# **Rational Numbers**



### ESSENTIAL QUESTION

How can you use rational numbers to solve real-world problems?



#### **Real-World Video**

In many competitive sports, scores are given as decimals. For some events, the judges' scores are averaged to give the athlete's final score.





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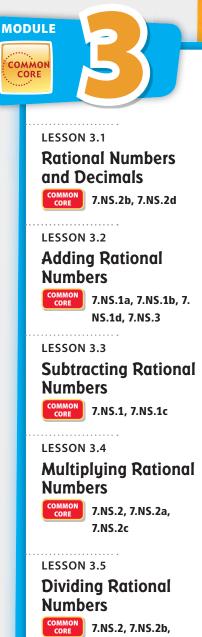
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Interactively explore key concepts to see how math works.



7.NS.2c

#### LESSON 3.6

Applying Rational Number Operations

COMMON CORE 7.NS.3, 7.EE.3





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# Are Ready

Complete these exercises to review skills you will need for this module.

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# **Multiply Fractions**

**EXAMPLE** 
$$\frac{3}{8} \times \frac{4}{9}$$
  $\frac{3}{8} \times \frac{4}{9} = \frac{1}{8} \times \frac{1}{4}$   
 $= \frac{1}{6}$ 

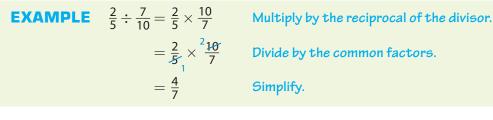
Divide by the common factors.

Simplify.

Multiply. Write the product in simplest form.

**1.**  $\frac{9}{14} \times \frac{7}{6}$  **2.**  $\frac{3}{5} \times \frac{4}{7}$  **3.**  $\frac{11}{8} \times \frac{10}{33}$  **4.**  $\frac{4}{9} \times 3$  **4.** 

# **Operations with Fractions**



Divide.

5.	$\frac{1}{2} \div \frac{1}{4}$ —	6.	$\frac{3}{8} \div \frac{13}{16}$ —	<b>7.</b> $\frac{2}{5} \div \frac{14}{15}$	8.	$\frac{4}{9} \div \frac{16}{27}$
9.	$\frac{3}{5} \div \frac{5}{6}$	10.	$\frac{1}{4} \div \frac{23}{24}$	<b>11.</b> $6 \div \frac{3}{5}$	12.	$\frac{4}{5} \div 10$

# **Order of Operations**

EXAMPLE	$50 - 3(3 + 1)^2$	To evaluate, first operate within parentheses.
	$50 - 3(4)^2$	Next simplify exponents.
	50 — 3(16)	Then multiply and divide from left to right.
	50 - 48	Finally add and subtract from left to right.
	2	

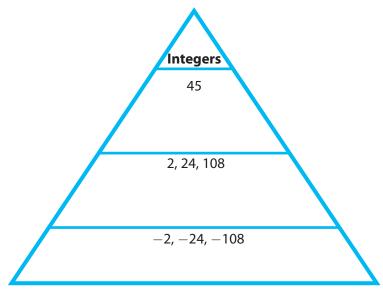
**Evaluate each expression.** 

**13.**  $21 - 6 \div 3$  \_\_\_\_ **14.**  $18 + (7 - 4) \times 3$  \_\_\_\_ **15.**  $5 + (8 - 3)^2$  \_\_\_\_ **16.** 9 + 18 ÷ 3 + 10 \_\_\_\_ **17.** 60 -  $(3 - 1)^4 \times 3$  \_\_\_\_ **18.** 10 - 16 ÷ 4 × 2 + 6 \_\_\_\_

Reading Start-Up

## **Visualize Vocabulary**

Use the 🖌 words to complete the graphic. You can put more than one word in each section of the triangle.



## **Understand Vocabulary**

### Complete the sentences using the preview words.

1. A decimal number for which the decimals come to an end is a

\_\_\_\_\_ decimal.

2. The \_\_\_\_\_\_, or \_\_\_\_\_, of a number is the same distance from 0 on a number line as the original number, but on the other side of 0.

### **Active Reading**

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the module, "Rational Numbers." Label the other flaps "Adding," "Subtracting," "Multiplying," and "Dividing." As you study each lesson, write important ideas, such as vocabulary and processes, on the appropriate flap.

# Vocabulary

### **Review Words**

- integers (enteros)
- negative numbers (números negativos) pattern (patrón)
- positive numbers (números positivos)
- whole numbers (números enteros)

#### **Preview Words**

additive inverse (inverso aditivo)

opposite (opuesto) rational number (número racional)

repeating decimal (decimal periódico)

terminating decimal (decimal finito)



# MODULE 3 Unpacking the Standards

Understanding the Standards and the vocabulary terms in the Standards will help you know exactly what you are expected to learn in this module.

#### COMMON CORE 7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers.

### **Key Vocabulary**

# rational number (número racional)

Any number that can be expressed as a ratio of two integers.

# What It Means to You

You will add, subtract, multiply, and divide rational numbers.

### **UNPACKING EXAMPLE 7.NS.3**

$-15 \cdot \frac{2}{3} - 12 \div 1\frac{1}{3}$
$-\frac{15}{1}\cdot\frac{2}{3}-\frac{12}{1}\div\frac{4}{3}$
$-\frac{15}{1}\cdot\frac{2}{3}-\frac{12}{1}\cdot\frac{3}{4}$
$-\frac{15^{5} \cdot 2}{1 \cdot 3} - \frac{12^{3} \cdot 3}{1 \cdot 4}$
$-\frac{10}{1}-\frac{9}{1}=-10-9=-19$

Write as fractions.

To divide, multiply by the reciprocal.

Simplify.

Multiply.

### COMMON 7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers.



# What It Means to You

You will solve real-world and mathematical problems involving the four operations with rational numbers.

### **UNPACKING EXAMPLE 7.NS.3**

In 1954, the Sunshine Skyway Bridge toll for a car was \$1.75. In 2012, the toll was  $\frac{5}{7}$  of the toll in 1954. What was the toll in 2012?

$$1.75 \cdot \frac{5}{7} = 1\frac{3}{4} \cdot \frac{5}{7}$$
$$= \frac{7}{4} \cdot \frac{5}{7}$$
$$= \frac{\frac{1}{7} \cdot 5}{4 \cdot \frac{7}{1}}$$
$$= \frac{5}{4} = 1.$$

Write the decimal as a fraction. Write the mixed number as an improper fraction.

Simplify.

25 Multiply, then write as a decimal.

The Sunshine Skyway Bridge toll for a car was \$1.25 in 2012.



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### **Rational Numbers** 7.NS.2d Convert a rational number to and Decimals a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. Also 7.NS.2b **ESSENTIAL QUESTION** How can you convert a rational number to a decimal? EXPLORE ACTIVITY COMMON 7.NS.2b, 7.NS.2d **Describing Decimal Forms** of Rational Numbers A rational number is a number that can be written as a ratio of two integers a and b, where b is not zero. For example, $\frac{4}{7}$ is a rational number, as is 0.37 because it can be written as the fraction $\frac{37}{100}$ .

A Use a calculator to find the equivalent decimal form of each fraction. Remember that numbers that repeat can be written as 0.333... or 0.3.

Fraction	$\frac{1}{4}$	<u>5</u> 8	$\frac{2}{3}$	<u>2</u> 9	<u>12</u> 5		
Decimal Equivalent						0.2	0.875

**B** Now find the corresponding fraction of the decimal equivalents given in the last two columns in the table. Write the fractions in simplest form.

**Conjecture** What do you notice about the digits after the decimal point in the decimal forms of the fractions? Compare notes with your neighbor and refine your conjecture if necessary.

### Reflect

- 1. Consider the decimal 0.101001000100001000001.... Do you think this decimal represents a rational number? Why or why not?
- 2. Do you think a negative sign affects whether or not a number is a rational number? Use  $-\frac{8}{5}$  as an example.

**3.** Do you think a mixed number is a rational number? Explain.



# Writing Rational Numbers as Decimals

You can convert a rational number to a decimal using long division. Some decimals are **terminating decimals** because the decimals come to an end. Other decimals are **repeating decimals** because one or more digits repeat infinitely.

### EXAMPLE 1

### CORE 7.NS.2d



Divide 5 by 16. Add a zero after the decimal point. Subtract 48 from 50. Use the grid to help you complete the long division.

Write each rational number as a decimal.

Add zeros in the dividend and continue dividing until the remainder is 0.

The decimal equivalent of  $\frac{5}{16}$  is 0.3125.

**B**  $\frac{13}{33}$ 

Divide 13 by 33. Add a zero after the decimal point.

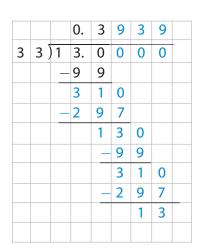
Subtract 99 from 130. Use the grid to help you complete the long division.

You can stop dividing once you discover a repeating pattern in the quotient.

Write the quotient with its repeating pattern and indicate that the repeating numbers continue.

The decimal equivalent of  $\frac{13}{33}$  is 0.3939..., or 0.39.

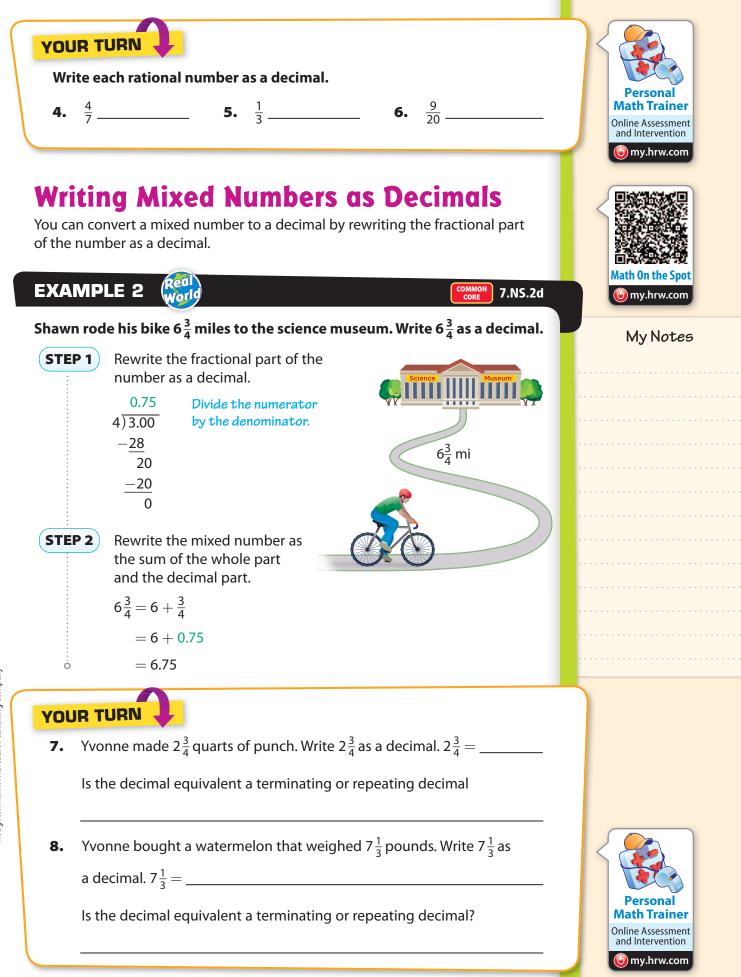
		0.	3	1	2	5
1		5.	0	0	0	0
	-	4	8			
			2	0		
		_	1	6		
				4	0	
			_	3	2	
					8	0
					8	0
						0



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Do you think that decimals that have repeating patterns always have the same number of digits in their pattern? Explain.

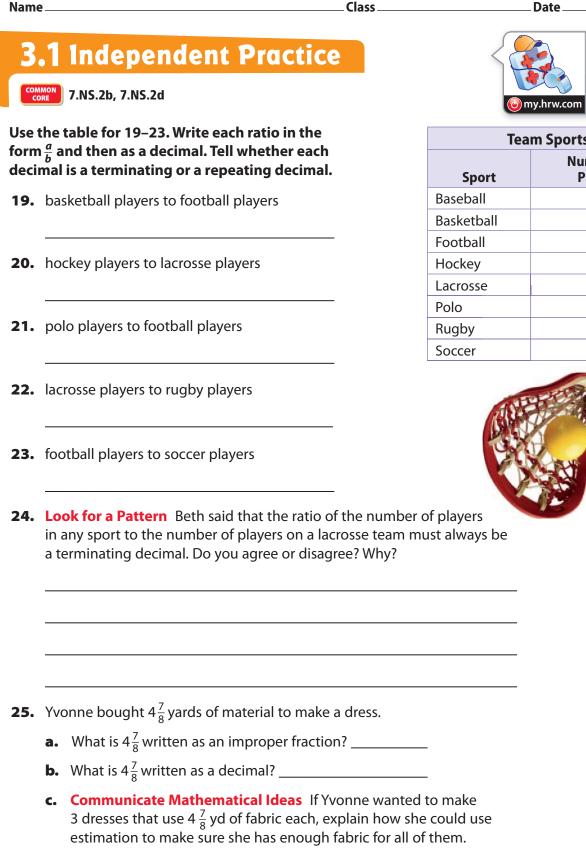


Lesson 3.1 63

# **Guided Practice**

	e each rational number as a erminating or a repeating (			I
1.	$\frac{3}{5} =$	<b>2.</b> $\frac{89}{100} =$	3.	<u>4</u> <u>12</u> =
4.	<u>25</u> <u>99</u> =	<b>5.</b> $\frac{7}{9} =$	6.	$\frac{9}{25} =$
7.	$\frac{1}{25} =$	<b>8.</b> $\frac{25}{176} =$	9.	12 1,000 =
Write	e each mixed number as a de	cimal. (Example 2)		
10.	$11\frac{1}{6} = $	<b>11.</b> $2\frac{9}{10} = $	12.	$8\frac{23}{100} =$
13.	$7\frac{3}{15} = $	<b>14.</b> $54\frac{3}{11} = $	15.	$3\frac{1}{18} =$
16.	Maggie bought $3\frac{2}{3}$ lb of app some apple pies. What is the apples written as a decimal	e weight of the ? (Example 2)	is the weight of H decimal? (Examp	Harry's dog written as a Ne 2)
	$3\frac{2}{3} =$		$12\frac{1}{8} = $	
	Tom is trying to write $\frac{3}{47}$ as a		ng division and divider	d
10.	until he got the quotient 0.0 the decimal doesn't seem to not rational. Do you agree of	0638297872, at whicl terminate or repeat	h point he stopped. Sin	ice

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**Team Sports** Number of **Players** 9 5 11 6 10 4 15 11



**26.** Vocabulary A rational number can be written as the ratio of one

\_\_\_\_\_to another and can be represented by a repeating

or \_\_\_\_\_ decimal.

- **27.** Problem Solving Marcus is  $5\frac{7}{24}$  feet tall. Ben is  $5\frac{5}{16}$  feet tall. Which of the two boys is taller? Justify your answer.
- **28.** Represent Real-World Problems If one store is selling  $\frac{3}{4}$  of a bushel of apples for \$9, and another store is selling  $\frac{2}{3}$  of a bushel of apples for \$9, which store has the better deal? Explain your answer.

# FOCUS ON HIGHER ORDER THINKING

- **29. Analyze Relationships** You are given a fraction in simplest form. The numerator is not zero. When you write the fraction as a decimal, it is a repeating decimal. Which numbers from 1 to 10 could be the denominator?
- **30.** Communicate Mathematical Ideas Julie got 21 of the 23 questions on her math test correct. She got 29 of the 32 questions on her science test correct. On which test did she get a higher score? Can you compare the fractions  $\frac{21}{23}$  and  $\frac{29}{32}$  by comparing 29 and 21? Explain. How can Julie compare her scores?

**31.** Look for a Pattern Look at the decimal 0.121122111222.... If the pattern continues, is this a repeating decimal? Explain.

# **Adding Rational Numbers**



Apply properties of operations as strategies to add and subtract rational numbers. *Also 7.NS.1 a, 7.NS.1 b, 7.NS.3* 

ESSENTIAL	QUESTION

How can you add rational numbers?

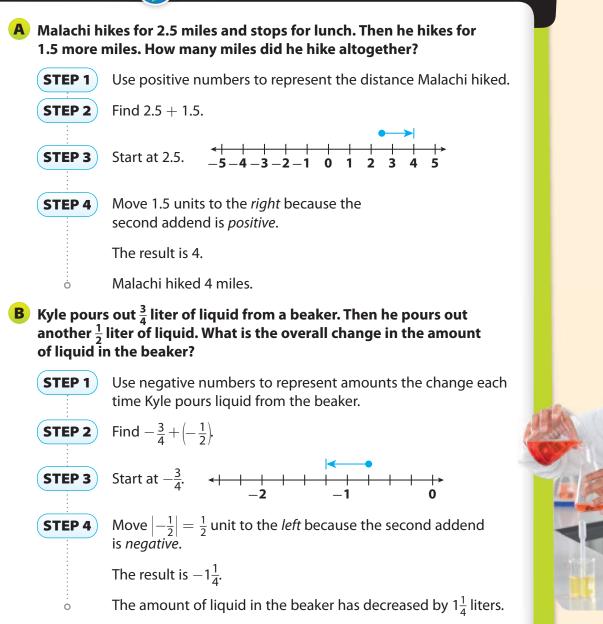
# Adding Rational Numbers with the Same Sign

To add rational numbers with the same sign, apply the rules for adding integers. The sum has the same sign as the sign of the rational numbers.

EXAMPLE 1

COMMON	7.NS.1k
CORE	





### Reflect

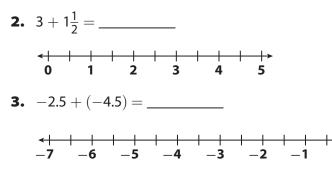
1. Explain how to determine whether to move right or left on the number line when adding rational numbers.





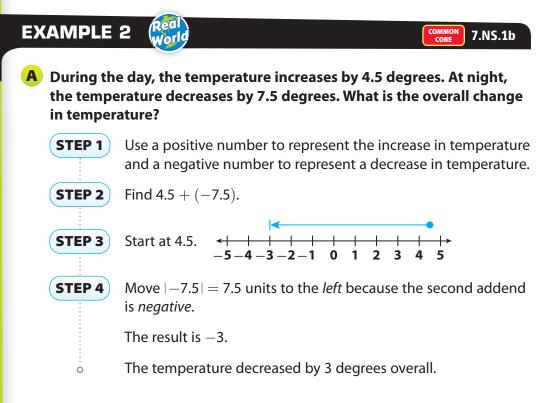
# YOUR TURN

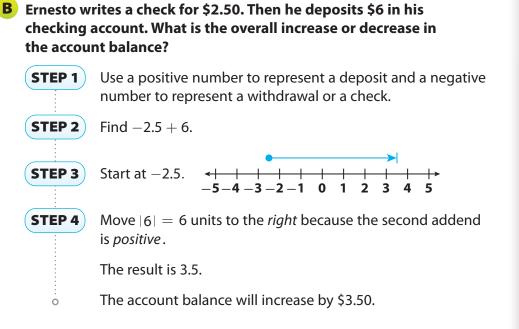
Use a number line to find each sum.



# Adding Rational Numbers with Different Signs

To add rational numbers with different signs, find the difference of their absolute values. Then use the sign of the rational number with the greater absolute value.

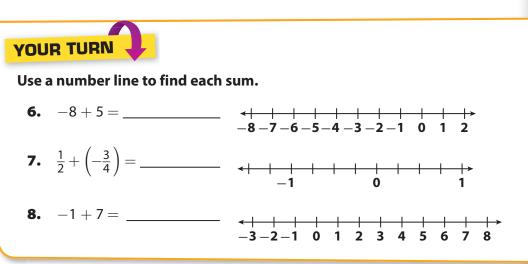


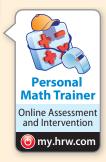


### Reflect

**4.** Do -3 + 2 and 2 + (-3) have the same sum? Does it matter if the negative number is the first addend or the second addend?

**5.** Make a Conjecture Do you think the sum of a negative number and a positive number will always be negative? Explain your reasoning.

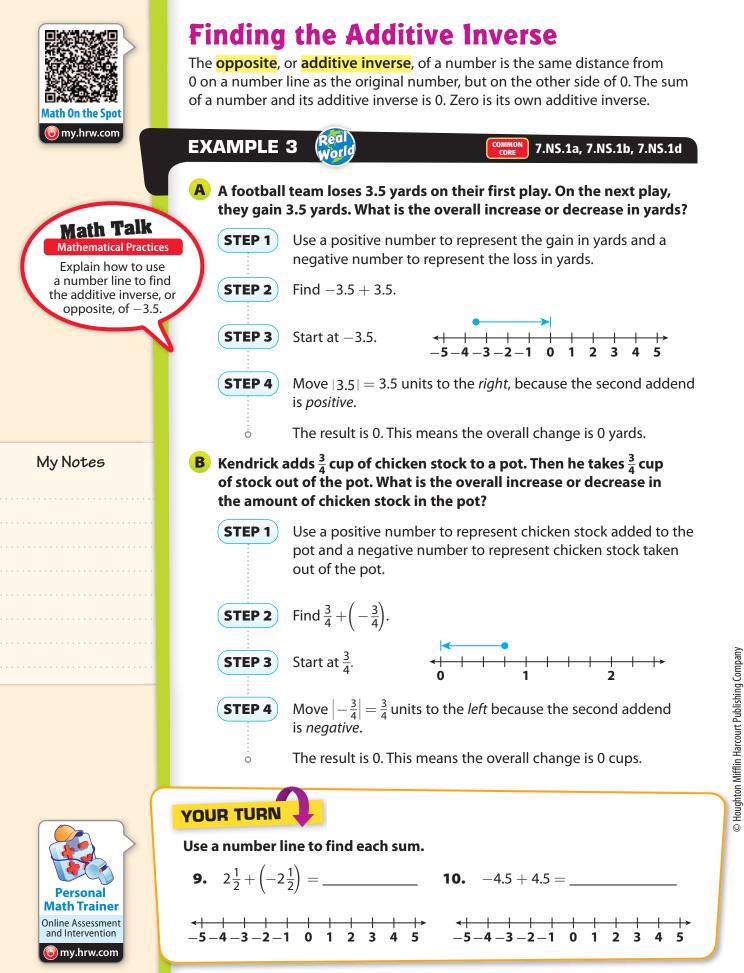




Animated

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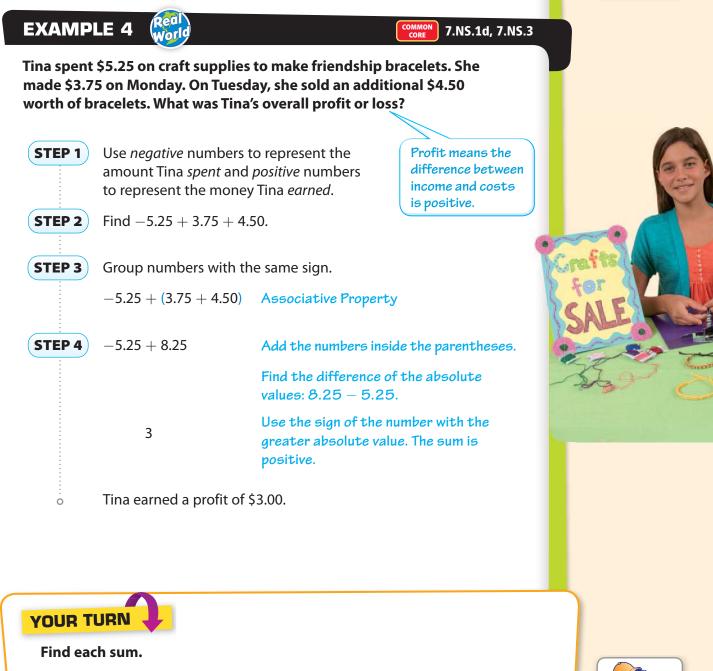
My Notes



# Adding Three or More Rational Numbers

Recall that the Associative Property of Addition states that you are adding more than two numbers, you can group any of the numbers together. This property can help you add numbers with different signs.





- **11.** -1.5 + 3.5 + 2 = \_\_\_\_\_\_ **12.**  $3\frac{1}{4} + (-2) + (-2\frac{1}{4}) =$  \_\_\_\_\_\_ **13.** -2.75 + (-3.25) + 5 = \_\_\_\_\_
  - **14.** 15 + 8 + (-3) = \_\_\_\_\_

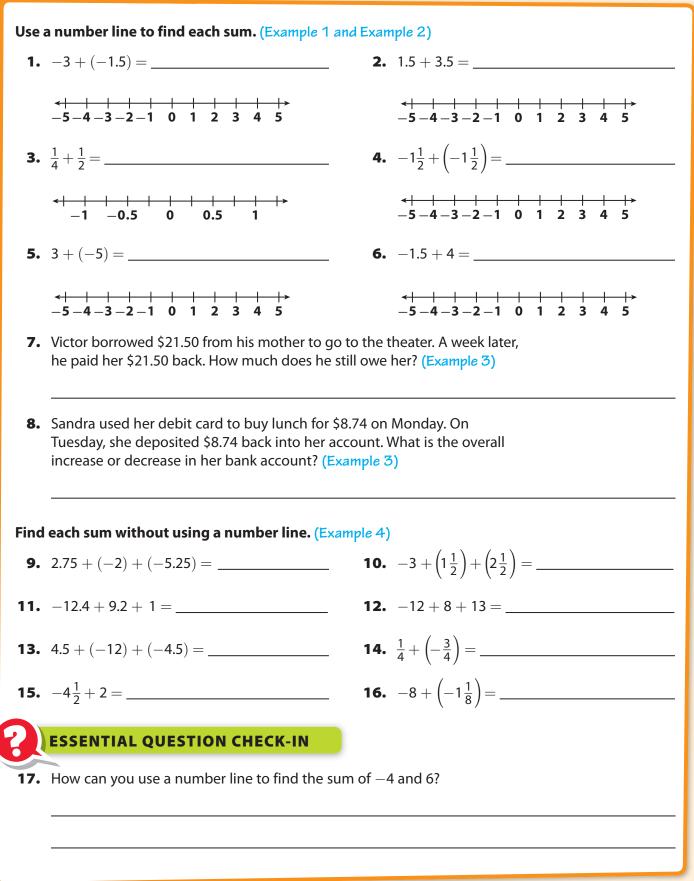
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### **Guided Practice**



### Class\_\_\_

3	<b>.2</b> Independent Practice		Personal Math Trainer
CON	MON         7.NS.1a, 7.NS.1b, 7.NS.1d, 7.NS.3	y.hrw.com	Online Assessment and Intervention
18.	Samuel walks forward 19 steps. He represents this movement with a positive 19. How would he represent the opposite of this number?		
19.	Julia spends \$2.25 on gas for her lawn mower. She earns \$15.00 mowing her neighbor's yard. What is Julia's profit?		
20.	A submarine submerged at a depth of -35.25 meters dives an additional 8.5 meters. What is the new depth of the submarine?		
21.	Renee hiked for $4\frac{3}{4}$ miles. After resting, Renee hiked back along the same route for $3\frac{1}{4}$ miles. How many more miles does Renee need to hike to return to the place where she started?		
22.	<b>Geography</b> The average elevation of the city of New Orleans, Louisiana, is 0.5 m below sea level. The highest point in Louisiana is Driskill Mountain at about 163.5 m higher than New Orleans. How high is Driskill Mountain?		
23.	<b>Problem Solving</b> A contestant on a game show has 30 points. She		

answers a question correctly to win 15 points. Then she answers a question incorrectly and loses 25 points. What is the contestant's final score?

# **Financial Literacy** Use the table for 24–26. Kameh owns a bakery. He recorded the bakery income and expenses in a table.

- **24.** In which months were the expenses greater than the income? Name the month and find how much money
  - was lost.
- **25.** In which months was the income greater than the expenses? Name the months and find how much money was gained.

Month	Income (\$)	Expenses (\$)
January	1,205	1,290.60
February	1,183	1,345.44
March	1,664	1,664.00
June	2,413	2,106.23
July	2,260	1,958.50
August	2,183	1,845.12

**26.** Communicate Mathematical Ideas If the bakery started with an extra \$250 from the profits in December, describe how to use the information in the table to figure out the profit or loss of money at the bakery by the end of August. Then calculate the profit or loss.

**27.** Vocabulary –2 is the \_\_\_\_\_ of 2.

**28.** The basketball coach made up a game to play where each player takes 10 shots at the basket. For every basket made, the player gains 10 points. For every basket missed, the player loses 15 points.

- a. The player with the highest score sank 7 baskets and missed 3. What was the highest score?
- b. The player with the lowest score sank 2 baskets and missed 8. What was the lowest score?
- c. Write an expression using addition to find out what the score would be if a player sank 5 baskets and missed 5 baskets.

H.O.1

FOCUS ON HIGHER ORDER THINKING

29. Communicate Mathematical Ideas Explain the different ways it is possible to add two rational numbers and get a negative number.

**30.** Explain the Error A student evaluated -4 + x for  $x = -9\frac{1}{2}$  and got an answer of  $5\frac{1}{2}$ . What might the student have done wrong?

**31.** Draw Conclusions Can you find the sum [5.5 + (-2.3)] + (-5.5 + 2.3)without performing any additions?

Work Area

# **Subtracting Rational Numbers**



Understand subtraction... as adding the additive inverse.... Show that the distance between two rational numbers...is the absolute value of their difference.....*Also 7.NS.*1

ESSENTIAL QUESTION

How do you subtract rational numbers?

-9 -8.5 -8 -7.5 -7 -6.5 -6 -5.5 -5 -4.5 -4

# **Subtracting Positive Rational Numbers**

To subtract rational numbers, you can apply the same rules you use to subtract integers.

EXAMPLE 1

YOUR TURN

COMMON CORE 7.NS.1 Math On the Spot

The temperature on an outdoor thermometer on Monday was 5.5 °C. The temperature on Thursday was 7.25 degrees less than the temperature on Monday. What was the temperature on Thursday?

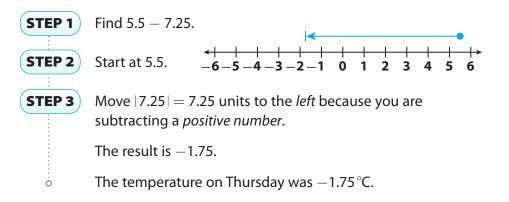
Subtract to find the temperature on Thursday.

Use a number line to find each difference.

**2.**  $1\frac{1}{2} - 2 =$  \_\_\_\_\_\_ **-1 0 1 2** 

**3.** -2.25 - 5.5 = -10 -9 -8 -7 -6 -5 -4 -3 -2 -1

**1.** -6.5 - 2 = \_\_\_\_\_



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# **Subtracting Negative Rational Numbers**

To subtract negative rational numbers, move in the opposite direction on the number line.

# EXAMPLE 2 Real

ORE 7.NS.1

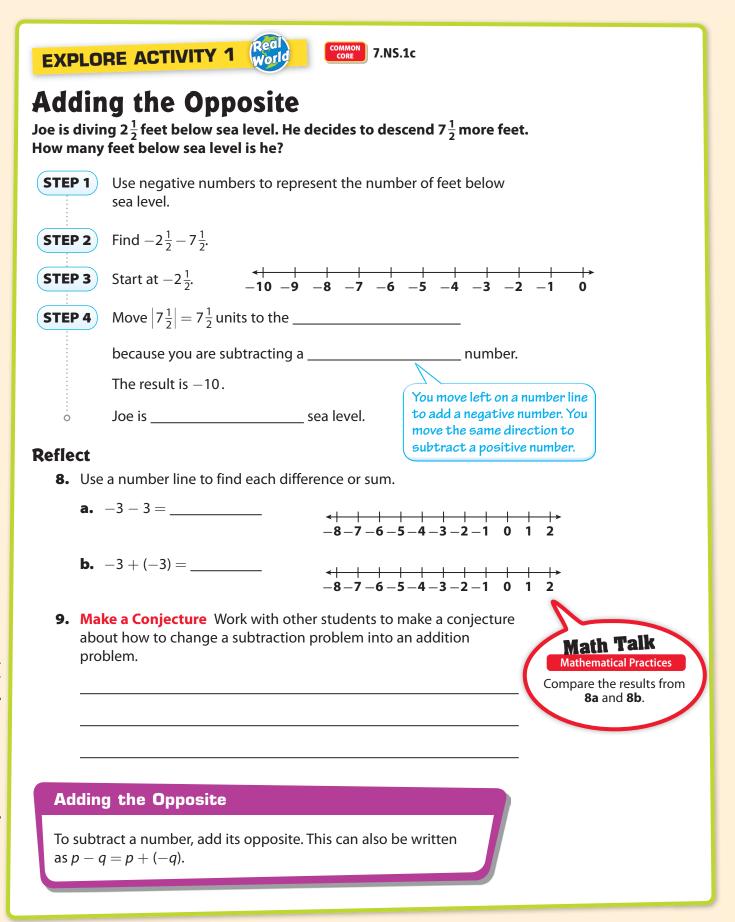
During the hottest week of the summer, the water level of the Muskrat River was  $\frac{5}{6}$  foot below normal. The following week, the level was  $\frac{1}{3}$  foot below normal. What is the overall change in the water level?

Subtract to find the difference in water levels.

STEP 1	Find $-\frac{1}{3} - (-\frac{5}{6})$ .							
STEP 2	Start at $-\frac{1}{3}$ . $-1$ 0 1							
STEP 3	Move $\left -\frac{5}{6}\right  = \frac{5}{6}$ to the <i>right</i> because you are subtracting a <i>negative</i> number.							
	The result is $\frac{1}{2}$ .							
: o	So, the water level changed $\frac{1}{2}$ foot.							
Reflect								
	rk with other students to compare addition of negative numbers a number line to subtraction of negative numbers on a number line.							
_								
<b>5.</b> Con	npare the methods used to solve Example 1 and Example 2.							
_								
YOUR TUR								
<b>Use a numb</b> <b>6.</b> 0.25 –	there line to find each difference. (-1.50) =							
++ _1	-+++++++++> 0 1 2							
<b>7.</b> $-\frac{1}{2} - (-\frac{1}{2})^{-1}$	$\left(-\frac{3}{4}\right) = $ $\left(+ + + + + + + + + + + + + + + + + + + $							

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		A State State
EXPLORE ACTIVITY 2 Real COMMON 7.NS.1c		
Finding the Distance between Two Numbers		
A cave explorer climbed from an elevation of -11 meters to an el of -5 meters. What vertical distance did the explorer climb? There are two ways to find the vertical distance.	evation	
<ul> <li>Start at</li> <li>Count the number of units on the vertical number line up to -5.</li> <li>The explorer climbed meters.</li> </ul>	$ \begin{array}{cccc} + & 0 \\ + & -1 \\ + & -2 \\ - & -3 \\ + & -4 \end{array} $	
<ul> <li>This means that the vertical distance between</li> <li>-11 meters and -5 meters is meters.</li> <li>B Find the difference between the two elevations and use absolute value to find the distance.</li> </ul>	+ -5 6 + -7 8 9 + -10	
-11 - (-5) = Take the absolute value of the difference because	-10	

distance traveled is always a nonnegative number.

|-11 - (-5) | = \_\_\_\_\_

The vertical distance is \_\_\_\_\_meters.

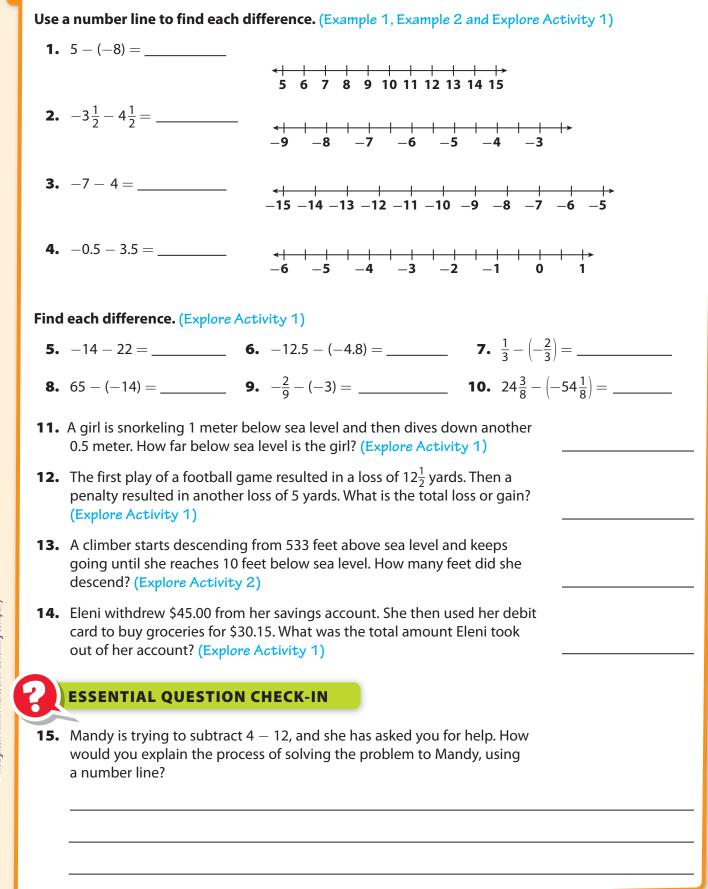
### Reflect

- **10.** Does it matter which way you subtract the values when finding distance? Explain.
- **11.** Would the same methods work if both the numbers were positive? What if one of the numbers were positive and the other negative?

### **Distance Between Two Numbers**

The distance between two values *a* and *b* on a number line is represented by the absolute value of the difference of *a* and *b*. Distance between *a* and b = |a - b| or |b - a|.

# **Guided Practice**



# **3.3 Independent Practice** 7.NS.1, 7.NS.1c

- **16.** Science At the beginning of a laboratory experiment, the temperature of a substance is -12.6 °C. During the experiment, the temperature of the substance decreases 7.5 °C. What is the final temperature of the substance?
- **17.** A diver went 25.65 feet below the surface of the ocean, and then 16.5 feet further down, he then rose 12.45 feet. Write and solve an expression to find the diver's new depth.
- A city known for its temperature extremes started the day at -5 degrees Fahrenheit. The temperature increased by 78 degrees Fahrenheit by midday, and then dropped 32 degrees by nightfall.
  - **a.** What expression can you write to find the temperature at nightfall?
  - **b.** What expression can you write to describe the overall change in temperature? *Hint*: Do not include the temperature at the beginning of the day since you only want to know about how much the temperature changed.
  - **c.** What is the final temperature at nightfall? What is the overall change in temperature?
- **19.** Financial Literacy On Monday, your bank account balance was -\$12.58. Because you didn't realize this, you wrote a check for \$30.72 for groceries.
  - a. What is the new balance in your checking account?
  - **b.** The bank charges a \$25 fee for paying a check on a negative balance. What is the balance in your checking account after this fee?
  - **c.** How much money do you need to deposit to bring your account balance back up to \$0 after the fee?

### Astronomy Use the table for problems 20–21.

**20.** How much deeper is the deepest canyon on Mars than the deepest canyon on Venus?

<b>Elevations on Planets</b>					
Lowest (ft) Highest (ft)					
Earth	-36,198	29,035			
Mars	Mars –26,000 70,				
Venus	Venus –9,500				



**21. Persevere in Problem Solving** What is the difference between Earth's highest mountain and its deepest ocean canyon? What is the difference between Mars' highest mountain and its deepest canyon? Which difference is greater? How much greater is it?

- **22.** Pamela wants to make some friendship bracelets for her friends. Each friendship bracelet needs 5.2 inches of string.
  - **a.** If Pamela has 20 inches of string, does she have enough to make bracelets for 4 of her friends?
  - **b.** If so, how much string would she had left over? If not, how much more string would she need?
- **23.** Jeremy is practicing some tricks on his skateboard. One trick takes him forward 5 feet, then he flips around and moves backwards 7.2 feet, then he moves forward again for 2.2 feet.
  - **a.** What expression could be used to find how far Jeremy is from his starting position when he finishes the trick?
  - **b.** How far from his starting point is he when he finishes the trick? Explain

- 24. Esteban has \$20 from his allowance. There is a comic book he wishes to buy that costs \$4.25, a cereal bar that costs \$0.89, and a small remote control car that costs \$10.99.
  - a. Does Esteban have enough to buy everything?
  - **b.** If so, how much will he have left over? If not, how much does he still need?

Work Area

FOCUS ON HIGHER ORDER THINKING

HOT

**25.** Look for a Pattern Show how you could use the Commutative Property to simplify the evaluation of the expression  $-\frac{7}{16} - \frac{1}{4} - \frac{5}{16}$ .

**26. Problem Solving** The temperatures for five days in Kaktovik, Alaska, are given below.

 $-19.6\ ^{\circ}F$ ,  $-22.5\ ^{\circ}F$ ,  $-20.9\ ^{\circ}F$ ,  $-19.5\ ^{\circ}F$ ,  $-22.4\ ^{\circ}F$ 

Temperatures for the following week are expected to be twelve degrees lower every day. What are the highest and lowest temperatures expected for the corresponding 5 days next week?

**27.** Make a Conjecture Must the difference between two rational numbers be a rational number? Explain.

**28.** Look for a Pattern Evan said that the difference between two negative numbers must be negative. Was he right? Use examples to illustrate your answer.

# **Multiplying Rational Numbers**



Apply and extend previous understandings of multiplication...and of fractions to multiply ...rational numbers. Also 7.NS.2a, 7.NS.2c

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**ESSENTIAL QUESTION** 

How do you multiply rational numbers?

# **Multiplying Rational Numbers** with Different Signs

The rules for the signs of products of rational numbers with different signs are summarized below. Let p and q be rational numbers.

Products of Rational Numbers			
Sign of Factor p	Sign of Factor <i>q</i>	Sign of Product <i>pq</i>	
+	_	-	
—	+	_	

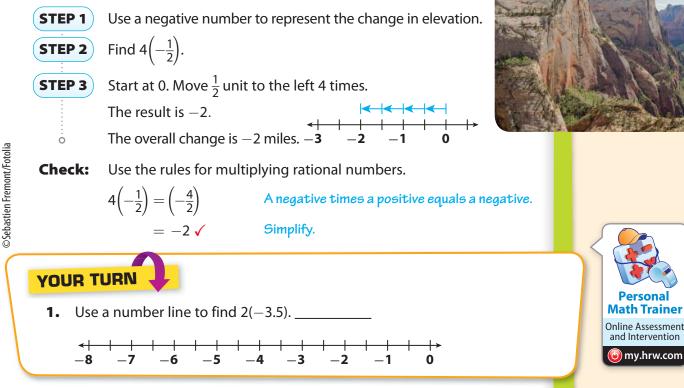
You can also use the fact that multiplication is repeated addition.



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COMMON CORE 7.NS.2, 7.NS.2a

Gina hiked down a canyon and stopped each time she descended  $\frac{1}{2}$  mile to rest. She hiked a total of 4 sections. What is her overall change in elevation?





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Personal

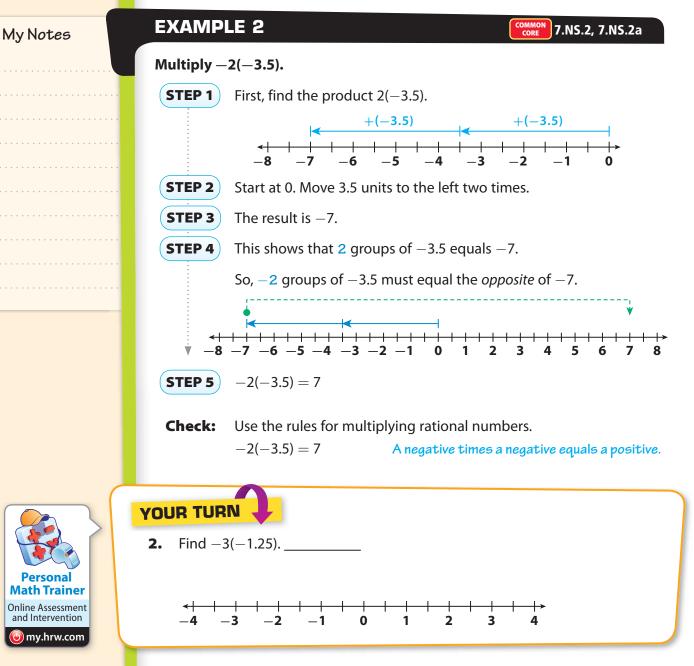


# Multiplying Rational Numbers with the Same Sign

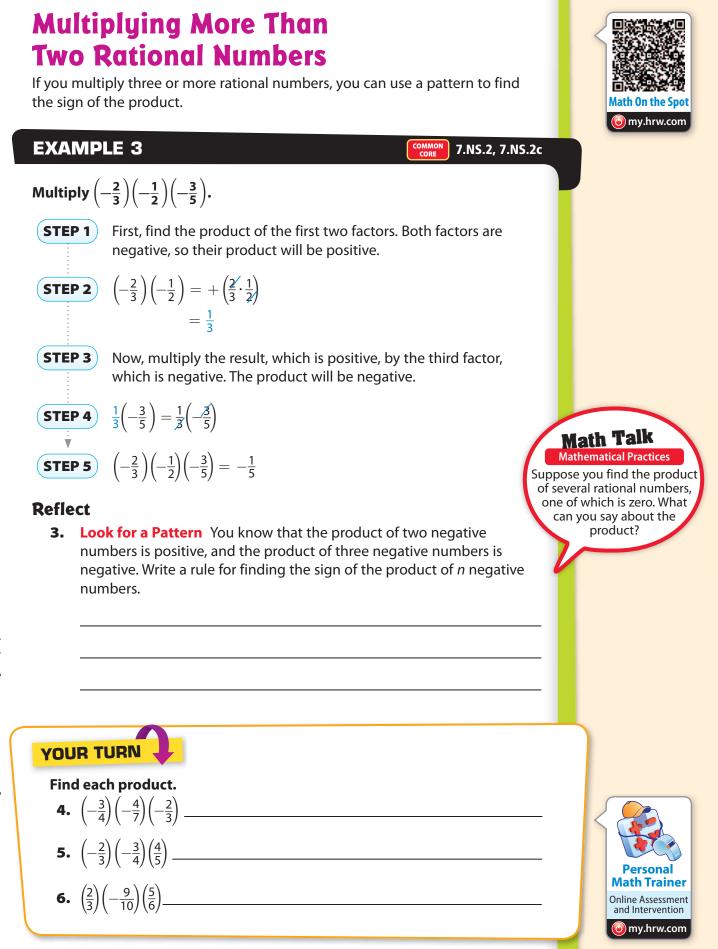
The rules for the signs of products with the same signs are summarized below.

Products of Rational Numbers					
Sign of Factor pSign of Factor qSign of Product pq					
+	+	+			
_	_	+			

You can also use a number line to find the product of rational numbers with the same signs.

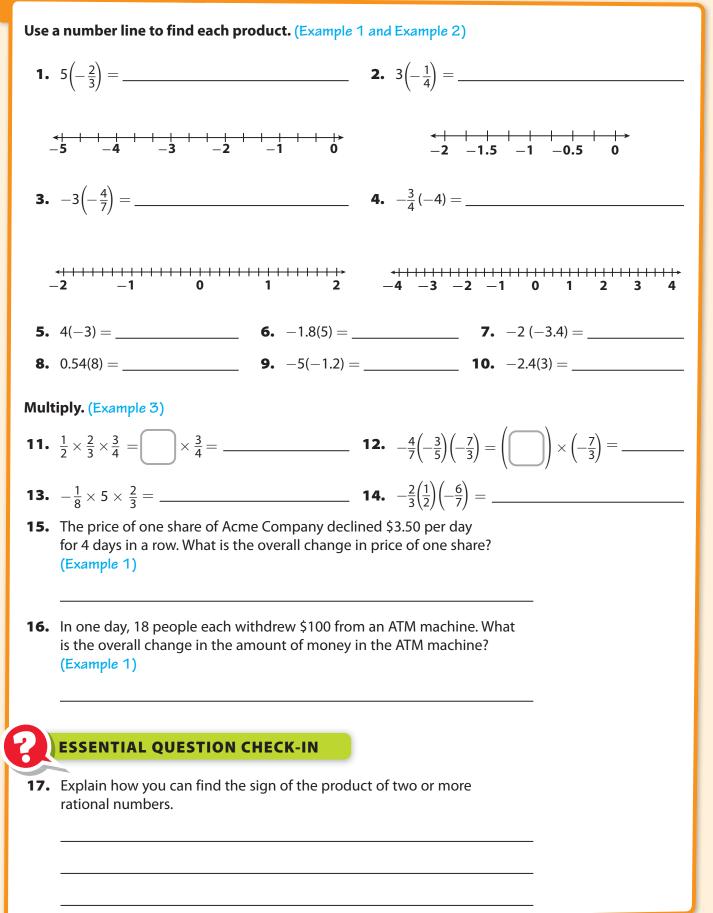


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Lesson 3.4 85

### **Guided Practice**



# **3.4 Independent Practice**

#### COMMON CORE 7.NS.2, 7.NS.2a, 7.NS.2c

- **18.** Financial Literacy Sandy has \$200 in her bank account.
  - a. If she writes 6 checks for exactly \$19.98, what expression describes the change in her bank account?
  - **b.** What is her account balance after the checks are cashed?

### **19.** Communicating Mathematical

**Ideas** Explain, in words, how to find the product of -4(-1.5) using a number line. Where do you end up?

- **20.** Greg sets his watch for the correct time on Wednesday. Exactly one week later, he finds that his watch has lost  $3\frac{1}{4}$  minutes. If his watch continues to lose time at the same rate, what will be the overall change in time after 8 weeks?
- **21.** A submarine dives below the surface, heading downward in three moves. If each move downward was 325 feet, where is the submarine after it is finished diving?
- 24. Kaitlin is on a long car trip. Every time she stops to buy gas, she loses 15 minutes of travel time. If she has to stop 5 times, how late will she be getting to her destination?

# 5.2a, 7.NS.2c



Date.



- 22. Multistep For Home Economics class, Sandra has 5 cups of flour. She made 3 batches of cookies that each used 1.5 cups of flour. Write and solve an expression to find the amount of flour Sandra has left after making the 3 batches of cookies.
- 23. Critique Reasoning In class, Matthew stated, "I think that a negative is like an opposite. That is why multiplying a negative times a negative equals a positive. The opposite of negative is positive, so it is just like multiplying the opposite of a negative twice, which is two positives." Do you agree or disagree with his reasoning What would you say in response to him?

HOT

25. The table shows the scoring system for quarterbacks in Jeremy's fantasy football league. In one game, Jeremy's quarterback had 2 touchdown passes, 16 complete passes, 7 incomplete passes, and 2 interceptions. How many total points did Jeremy's quarterback score?

FOCUS ON HIGHER ORDER THINKING

Quarterback Scoring		
Action Points		
Touchdown pass	6	
Complete pass	0.5	
Incomplete pass -0.5		
Interception	-1.5	

Work Area

**26. Represent Real-World Problems** The ground temperature at Brigham Airport is 12°C. The temperature decreases by 6.8 °C for every increase of 1 kilometer above the ground. What is the overall change in temperature outside a plane flying at an altitude of 5 kilometers above Brigham Airport?

**27. Identify Patterns** The product of four numbers, *a*, *b*, *c*, and *d*, is a negative number. The table shows one combination of positive and negative signs of the four numbers that could produce a negative product. Complete the table to show the seven other possible combinations.

а	b	c	d
+	+	+	_

**28.** Reason Abstractly Find two integers whose sum is -7 and whose product is 12. Explain how you found the numbers.

6	LESSON Dividing Rational 3.5 Numbers	<b>COMMON</b> <b>7.NS.2</b> Apply and extend previous understandings of multiplication and division and of fractions todivide rational numbers. <i>Also</i> <i>7.NS.2b, 7.NS.2c</i>
	How do you divide rational numbers?	
	EXPLORE ACTIVITY 1 Real COMMON 7.NS.2, 7.NS.2b	
	<b>Dividing Rational Numbers</b> A diver needs to descend to a depth of 100 feet below sea level. She wants to do it in 5 equal descents. How far should she travel in each descent?	350
	A To solve this problem, you can set up a division problem: $\frac{-100}{}$ = ?	
	B Rewrite the division problem as a multiplication problem. Think: Some number multiplied by 5 equals – 100.	
	×?=-100	
	C Remember the rules for integer multiplication. If the product is	
ock	negative, one of the factors must be negative. Since is	
es/Comstock	positive, the unknown factor must be <b>positive / negative.</b>	
er Image	<b>D</b> You know that 5 $\times$ = 100. So, using the rules for integer	
© Jupite	multiplication you can say that 5 $\times$ = -100.	
mpany •	The diver should descend feet in each descent.	
fflin Harcourt Publishing Company • © Jupiter Imag Houghton Mifflin Harcourt	<ul><li><b>Reflect</b></li><li>1. What do you notice about the quotient of two rational numbers with different signs?</li></ul>	

**2.** What do you notice about the quotient of two rational numbers with the same sign? Does it matter if both signs are positive or both are negative?

EXPLORE ACTIVITY 1 (cont'd)

Let *p* and *q* be rational numbers.

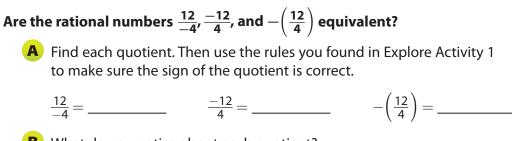
Quotients of Rational Numbers					
Sign of Dividend $p$ Sign of Divisor $q$ Sign of Quotient $\frac{p}{q}$					
+	_	_			
_	+	-			
+	+	+			
—	—	+			

Also,  $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$ , for q not zero.

EXPLORE ACTIVITY 2 COMMON 7.NS.2b

# **Placement of Negative Signs in** Quotients

Quotients can have negative signs in different places.



B What do you notice about each quotient?

C The rational numbers are / are not equivalent.

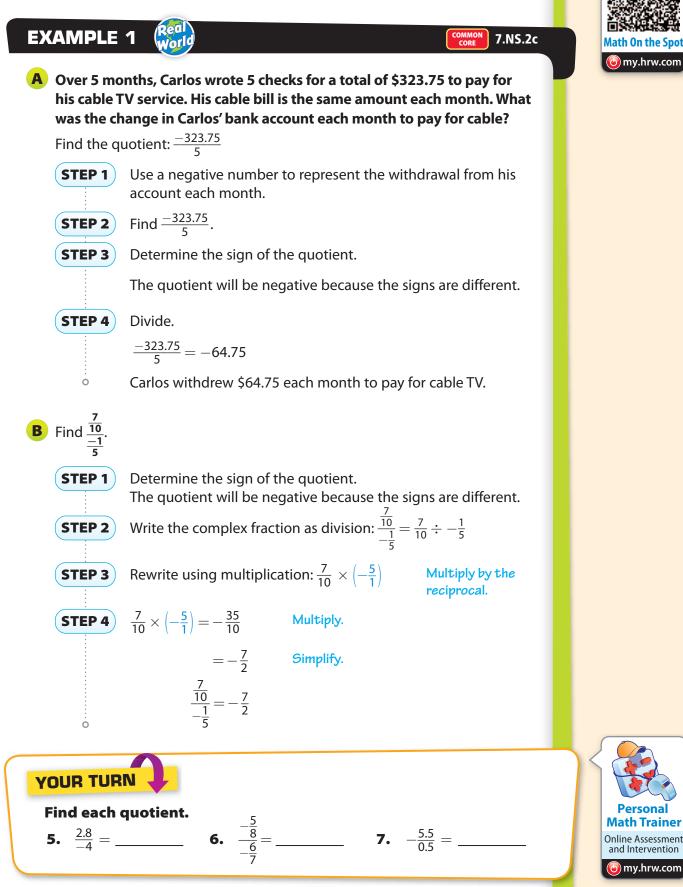
**D** Conjecture Explain how the placement of the negative sign in the rational number affects the sign of the guotients.

### Reflect

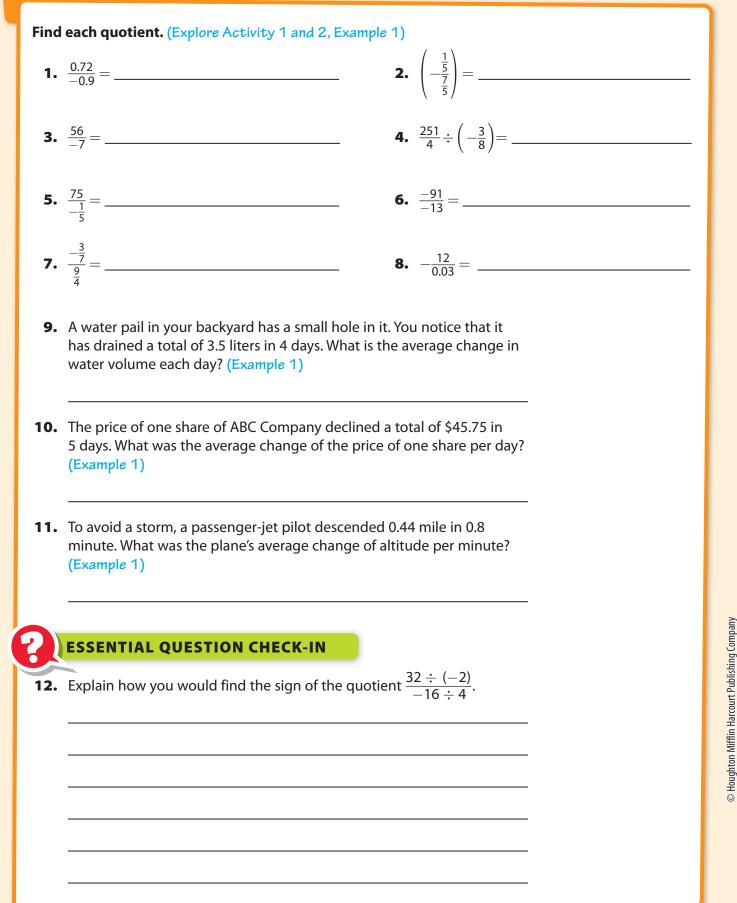
Write two equivalent expressions for each quotient.

# **Quotients of Rational Numbers**

The rules for dividing rational numbers are the same as dividing integers.



## **Guided Practice**



#### Class



- **23.** The running back for the Bulldogs football team carried the ball 9 times for a total loss of  $15\frac{3}{4}$  yards. Find the average change in field position on each run.
- 24. The 6:00 a.m. temperatures for four consecutive days in the town of Lincoln were -12.1 °C, -7.8 °C, -14.3 °C, and -7.2 °C. What was the average 6:00 a.m. temperature for the four days?
- **25. Multistep** A seafood restaurant claims an increase of \$1,750.00 over its average profit during a week where it introduced a special of baked clams.
  - **a.** If this is true, how much extra profit did it receive per day?
  - **b.** If it had, instead, lost \$150 per day, how much money would it have lost for the week?
  - **c.** If its total loss was \$490 for the week, what was its average daily change?
- 26. A hot air balloon descended 99.6 meters in 12 seconds. What was the balloon's average rate of descent in meters per second?



**Prove** 7.NS.2, 7.NS.2b, 7.NS.2c **13.**  $\frac{5}{-\frac{2}{8}} =$  **14.**  $5\frac{1}{3} \div (-1\frac{1}{2}) =$  **15.**  $\frac{-120}{-6} =$  **16.**  $\frac{-\frac{4}{5}}{-\frac{2}{-3}} =$  **17.**  $1.03 \div (-10.3) =$  **18.**  $\frac{-0.4}{80} =$  **19.**  $1 \div \frac{9}{5} =$  **20.**  $\frac{\frac{-1}{4}}{\frac{23}{24}} =$ 

**3.5** Independent Practice

- **21.**  $\frac{-10.35}{-2.3} =$  \_\_\_\_\_
- **22.** Alex usually runs for 21 hours a week, training for a marathon. If he is unable to run for 3 days, describe how to find out how many hours of training time he loses, and write the appropriate integer to describe how it affects his time.

**27.** Sanderson is having trouble with his assignment. His shown work is as follows:

$$\frac{\frac{3}{4}}{\frac{4}{3}} = -\frac{3}{4} \times \frac{4}{3} = -\frac{12}{12} = -1$$

However, his answer does not match the answer that his teacher gives him. What is Sanderson's mistake? Find the correct answer.

**28.** Science Beginning in 1996, a glacier lost an average of 3.7 meters of thickness each year. Find the total change in its thickness by the end of 2012.



**29.** Represent Real-World Problems Describe a real-world situation that can be represented by the quotient  $-85 \div 15$ . Then find the quotient and explain what the quotient means in terms of the real-world situation.

**30.** Construct an Argument Divide 5 by 4. Is your answer a rational number? Explain.

**31. Critical Thinking** Should the quotient of an integer divided by a nonzero integer always be a rational number? Why or why not?

Work Area

# **3.6** Applying Rational Number Operations

COMMON CORE 7.EE.3 Solve ... problems ... with positive and negative rational numbers in any form ... using tools strategically. *Also* 7.NS.3

ESSENTIAL QUESTION

How do you use different forms of rational numbers and strategically choose tools to solve problems?

# **Assessing Reasonableness of Answers**

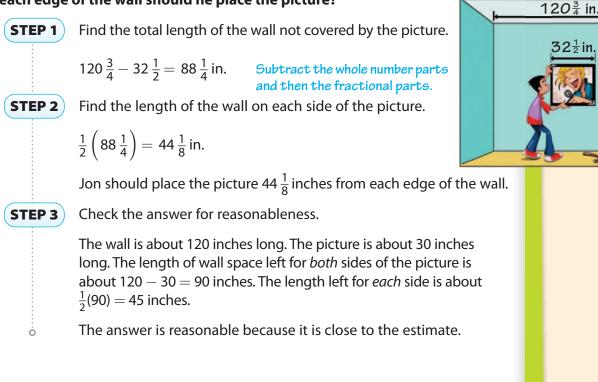
Even when you understand how to solve a problem, you might make a careless solving error. You should always check your answer to make sure that it is reasonable.

EXAMPLE 1 (Real

MON 7.EE.3, 7.NS.3



Jon is hanging a picture. He wants to center it horizontally on the wall. The picture is  $32\frac{1}{2}$  inches long, and the wall is  $120\frac{3}{4}$  inches long. How far from each edge of the wall should he place the picture?



YOUR TURN

**1.** A 30-minute TV program consists of three commercials, each  $2\frac{1}{2}$  minutes long, and four equal-length entertainment segments. How long is each

entertainment segment? \_\_\_\_\_

Personal Math Trainer Online Assessment

and Intervention



### My Notes

# **Using Rational Numbers in Any Form**

You have solved problems using integers, positive and negative fractions, and positive and negative decimals. A single problem may involve rational numbers in two or more of those forms.

#### Problem **EXAMPLE 2**

Alana uses  $1\frac{1}{4}$  cups of flour for each batch of blueberry muffins she makes. She has a 5-pound bag of flour that cost \$4.49 and contains seventy-six  $\frac{1}{4}$ -cup servings. How many batches can Alana make if she uses all the flour? How much does the flour for one batch cost?

### **Analyze Information**

Identify the important information.

- Each batch uses 1<sup>1</sup>/<sub>4</sub> cups of flour.
  Seventy-six <sup>1</sup>/<sub>4</sub> -cup servings of flour cost \$4.49.



7.EE.3, 7.NS.3

### **Formulate a Plan**

Use logical reasoning to solve the problem. Find the number of cups of flour that Alana has. Use that information to find the number of batches she can make. Use that information to find the cost of flour for each batch.

### Solve

Number of cups of flour in bag:

76 cups  $\times \frac{1}{4}$  cup per serving = 19 cups

Number of batches Alana can make:

total cups of flour  $\div \frac{\text{cups of flour}}{\text{batch}} = 19 \text{ cups} \div \frac{1.25 \text{ cups}}{1 \text{ batch}}$  $= 19 \div 1.25$ = 15.2

Write 17 as a decimal

Alana cannot make 0.2 batch. The recipe calls for one egg, and she cannot divide one egg into tenths. So, she can make 15 batches.

Cost of flour for each batch:  $$4.49 \div 15 = $0.299$ , or about \$0.30.

### **Justify and Evaluate**

A bag contains about 80 guarter cups, or about 20 cups. Each batch uses about 1 cup of flour, so there is enough flour for about 20 batches. A bag costs about \$5.00, so the flour for each batch costs about  $5.00 \div 20 = 0.25$ . The answers are close to the estimates, so the answers are reasonable.



**2.** A 4-pound bag of sugar contains 454 one-teaspoon servings and costs \$3.49. A batch of muffins uses  $\frac{3}{4}$  cup of sugar. How many batches can you make if you use all the sugar? What is the cost of sugar for each

batch? (1 cup = 48 teaspoons) \_\_\_\_\_

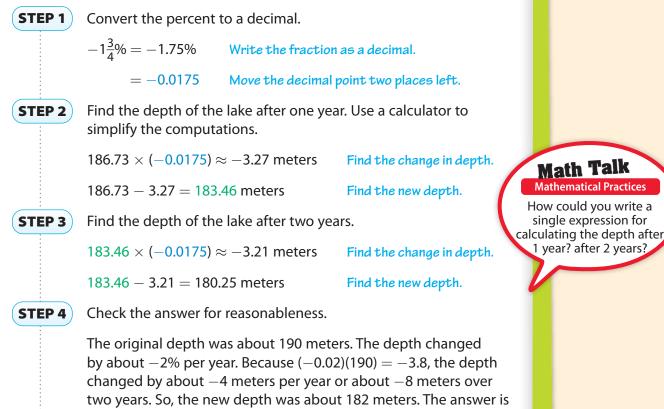
# **Using Tools Strategically**

A wide variety of tools are available to help you solve problems. Rulers, models, calculators, protractors, and software are some of the tools you can use in addition to paper and pencil. Choosing tools wisely can help you solve problems and increase your understanding of mathematical concepts.

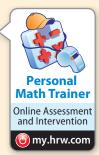
EXAMPLE 3

COMMON 7.EE.3, 7.NS.3

The depth of Golden Trout Lake has been decreasing in recent years. Two years ago, the depth of the lake was 186.73 meters. Since then the depth has been changing at an average rate of  $-1\frac{3}{4}$ % per year. What is the depth of the lake today?

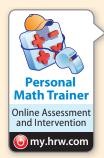


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# YOUR TURN

**3.** Three years ago, Jolene bought \$750 worth of stock in a software company. Since then the value of her purchase has been increasing at an average rate

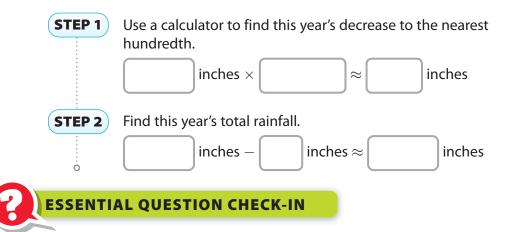
of  $12\frac{3}{5}\%$  per year. How much is the stock worth now?

# **Guided Practice**

**1.** Mike hiked to Big Bear Lake in 4.5 hours at an average rate of  $3\frac{1}{5}$  miles per hour. Pedro hiked the same distance at a rate of  $3\frac{3}{5}$  miles per hour. How long did it take Pedro to reach the lake? (Example 1 and Example 2)

	STEP 1	Find the distance Mike hiked.		
		4.5 h × miles per hour = miles		
	STEP 2	Find Pedro's time to hike the same distance.		
	0	miles ÷ miles per hour = hours		
2.		ear, Greenville had averaged 25.68 inches of rainfall per year for a century. This year's total rainfall showed a change of $-2\frac{3}{8}\%$ with		

respect to the previous average. How much rain fell this year? (Example 3)



3. Why is it important to consider using tools when you are solving a problem?

Lesson 3.6

99

# **3.6 Independent Practice**

COMMON 7.NS.3, 7.EE.3

### Solve, using appropriate tools.

**4.** Three rock climbers started a climb with each person carrying 7.8 kilograms of climbing equipment. A fourth climber with no equipment joined the group. The group divided the total weight of climbing equipment equally among the four climbers. How much

did each climber carry? \_

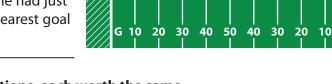
**5.** Foster is centering a photo that is  $3\frac{1}{2}$  inches wide on a scrapbook page that is 12 inches wide. How far from each side of the page

should he put the picture? \_

**6.** Diane serves breakfast to two groups of children at a daycare center. One box of Oaties contains 12 cups of cereal. She needs  $\frac{1}{3}$  cup for each younger child and  $\frac{3}{4}$  cup for each older child. Today's group includes 11 younger children and 10 older children. Is one box of Oaties enough for everyone?

Explain. \_

7. The figure shows how the yard lines on a football field are numbered. The goal lines are labeled G. A referee was standing on a certain yard line as the first quarter ended. He walked  $41\frac{3}{4}$  yards to a yard line with the same number as the one he had just left. How far was the referee from the nearest goal



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50 30 40 20 40 30 50

In 8–10, a teacher gave a test with 50 questions, each worth the same number of points. Donovan got 39 out of 50 questions right. Marci's score was 10 percentage points higher than Donovan's.

**8.** What was Marci's score? Explain.

line?

9. How many more questions did Marci answer correctly? Explain.

**10.** Explain how you can check your answers for reasonableness.



Date



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Class \_\_\_\_

## For 11–13, use the expression 1.43 $\times \left(-\frac{19}{37}\right)$ .

- **11.** Critique Reasoning Jamie says the value of the expression is close to -0.75. Does Jamie's estimate seem reasonable? Explain.
- **12.** Find the product. Explain your method.
- **13.** Does your answer to Exercise 12 justify your answer to Exercise 11?



**14.** Persevere in Problem Solving A scuba diver dove from the surface of the ocean to an elevation of  $-79\frac{9}{10}$  feet at a rate of -18.8 feet per minute. After spending 12.75 minutes at that elevation, the diver ascended to an elevation of  $-28\frac{9}{10}$  feet. The total time for the dive so far was  $19\frac{1}{8}$  minutes. What was

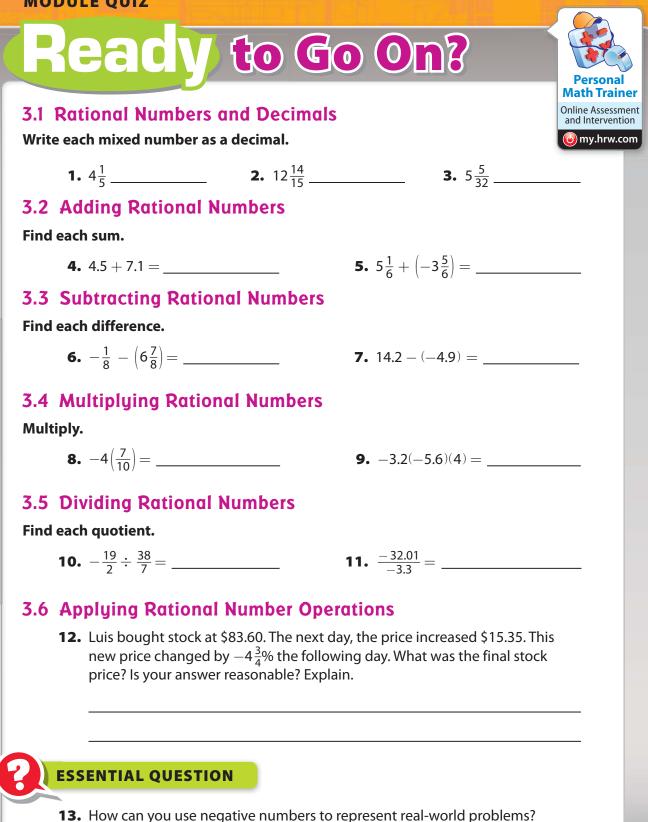
the rate of change in the diver's elevation during the ascent? \_

**15.** Analyze Relationships Describe two ways you could evaluate 37% of the sum of  $27\frac{3}{5}$  and 15.9. Tell which method you would use and why.

**16. Represent Real-World Problems** Describe a real-world problem you could solve with the help of a yardstick and a calculator.

### Work Area

### **MODULE QUIZ**





# MODULE 3 MIXED REVIEW Assessment Readiness



### **Selected Response**

- **1.** What is  $-7\frac{5}{12}$  written as a decimal?
  - (A) −7.25
  - **B** −7.333...
  - € –7.41666…
  - **(D)** −7.512
- 2. Glenda began the day with a golf score of -6 and ended with a score of -10. Which statement represents her golf score for that day?
  - (A) -6 (-10) = 4
  - **B** -10 (-6) = -4
  - $\bigcirc$  -6 + (-10) = -16

**(D)** 
$$-10 + (-6) = -16$$

**3.** A submersible vessel at an elevation of -95 feet descends to 5 times that elevation. What is the vessel's new elevation?

( <b>A</b> ) −475 ft	🔘 19 ft
	<b>D</b> 475 ft

**4.** The temperature at 7 P.M. at a weather station in Minnesota was −5 °F. The temperature began changing at the rate of −2.5 °F per hour. What was the temperature at 10 P.M.?

(A) −15 °F	.5 °F
------------	-------

- **B** −12.5 °F **D** 5 °F
- **5.** What is the sum of -2.16 and -1.75?
  - (A) 0.41
    (C) −0.41
  - **(B)** 3.91 **(D)** −3.91

6. On Sunday, the wind chill temperature reached -36 °F. On Monday, the wind chill temperature only reached  $\frac{1}{4}$  of Sunday's wind chill temperature. What was the lowest wind chill temperature on Monday?

(A) 
$$-9 \,^{\circ}F$$
 (C)  $-40 \,^{\circ}F$   
(B)  $-36\frac{1}{4} \,^{\circ}F$  (D)  $-144 \,^{\circ}F$ 

7. The level of a lake was 8 inches below normal. It decreased  $1\frac{1}{4}$  inches in June and  $2\frac{3}{8}$  inches more in July. What was the new level with respect to the normal level?

(A) 
$$-11\frac{5}{8}$$
 in. (C)  $-9\frac{1}{8}$  in.  
(B)  $-10\frac{5}{8}$  in. (D)  $-5\frac{3}{8}$  in.

### Mini-Task

- **8.** The average annual rainfall for a town is 43.2 inches.
  - **a.** What is the average *monthly* rainfall?
  - **b.** The difference of a given month's rainfall from the average monthly rainfall is called the *deviation*. What is the deviation for each month shown?

Town's Rainfall in Last Three Months			
Month	May	June	July
Rain (in.)	$2\frac{3}{5}$	$\frac{7}{8}$	$4\frac{1}{4}$

c. The average monthly rainfall for the previous 9 months was 4 inches. Did the town exceed its average annual rainfall? If so, by how much?