A64824A950&t= B5929E69BBBD49D8AD 68A886C38ACADC Probability distributions https://education.ti.com/en /timathnspired/us/detail?id =3C2942AB15014EAA99 DC8E921DA5B3DE&t= B5929E69BBBD49D8AD 68A886C38ACADC Tossing coins https://education.ti.com/en /timathnspired/us/detail?id =4FCE534705A3475FA7 A3E7F81373ED0E&t=B5 929E69BBBD49D8AD68 A886C38ACADC Random samples https://education.ti.com/en /timathnspired/us/detail?id =511DE97F24F54F57A1 F416DCB147C567&t=70 97DC8CA7E749BA967D 37E2F8BA239B Interpreting data: muddying the waters http://map.mathshell.org/l essons.php?unit=9400&co llection=8&redir=1 Representing data with frequency graphs

differences between parameters are significant. **S-IC.B.6.** Evaluate reports based on data. **S-ID. C.8.** Compute (using technology) and interpret the correlation coefficient of a linear fit. **S-ID. C.9.** Distinguish between correlation and causation.		http://map.mathshell.org/lessons.php?unit=9415&collection=8 Representing data with box plots http://map.mathshell.org/lessons.php?unit=9420&collection=8 Representing conditional probabilities 1 http://map.mathshell.org/lessons.php?unit=9425&collection=8 Unit 11 – Probability https://emathinstruction.com/algebra-2-trigonometry/unit-11-probability/ Unit 12 – Statistics https://emathinstruction.com/algebra-2-trigonometry/unit-12-statistics/

Modifications:

- New Jersey Department of Education – Instructional Supports and Scaffolds
- Suggested Strategies for English Language Learners
- Secondary activities were created to allow for greater personalized learning to meet the needs of all learners including students with gifts and talents

Key Vocabulary: sample space, event, outcome, probability function, probability distribution, Venn diagram, tree diagram, conditional probability, categorical variable, quantitative variable, statistics, distribution, pie chart, bar graph, circle graph, stem-and-leaf plots, association, independent variables, frequency tables, histograms, frequency distribution, unimodal, bimodal, symmetric, mode, line graph, time plot,

descriptive statistics, inferential statistics, median, first quartile, second quartile, third quartile, range, interquartile range, box-and-whisker plot, mean, standard deviation, variance, Gaussian curve, Normal curve, random variable, bimodal distribution, positive association, positive correlation, negative association, negative correlation, bias, randomization, placebo, average velocity, instantaneous velocity, limit, slope, tangent lines, secant lines, the derivative, the definite integral, left-hand limit, right-hand limit, two-sided limit, numerical integral, continuity, discontinuity.