CONNECTED MATHEMATICS® 3

Accentuate the Negative

Integers and Rational Numbers

Glenda Lappan, Elizabeth Difanis Phillips,
James T. Fey, Susan N. Friel

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Accentuate the Negative

Integers and Rational Numbers

Look for these icons that point to digital resources. Launch Tools ACTIVe-book Student Activities

Looking Ahead

A person goes from a sauna at 115°F to an outside temperature of $^{-}30$°F. **What** is the change in temperature?

A racetrack is marked by a number line measured in meters. Hahn runs from the 15-meter line to the $^{-}15$-meter line in 8 seconds. At **what** rate (meters per second), and in what direction, does he run?

Water flows into and out of a water tower at different rates throughout the day. **When** is the water in the water tower at its highest level?

Most of the numbers you have worked with in math class have been greater than or equal to zero. However, numbers less than zero can provide important information. Winter temperatures in many places fall below 0°F. Businesses that lose money have profits less than $0. Scores in games or sports can be less than zero.

Numbers greater than zero are called *positive numbers*. Numbers less than zero are called *negative numbers*. In *Accentuate the Negative*, you will work with both positive and negative numbers. You will study *integers* and *rational numbers,* two specific sets of numbers that include positive and negative numbers. You will explore models that help you think about adding, subtracting, multiplying, and dividing these numbers. You will also learn more about the properties of operations on positive and negative numbers.

In *Accentuate the Negative,* you will solve problems similar to those on the previous page that require understanding and skill in working with positive and negative numbers.

Mathematical Highlights

Integers and Rational Numbers

In *Accentuate the Negative,* you will extend your knowledge of negative numbers. You will use negative numbers to solve problems.

You will learn how to

- Use appropriate notation to indicate positive and negative numbers and zero
- Compare and order rational numbers and locate them on a number line
- Understand the relationship between a number and its opposite (additive inverse)
- Relate direction and distance to the number line
- Develop and use different models (number line, chip model) for representing addition, subtraction, multiplication, and division
- Develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers
- Interpret and write mathematical sentences to show relationships and solve problems
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations
- Use parentheses and the Order of Operations in computations
- Use the commutative properties of addition and multiplication
- Apply the Distributive Property to simplify expressions and solve problems
- Use models and rational numbers to represent and solve problems

ASK YOURSELF

When you encounter a new problem, it is a good idea to ask yourself questions. In this Unit, you might ask questions such as:

How do negative and positive numbers and zero help describe the situation?

What will addition, subtraction, multiplication, or division of rational numbers tell about the problem?

What model(s) for positive and negative numbers and zero help show relationships in the problem situation?

Common Core State Standards

Mathematical Practices and Habits of Mind

In the *Connected Mathematics* curriculum you will develop an understanding of important mathematical ideas by solving problems and reflecting on the mathematics involved. Every day, you will use "habits of mind" to make sense of problems and apply what you learn to new situations. Some of these habits are described by the *Common Core State Standards for Mathematical Practices* (MP).

MP1 Make sense of problems and persevere in solving them.

When using mathematics to solve a problem, it helps to think carefully about

- data and other facts you are given and what additional information you need to solve the problem;
- strategies you have used to solve similar problems and whether you could solve a related simpler problem first;
- how you could express the problem with equations, diagrams, or graphs;
- whether your answer makes sense.

MP2 Reason abstractly and quantitatively.

When you are asked to solve a problem, it often helps to

- focus first on the key mathematical ideas;
- check that your answer makes sense in the problem setting;
- use what you know about the problem setting to guide your mathematical reasoning.

MP3 Construct viable arguments and critique the reasoning of others.

When you are asked to explain why a conjecture is correct, you can

- show some examples that fit the claim and explain why they fit;
- show how a new result follows logically from known facts and principles.

When you believe a mathematical claim is incorrect, you can

- show one or more counterexamples—cases that don't fit the claim;
- find steps in the argument that do not follow logically from prior claims.

MP4 Model with mathematics.

When you are asked to solve problems, it often helps to

- think carefully about the numbers or geometric shapes that are the most important factors in the problem, then ask yourself how those factors are related to each other;
- express data and relationships in the problem with tables, graphs, diagrams, or equations, and check your result to see if it makes sense.

MP5 Use appropriate tools strategically.

When working on mathematical questions, you should always

- decide which tools are most helpful for solving the problem and why;
- try a different tool when you get stuck.

MP6 Attend to precision.

In every mathematical exploration or problem-solving task, it is important to

- think carefully about the required accuracy of results: is a number estimate or geometric sketch good enough, or is a precise value or drawing needed?
- report your discoveries with clear and correct mathematical language that can be understood by those to whom you are speaking or writing.

MP7 Look for and make use of structure.

In mathematical explorations and problem solving, it is often helpful to

- look for patterns that show how data points, numbers, or geometric shapes are related to each other;
- use patterns to make predictions.

MP8 Look for and express regularity in repeated reasoning.

When results of a repeated calculation show a pattern, it helps to

- express that pattern as a general rule that can be used in similar cases;
- look for shortcuts that will make the calculation simpler in other cases.

You will use all of the Mathematical Practices in this Unit. Sometimes, when you look at a Problem, it is obvious which practice is most helpful. At other times, you will decide on a practice to use during class explorations and discussions. After completing each Problem, ask yourself:

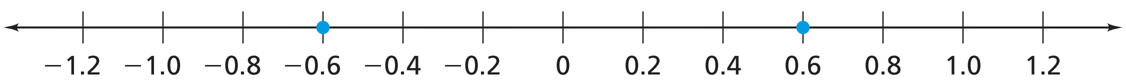
- What mathematics have I learned by solving this Problem?
- What Mathematical Practices were helpful in learning this mathematics?

Investigation 1

Extending the Number System

One of the most useful representations of numbers is a number line. A number line displays numbers in order so that their relationship to each other is clear. You can determine whether numbers are less than or greater than other numbers by looking at their positions on a number line.

A number line also illustrates the relationships between signed numbers.



- What is the relationship between $^{-}0.6$ and 0.6?
- Which number is greater, $^{-}2.3$ or 1.2?
- How can you use a number line to help you list $^{-}2.3$, $^{-}3.5$, and 1.7 in order?

As you work on this Investigation, use number lines to help you think and reason about mathematical situations.

Common Core State Standards

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.1a Describe situations in which opposite quantities combine to make 0.

7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number . . .

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

7.EE.B.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Also 7.NS.A.1b, 7.NS.A.1c, 7.EE.B.4

You have worked with whole numbers, fractions, and decimals in earlier units. In this Unit, you will work with integers. **Integers** are the set of whole numbers, their opposites, and zero. Integers and fractions, (and their equivalent decimals), are called *rational* numbers.

Problem 1.1 involves a game with positive and negative scores. As you work through the Problem, think about which operations you use to keep track of the scores. Notice how the score goes higher or lower depending on whether a team answers a question correctly or incorrectly.

1.1 Playing Math Fever

Using Positive and Negative Numbers

Ms. Bernoski's math classes often play Math Fever, a game similar to a popular television game show. The game board is shown. Below each category name are five cards. The front of each card shows a point value. The back of each card has a question related to the category. Cards with higher point values have more difficult questions.

Math Fever

Operations With Fractions	Similarity	Probability	Area and Perimeter	Tiling the Plane	Factors and Multiples
50	50	50	50	50	50
100	100	100	100	100	100
150	150	150	150	150	150
200	200	200	200	200	200
250	250	250	250	250	250

Math Fever is played in teams. One team starts the game by choosing a card. The teacher asks the question on the back of the card. The first team to answer the question correctly gets the point value on the card. The card is then removed from the board. If a team answers the question incorrectly, the point value is subtracted from their score. The other teams may then try to answer the question. The team that answers correctly chooses the next card.

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Problem 1.1

A At one point in a game, the scores are as follows:

Super Brains	Rocket Scientists	Know-It-Alls
$^{-}300$	150	$^{-}500$

1. Which team has the highest score? Which team has the lowest score? Explain how you decided.
2. Find the difference in points for each pair of teams.
3. Use *number sentences* to describe two possible ways that each team reached its score.

B The current scores are $^{-}300$ for Super Brains, 150 for Rocket Scientists, and $^{-}500$ for Know-It-Alls.

1. Write a number sentence to represent each sequence of points. Start with the current score for each team.

   a. **Super Brains**

Point Value	Answer
200	Correct
150	Incorrect
50	Correct
50	Correct

   b. **Rocket Scientists**

Point Value	Answer
50	Incorrect
200	Incorrect
100	Correct
150	Incorrect

   c. **Know-It-Alls**

Point Value	Answer
100	Incorrect
200	Correct
150	Incorrect
50	Incorrect

2. At this point in the game, which team has the highest score? Which team has the lowest score?
3. Find the difference in points for each pair of teams.

continued on the next page >

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Problem 1.1 continued

C The number sentences below describe what happens at a particular point during a game of Math Fever. For each number sentence:

- Find the missing number.
- Explain what the sentence tells about a team's performance and overall score.

1. BrainyActs: $^-200 + 150 - 100 = \blacksquare$
2. Xtremes: $450 - 300 = \blacksquare$
3. ExCells: $300 - 450 = \blacksquare$
4. AmazingMs: $^-350 + \blacksquare = {}^-150$

D Sam forgot to record a score. Sam wrote this number sentence:

$$^-350 + \blacksquare = {}^-450$$

What score goes in the box?

E 1. Find three different pairs of numbers that have a sum of $^-150$.

$$\blacksquare + \blacksquare = {}^-150$$

2. Does the order of the addends matter? Explain your reasoning.

F Luisa answers a 300-point question correctly and a 400-point question incorrectly. Luisa and Sam use different methods to keep score:

Luisa's Method		**Sam's Method**
$300 - 400 = {}^-100$	OR	$300 + {}^-400 = {}^-100$

Who is correct? Which methods work for other pairs of scores? Explain your reasoning.

ACE Homework starts on page 20.

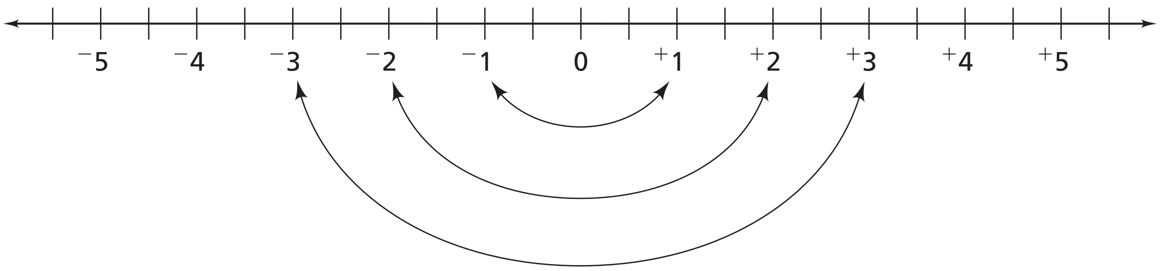
1.2 Extending the Number Line

Rational numbers are numbers that can be expressed as one integer *a* divided by another integer *b*, where *b* is not zero. You can write a rational number in the form $\frac{a}{b}$ or in decimal form.

For a rational number, $\frac{a}{b}$, why does *b* have to be nonzero?

- Are integers rational numbers? Explain.
- Is zero a rational number? Explain.

Each **negative number** can be paired with a **positive number.** These two numbers are called **opposites** because they are the same distance from zero on the number line, but in different directions.



Opposites

To avoid confusion with operation signs, you can use raised signs to show negative rational numbers, such as $^{-}150$. If a rational number does not have a sign, you can assume it is positive. For example, 150 is the same as $^{+}150$.

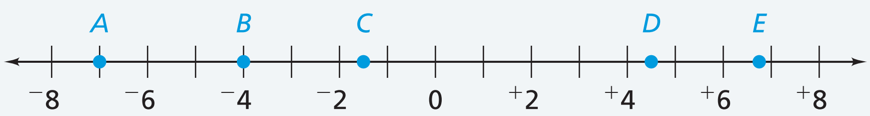
- Where would the following pairs of numbers be located on the number line?

$$7 \text{ and } {}^-7;\ \frac{21}{2} \text{ and } {}^-\frac{21}{2};\ {}^-3\frac{1}{2} \text{ and } 3\frac{1}{2};\ {}^-\frac{1}{2} \text{ and } \frac{1}{2}$$

- How would you graph the set of all numbers less than 4 on a number line? The numbers between 1 and ${}^-15\frac{1}{2}$?

Problem 1.2

A **1.** Estimate values for points A–E.



2. For each value you estimated in part (1), state the number's opposite.

3. A thermometer can be thought of as part of a vertical number line on which values above zero are positive. Sketch a thermometer (vertical number line), and place the following temperatures on it. Explain how you decided where each temperature should be placed.

0°F $^+115$°F $^-15$°F $^-32.5$°F $^+40$°F $^+113.2$°F $^-32.7$°F

4. How do the number lines from parts (1) and (3) help you find which of two numbers is greater?

B For each pair of temperatures, identify which temperature is further from $^-2$°F. Explain how you decided.

1. $^+6$°F or $^-6$°F?

2. $^-7$°F or $^+3$°F?

3. $^+2$°F or $^-7$°F?

4. $^-10$°F or $^+7$°F?

C Identify the temperature that is halfway between each pair of temperatures. Explain your reasoning.

1. 0°F and $^+10$°F

2. $^-5$°F and $^+15$°F

3. $^+5$°F and $^-15$°F

4. $^-8$°F and $^+8$°F

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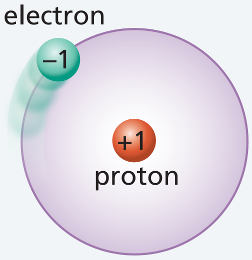
Problem 1.2 continued

D Integers are also used in chemistry. For example, a hydrogen atom has one proton, which has a charge of $^{+}1$, and one electron, which has a charge of $^{-}1$. The total charge of a hydrogen atom is $^{+}1 + {}^{-}1$, or 0. Describe three more real-life situations in which opposite quantities combine to make 0.



E Recall that the graph of an inequality is a sketch on a number line on which possible answers are shaded. For each part, graph the possible solutions for x on a number line.

1. x is positive.
2. x is less than or equal to $^{-}5$.
3. $x < {}^{-}7$
4. $x \geq 5$
5. $6 < x$
6. $^{-}1 \leq x$

F Find the values of x that satisfy the inequality. Then graph the solutions.

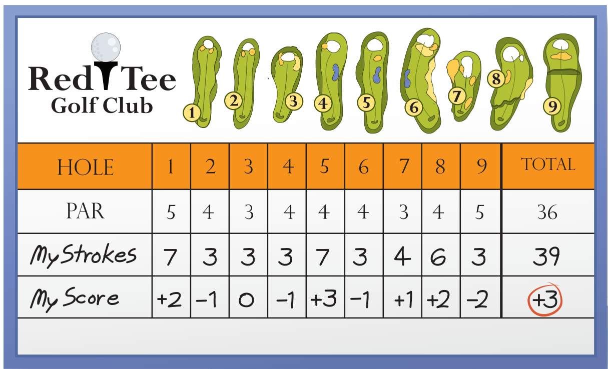
1. $x + 5 > 0$
2. $x - 1 \leq 0$
3. $3x < 9$

G Describe how you drew your graphs for Questions E and F.

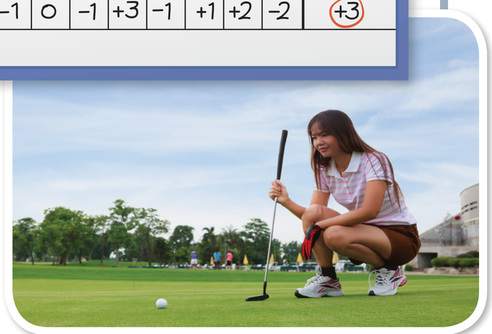
ACE Homework starts on page 20.

Did You Know?

In golf, scores can be negative. Each golf hole has a value called *par*. Par is the number of strokes a golfer usually needs to complete the hole. For example, a good golfer should be able to complete a par-4 hole in four strokes or less. If a golfer completes the hole in six strokes, then the score for that hole is "two over par" ($^{+}2$). A player's score for a round of golf is the total number of strokes above or below par. A winning score at a golf tournament is often negative. The lower the score, the better!



HOLE	1	2	3	4	5	6	7	8	9	TOTAL
PAR	5	4	3	4	4	4	3	4	5	36
My Strokes	7	3	3	3	7	3	4	6	3	39
My Score	+2	–1	0	–1	+3	–1	+1	+2	–2	+3



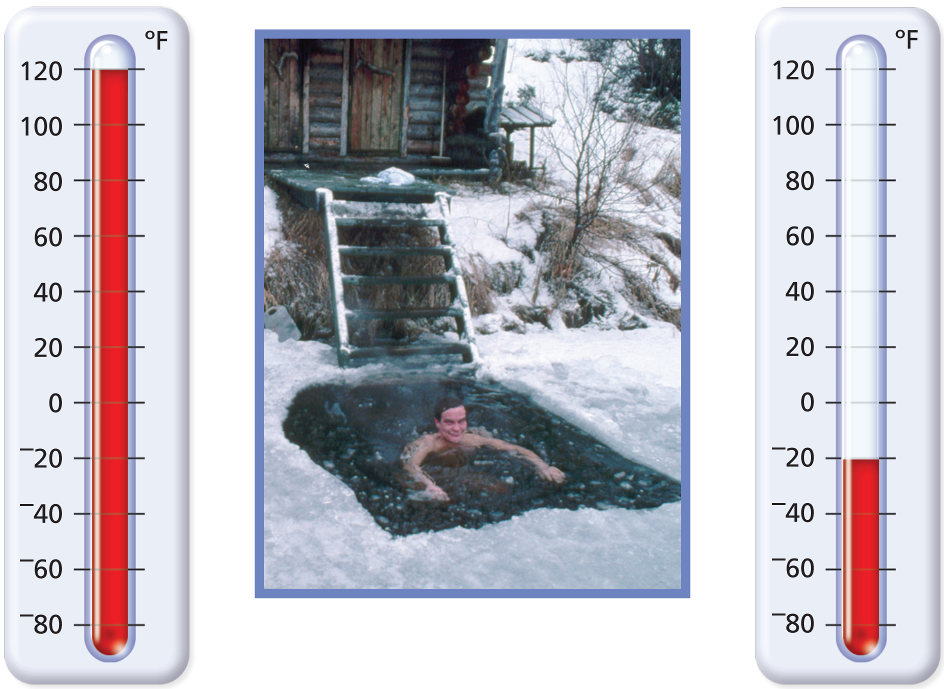
1.3 From Sauna to Snowbank

Using a Number Line

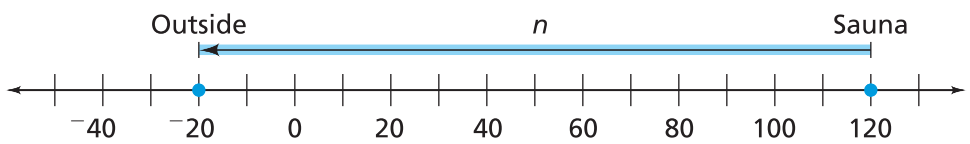
In Finland, people sit for a short time in sauna houses that are heated up to temperatures as high as 120°F. Then they go outside, where the temperature may be as low as $^{-}20$°F, to cool off.

The two thermometers shown are similar to vertical number lines. On a thermometer, a move down shows a decrease in value. The temperatures get colder. A move up shows an increase in value. The temperatures get hotter.

Inside the Sauna **Outside in Snow**



One horizontal number line can show the same information as the two thermometers.

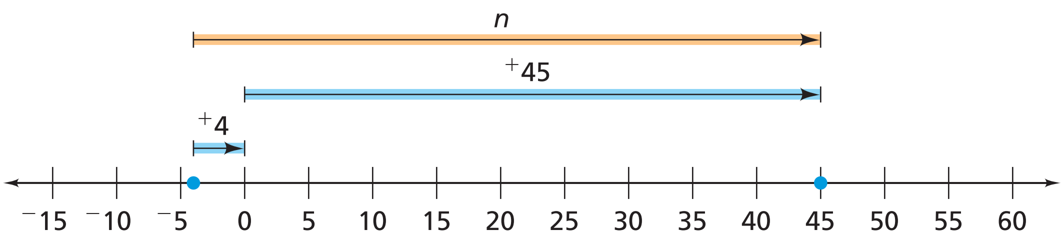


- What does n represent?
- What does the number sentence $120 + n = {}^{-}20$ tell you?
- What does the number sentence ${}^{-}20 + n = 120$ tell you?

On a number line, a move to the left is a move in a negative direction. The numbers decrease in value. A move to the right is a move in a positive direction. The numbers increase in value.

The National Weather Service keeps records of temperature changes. The world record for the fastest rise in outside air temperature occurred in Spearfish, South Dakota, on January 22, 1943. The temperature rose from $^-4$°F to 45°F in two minutes.

- What was the temperature change over those two minutes?
- How could you show this change, *n*, on the number line?



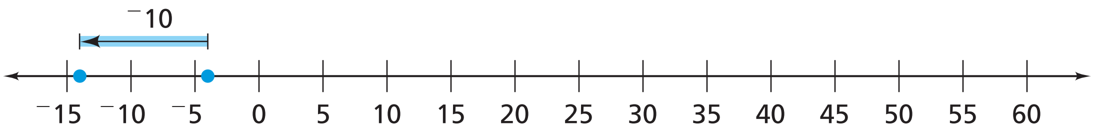
From $^-4$°F to 0°F is a change of $^+4$°F. From 0°F to 45°F is a change of $^+45$°F. The total change is $^+49$°F. The following number sentences show this.

$$^-4 + n = {}^+45$$

$$^-4 + {}^+49 = {}^+45$$

The sign of the change in temperature shows the direction of the change. In this case, $^+49$ means the temperature increased 49°F.

If the temperature had instead dropped 10°F from $^-4$°F, you would write the change as $^-10$°F. The final temperature would be $^-14$°F.



$$^-4 + {}^-10 = n$$

$$^-4 + {}^-10 = {}^-14$$

- If the current temperature is 5°F, what change in temperature would result in a final temperature of $^-25$°F?

Problem 1.3

Sketch number lines for Questions A–D. Write number sentences for Questions A–E.

A A person goes from a sauna at 115°F to an outside temperature of $^{-}30$°F. What is the change in temperature?

B The temperature reading on a thermometer is 25°F at noon. During the afternoon, the temperature changes. What is the new reading for each temperature change?

1. rises 10°F **2.** falls 2°F **3.** falls 30°F

C The temperature reading on a thermometer is $^{-}15$°F. What is the new reading for each temperature change?

1. $^{+}3$°F **2.** $^{-}10$°F **3.** $^{+}40$°F

D What is the change in temperature when the thermometer reading moves from the first temperature to the second temperature?

1. 20°F to $^{-}10$°F **2.** $^{-}20$°F to $^{-}10$°F

3. $^{-}20$°F to 10°F **4.** $^{-}10$°F to $^{-}20$°F

5. 20°F to 10°F **6.** 10°F to 20°F

7. Describe a strategy for finding the difference of two temperatures.

E **1.** The temperature was $^{-}5$°F when Sally went to school on Monday. The temperature rose 20°F during the day, but fell 25°F during the night. A heat wave increased the temperature 40°F on Tuesday, but then an arctic wind overnight decreased the temperature 70°F! What was the temperature on Wednesday? Explain how you found your answer.

2. Sally's work for finding Monday's temperature changes in part (1) is shown below. Do you agree with Sally's computation? Explain your reasoning.

$$
\begin{aligned}
^{-}5 + 20 + {}^{-}25 &= {}^{+}15 + {}^{-}25 \\
&= {}^{+}15 - 25 \\
&= {}^{-}10
\end{aligned}
$$

ACE Homework starts on page 20.

1.4 In the Chips

Using a Chip Model

When business records were kept by hand, accountants used red ink for expenses and black ink for income. If your income was greater than your expenses, you were "in the black." If your expenses were greater than your income, you were "in the red." You wanted to be "in the black."

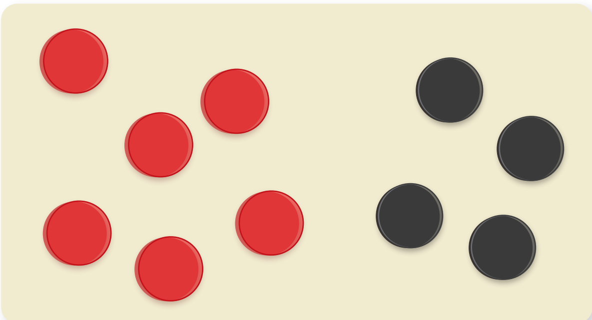
Julia has this problem to solve:

> Tate owes his sister \$6 for helping him cut the lawn. He earns \$4 delivering papers. Is Tate "in the red" or "in the black"?

To solve this problem, Julia uses red and black chips to model income and expenses. Each black chip represents $^{+}1$ dollar of income. Each red chip represents $^{-}1$ dollar of income (expenses).

Julia puts chips on the board to represent the situation.

Julia's Chip Board



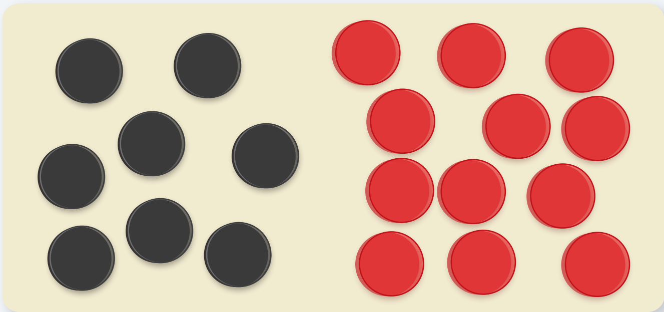
She decides that Tate is "in the red" 2 dollars, or has $^{-}2$ dollars. She writes

$$^{-}6 + {}^{+}4 = {}^{-}2$$

- Why do you think Julia concludes that $^{-}6 + {}^{+}4 = {}^{-}2$?
- What is another way to show a total value of $^{-}2$ on the chip board?
- What are some ways to show a total value of zero?
- Julia changes the board by adding one red chip and one black chip. By how much has Julia changed the total value?
- What groups of red and black chips can you add to the board that will not change the total value on the board?

Problem 1.4

A Use this chip board as the starting value for each part.



Write a number sentence to show the total value on the chip board for each move.

1. original chip board
2. add 5 black chips
3. remove 5 red chips
4. remove 3 black chips
5. add 3 red chips
6. What patterns do you see?

B Start with the original chip board from Question A.

1. Describe three ways to get a total value of $^-2$.
2. Describe three ways to get a total value of 0.
3. Describe three ways to get a total value of $^-4$.

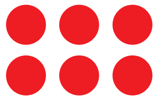
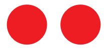
C Give three combinations of red and black chips (using at least one of each color) that will equal each value.

1. 0
2. $^+12$
3. $^-7$
4. $^-125$

continued on the next page >

Problem 1.4 continued

D Find the missing part for each chip problem. Write a number sentence for each problem.

	Start With	Rule	End With	Number Sentence
1.		Add 5 	■	■
2.		Subtract 3 	■	■
3.		■		■
4.	■	Subtract 3 		■

E Describe a chip board display that matches each number sentence. Find the missing value in each case.

1. $^{+}3 - {}^{+}2 =$ ■
2. $^{-}4 - {}^{+}2 =$ ■
3. $^{-}4 - {}^{-}2 =$ ■
4. $^{+}7 +$ ■ $= {}^{+}1$
5. $^{-}3 - {}^{+}5 =$ ■
6. ■ $- {}^{-}2 = {}^{+}6$

F Nadie has a chip board with 4 red chips. She needs to subtract 2 black chips, but there are no black chips on the board. Nadie says, “It is impossible to subtract 2 black chips. There are none on the board!” What can Nadie do to the chip board so that she can subtract 2 black chips? Explain your reasoning.

ACE Homework starts on page 20.

A C E Applications | Connections | Extensions

Applications

For Exercises 1–4, describe a sequence of five correct or incorrect answers that would produce each Math Fever score. Write a number sentence for each score.

1. Super Brains: 300

2. Rocket Scientists: $^{-}200$

3. Know-It-Alls: $^{-}250$

4. Teacher's Pets: 0

5. Multiple Choice Which numbers are listed from least to greatest?

A. 300, 0, $^{-}200$, $^{-}250$

B. $^{-}250$, $^{-}200$, 0, 300

C. 0, $^{-}200$, $^{-}250$, 300

D. $^{-}200$, $^{-}250$, 300, 0

For Exercises 6–8, find each Math Fever team's score. Write a number sentence for each team. Assume that each team starts with 0 points.

6. **Protons**

Point Value	Answer
250	Correct
100	Correct
200	Correct
150	Incorrect
200	Incorrect

7. **Neutrons**

Point Value	Answer
200	Incorrect
50	Correct
250	Correct
150	Incorrect
50	Incorrect

8. **Electrons**

Point Value	Answer
50	Incorrect
200	Incorrect
100	Correct
200	Correct
150	Incorrect

For each set of rational numbers in Exercises 9 and 10, draw a number line and locate the points. Remember to choose an appropriate scale.

9. $^{-}\frac{2}{8}, \frac{1}{4}, {}^{-}1.5, 1\frac{3}{4}$

10. $^{-}1.25, {}^{-}\frac{1}{3}, 1.5, {}^{-}\frac{1}{6}$

11. Order the numbers from least to greatest.

$23.6 \qquad {}^{-}45.2 \qquad 50 \qquad {}^{-}0.5 \qquad 0.3 \qquad \frac{3}{5} \qquad -\frac{4}{5}$

Copy each pair of numbers in Exercises 12–19. Then insert $<$, $>$, or $=$ to make each a true statement.

12. 3 ■ 0

13. $^{-}23.4$ ■ 23.4

14. 46 ■ $^{-}79$

15. $^{-}75$ ■ $^{-}90$

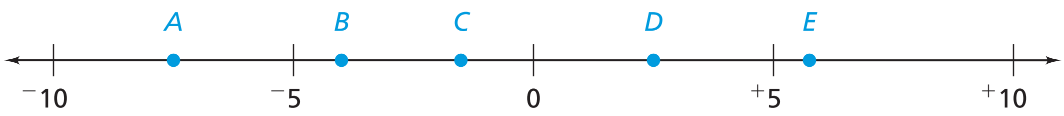
16. $^{-}300$ ■ 100

17. $^{-}1{,}000$ ■ $^{-}999$

18. $^{-}1.73$ ■ $^{-}1.730$

19. $^{-}4.3$ ■ $^{-}4.03$

20. **a.** Estimate values for points A–E.



b. On a copy of the number line, graph the following numbers.

$^{-}9 \qquad 10.5 \qquad \frac{1}{2} \qquad -\frac{5}{2}$

c. Describe the location of a number and its opposite on the number line.

21. For each pair of numbers, identify which number is farther from $^{+}1$. Explain your reasoning.

a. $^{-}7$ or $^{+}3$

b. $^{-}10$ or $^{+}7$

22. Identify the temperature that is halfway between each pair of temperatures.

a. $^{-}23$°F and $^{+}23$°F

b. $^{-}20$°F and $^{+}10$°F

c. $^{+}20$°F and $^{-}10$°F

Did You Know?

The record high and low temperatures in the United States are 134°F in Death Valley, California and $^{-}80$°F in Prospect Creek in the Endicott Mountains of Alaska. Imagine going from 134°F to $^{-}80$°F in an instant!

For Exercises 23–30, graph each statement on a number line.

23. x is less than 7.

24. x is greater than or equal to $^{-}7$.

25. $x < {}^{-}2$

26. $x \geq {}^{-}1$

27. $x \leq 8$

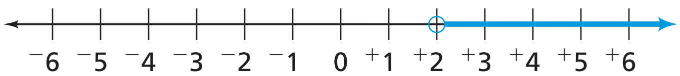
28. $x < 5$

29. $^{-}3 < x < 5$

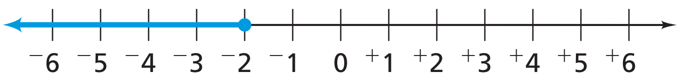
30. $x > {}^{-}6$

For Exercises 31–34, write an inequality for each set of numbers on the number line.

31.



32.



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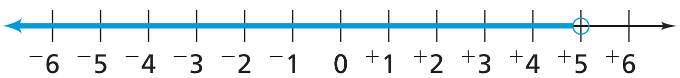
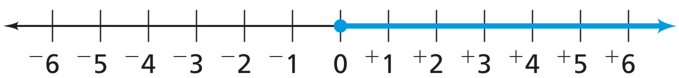


34.

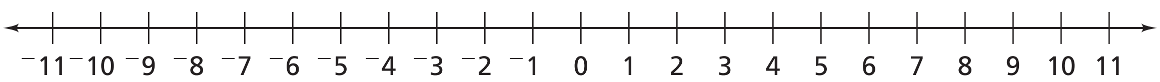


35. The school cafeteria can hold at most 150 people.

a. Write a number sentence to represent the number of people that can be in the cafeteria at any time during the day.

b. Graph your answer to part (a) on a number line.

For Exercises 36–45, follow the steps using the number line. What is the final position?



36. Start at 8. Add $^{-}7$.

37. Start at $^{-}8$. Add 10.

38. Start at $^{-}3$. Add $^{-}5$.

39. Start at 7. Add $^{-}7$.

40. Start at $^{-}2$. Add 12.

41. Start at 3. Subtract 5.

42. Start at $^{-}2$. Subtract 2.

43. Start at 4. Subtract 7.

44. Start at 0. Subtract 5.

45. Start at $^{-}8$. Subtract 3.

46. **a.** What are the opposites of 3, 7.5, and $^{-}2\frac{2}{3}$?

b. For each number in part (a), find the sum of that number and its opposite.

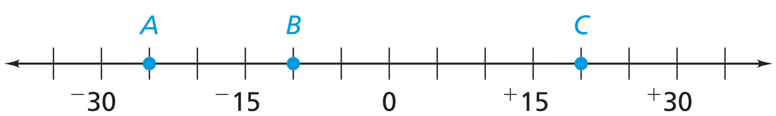
47. The greatest recorded one-day temperature change occurred in Browning, Montana (bordering Glacier National Park), from January 23–24, 1916. The temperature fell from 44°F to $^{-}56$°F in less than 24 hours.

a. What was the temperature change that day?

b. Write a number sentence to represent the change.

c. Show the temperature change on a number line.

48. Find the value for each labeled point on the number line. Then use the values to calculate each change.



a. *A* to *B*

b. *A* to *C*

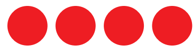
c. *B* to *C*

d. *C* to *A*

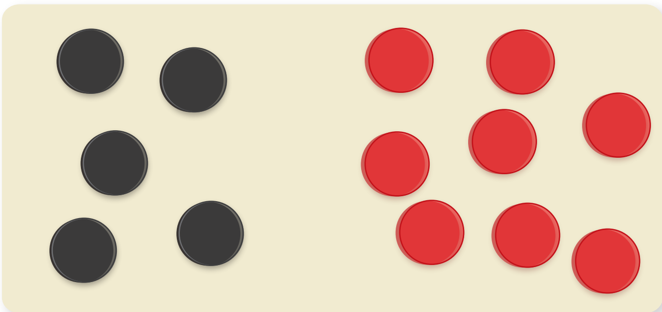
e. *B* to *A*

f. *C* to *B*

For Exercises 49–52, find the missing part for each chip problem. Write a number sentence for each problem.

	Start With	Rule	End With	Number Sentence
49.		Add 5 	■	■
50.		Subtract 3 	■	■
51.		■		■
52.	■	Subtract 3 		■

53. Write a story problem for this situation. Find the value represented by the chips on the board.



For Exercises 54 and 55, use the chip board from Exercise 53.

54. What is the new overall value of the board when you

a. remove 3 red chips?

b. then add 3 black chips?

c. then add 200 black chips and 195 red chips?

55. Describe three different ways to change the numbers of black and red chips on the original board, but leave the value of the board unchanged.

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Connections

56. In a football game, one team makes seven plays in the first quarter. The results of those plays are (in order): gain of 7 yards, gain of 2 yards, loss of 5 yards, loss of 12 yards, gain of 16 yards, gain of 8 yards, loss of 8 yards.

a. What is the overall gain (or loss) from all seven plays?

b. What is the average gain (or loss) per play?

For Exercises 57 and 58, find the total number of strokes over or under par for each golf player. Write number sentences with positive and negative integers to show each result.

	Player	Round 1	Round 2	Round 3	Round 4
57.	Elijah Sparks	4 over par	6 under par	3 under par	1 over par
58.	Keiko Aida	2 under par	1 under par	5 over par	5 under par

For Exercises 59–64, draw a number line and label it with an appropriate scale. Graph and name the two numbers described on the number line.

59. two fractions between 0 and 1

60. two fractions between 2 and 3

61. two fractions between $^{-}1$ and 0

62. two decimals between $^{-}3$ and $^{-}2$

63. two decimals between 4 and 5

64. two decimals between $^{-}4$ and $^{-}3$

There is always a rational number between two other rational numbers. For Exercises 65–67, graph the two numbers on a number line. Then graph and label a point between the two numbers.

65. 1.4 and 1.5

66. $^{-}1.42$ and $^{-}1.4$

67. $^{-}5\frac{1}{2}$ and $^{-}5\frac{1}{4}$

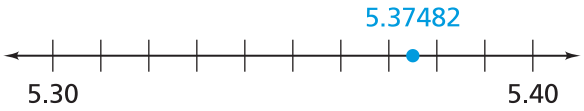
For Exercises 68 and 69, copy the number line below.

68. Label the first tick mark 28.36 and the last tick mark 28.37. Label the appropriate tick mark for 28.369. Then label the remaining tick marks.

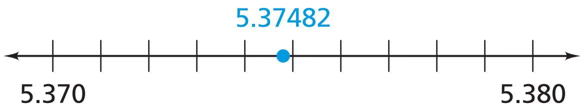
69. Label the first tick mark $^{-}7.7$ and the last tick mark $^{-}7.6$. Label the appropriate tick mark for $^{-}7.65$. Then label the remaining tick marks.

For Exercises 70 and 71, label the tick marks on each number line. Explain why you labeled them that way.

70.



71.



For Exercises 72–75, order the numbers from least to greatest.

72. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{5}{9}$, $\frac{9}{25}$

73. 20.33, 2.505, 23.30, 23

74. 1.52, $1\frac{4}{7}$, 2, $\frac{9}{6}$

75. 3, $\frac{19}{6}$, $2\frac{8}{9}$, 2.95

For Exercises 76 and 77, use the following. The highest point on earth is the top of Mount Everest. It is 29,035 feet above sea level. The lowest exposed land is the shore of the Dead Sea. It is 1,310 feet below sea level.

76. Multiple Choice What is the change in elevation from the top of Everest to the shore of the Dead Sea?

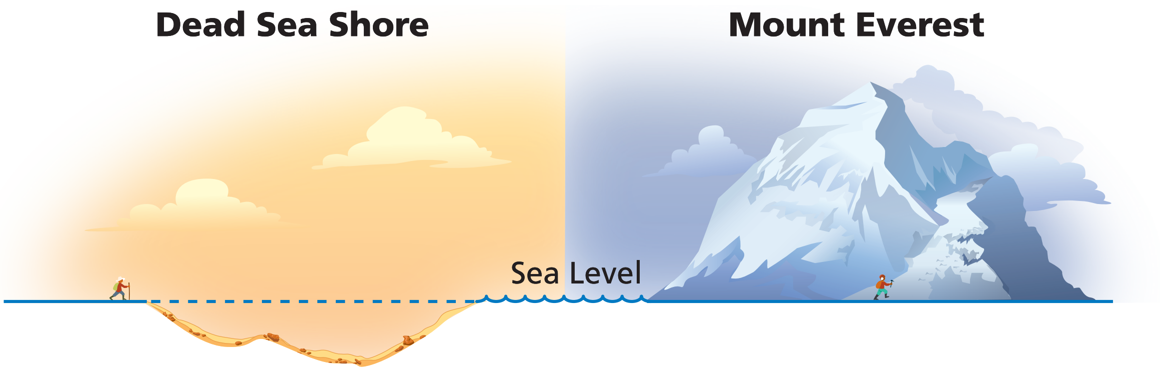
F. $^-30{,}345$ feet **G.** $^-27{,}725$ feet

H. 27,725 feet **J.** 30,345 feet

77. Multiple Choice What is the change in elevation from the shore of the Dead Sea to the top of Everest?

A. $^-30{,}345$ feet **B.** $^-27{,}725$ feet

C. 27,725 feet **D.** 30,345 feet



Extensions

78. At the start of December, Kenji had a balance of \$595.50 in his checking account. The following is a list of transactions he made during the month.

Date	Transaction	Balance
December 1		\$595.50
December 5	Writes a check for \$19.95	
December 12	Writes a check for \$280.88	
December 15	Deposits \$257.00	
December 17	Writes a check for \$58.12	
December 21	Withdraws \$50.00	
December 24	Writes checks for \$17.50, \$41.37, and \$65.15	
December 26	Deposits \$100.00	
December 31	Withdraws \$50.00	

a. Copy and complete the table.

b. What was Kenji's balance at the end of December?

c. When was his balance the greatest? When was his balance the least?

For Exercises 79–84, find all the values of *x* that satisfy the statement. Then sketch the solution on a number line.

79. $x + 2$ is negative.

80. $x - 5$ is greater than 0.

81. $x + 3 < 1$

82. $x + 3 \geq 2$

83. $3 - x < 0$

84. $6 \leq x - 4$

For Exercises 85–87, find the missing temperature in each situation.

85. On Monday, the high temperature was 20°C. The low temperature was $^{-}15$°C. What temperature is halfway between the high and the low?

86. On Tuesday, the low temperature was $^{-}8$°C. The temperature halfway between the high and the low is 5°C. What was the high temperature?

87. On Wednesday, the high temperature was $^{-}10$°C. The low temperature was $^{-}15$°C. What temperature is halfway between the high and the low?

Find values for *A* and *B* that make each mathematical sentence true.

88. $^{+}A + {}^{-}B = {}^{-}1$

89. $^{-}A + {}^{+}B = 0$

90. $^{-}A - {}^{-}B = {}^{-}2$

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Mathematical Reflections 1

In this Investigation, you learned ways to order and operate with positive and negative numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **How** do you decide which of two numbers is greater when
   - **a.** both numbers are positive?
   - **b.** both numbers are negative?
   - **c.** one number is positive and one number is negative?
2. **How** does a number line help you compare numbers?
3. **When** you add a positive number and a negative number, how do you determine the sign of the answer?
4. If you are doing a subtraction problem on a chip board, and the board does not have enough chips of the color you wish to subtract, **what** can you do to make the subtraction possible?

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Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Nick described his thoughts in the following way:

> *We used the number line to determine the temperature in Problem 1.3, Question E. We started at $^{-}5^{\circ}$F. Since the temperature rose 20°F during the day, we moved 20 tick marks to the right, which put us at 15°F. Since the temperature fell 25°F during the night, we moved 25 tick marks to the left and landed on $^{-}10^{\circ}$F. Then we moved 40 tick marks to the right to 30°F because of the heat wave. The temperature fell 70°F, so we moved to the left 70 tick marks. We are now at $^{-}40^{\circ}$F. We think it is unusual for the temperature to drop this much overnight and wonder where Sally lives.*
>
> **Common Core Standards for Mathematical Practice**
>
> **MP5** Use appropriate tools strategically.

- What other Mathematical Practices can you identify in Nick's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 2

Adding and Subtracting Rational Numbers

In Investigation 1 you used number lines and chip boards to model rational numbers. Now, you will develop algorithms for adding and subtracting rational numbers.

An **algorithm** is a plan, or a series of steps, for doing a computation. In an effective algorithm, the steps lead to a correct answer, no matter what numbers you use. Your class may develop more than one algorithm for each operation. Set a goal to understand and skillfully use at least one algorithm for adding rational numbers and one algorithm for subtracting rational numbers.

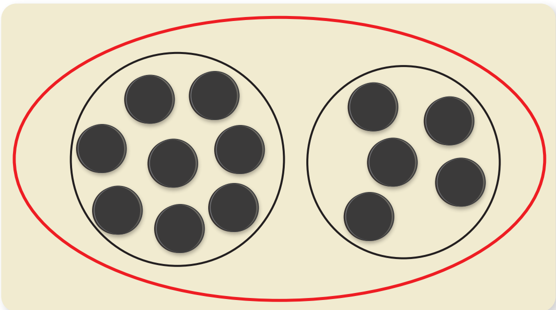
2.1 Extending Addition to Rational Numbers

There are two common ways that number problems lead to addition calculations like $8 + 5$. The first involves combining two similar sets of objects, as in this example:

> Linda has 8 video games, and her friend has 5.
> Together they have $8 + 5 = 13$ games.

You can represent this situation on a chip board.

$8 + 5 = 13$



Common Core State Standards

7.NS.A.1b Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses) . . .

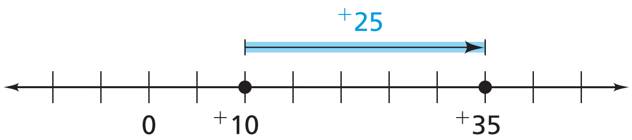
7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Also 7.NS.A.1, 7.NS.A.1d, 7.NS.A.3, 7.EE.B.3

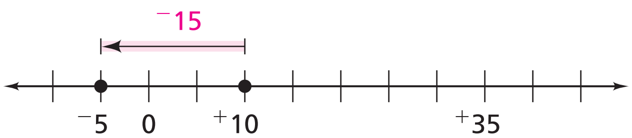
Number problems also lead to addition calculations when you add to a starting number. Here is an example:

> At a desert weather station, the temperature at sunrise was 10°C. It rose 25°C by mid-day. The temperature at noon was 10°C + 25°C = 35°C.

You can represent this situation on a number line. The starting point is $^{+}10$. The change in distance and direction is $^{+}25$. The sum ($^{+}35$) is the result of moving a distance of 25 to the right.



Suppose, instead of rising 25°C, the temperature fell 15°C. The next number line shows that $^{+}10°C + {^{-}15}°C = {^{-}5}°C$.



Suppose that the temperature change one day is $^{-}25$°C. What could the original temperature and the final temperature be for that day?

Use these ideas about addition as you develop an algorithm for addition of integers.

How can you predict whether the sum of two integers is 0, positive, or negative? Explain.

Problem 2.1

Use chip boards or number line models to solve these problems.

A **1.** Find the sums in each group.

Group 1	Group 2
${}^{+}2 + {}^{+}8$	${}^{+}2 + {}^{-}8$
${}^{-}2 + {}^{-}8$	${}^{-}2 + {}^{+}8$
${}^{+}8 + {}^{+}12$	${}^{+}8 + {}^{-}12$
${}^{-}8 + {}^{-}12$	${}^{-}8 + {}^{+}12$

2. What do the examples in each group have in common?

3. Write two new problems that belong to each group.

4. Describe an algorithm for adding the integers in each group.

B You know that ${}^{-}5 + {}^{-}3 = {}^{-}8$. Use this information to help you solve the following related problems.

1. ${}^{-}5\frac{1}{4} + {}^{-}3$

2. ${}^{-}5\frac{1}{5} + {}^{-}3\frac{3}{5}$

3. ${}^{-}5\frac{1}{3} + {}^{-}3\frac{2}{3}$

C You know that ${}^{-}8 + {}^{+}5 = {}^{-}3$. Use this information to help you solve the following related problems.

1. ${}^{-}8.35 + {}^{+}5$

2. ${}^{-}8.55 + {}^{+}5.3$

3. ${}^{-}8.65 + {}^{+}5.25$

4. Does your algorithm for adding integers from Question A work with fractions and decimals? Explain.

continued on the next page >

Problem 2.1 *continued*

D For parts (1)–(3), decide whether or not the expressions are equal.

1. $^{-}4 + ^{+}6$ and $^{+}6 + ^{-}4$
2. $^{+}2\frac{2}{3} + ^{-}5\frac{7}{8}$ and $^{-}5\frac{7}{8} + ^{+}2\frac{2}{3}$
3. $^{-}7\frac{2}{3} + ^{-}1\frac{1}{6}$ and $^{-}1\frac{1}{6} + ^{-}7\frac{2}{3}$
4. The property of rational numbers that you have observed in these pairs of problems is called the **Commutative Property** of addition. Explain why addition is commutative. Give examples using number lines or chip boards.

E
1. Find the sums in Group 3.
2. What do the examples in Group 3 have in common?
3. Write three new problems that belong to Group 3.

Group 3
$^{-}5 + ^{+}5$
$^{+}9.4 + ^{-}9.4$
$^{+}2\frac{1}{4} + ^{-}2\frac{1}{4}$

F Write a story to match each number sentence. Find the solutions.

1. $^{+}50 + ^{-}50 = \blacksquare$
2. $^{-}15 + \blacksquare = ^{+}25$
3. $^{-}300 + ^{+}250 = \blacksquare$

G
1. Use properties of addition to find each value.
   - **a.** $^{+}17 + ^{-}17 + ^{-}43$
   - **b.** $^{+}47 + ^{+}62 + ^{-}47$
2. Luciana claims that if you add numbers with the same sign, the sum is always greater than each of the addends. Is she correct? Explain.

ACE Homework starts on page 44.

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2.2 Extending Subtraction to Rational Numbers

In Problem 2.1, you explored some important properties of rational numbers. You found that the Commutative Property is true for addition of rational numbers.

You also found that the sum of an integer and its opposite is 0.

$$50 + {}^{-}50 = 0 \qquad {}^{-}17 + 17 = 0$$

Numbers such as 50 and -50 are **additive inverses** of each other. Their sum is 0. Zero is the **additive identity** for rational numbers. This means that zero added to a number does not change the value of the number.

$${}^{-}7 + 0 = {}^{-}7 \qquad \frac{1}{2} + 0 = \frac{1}{2}$$

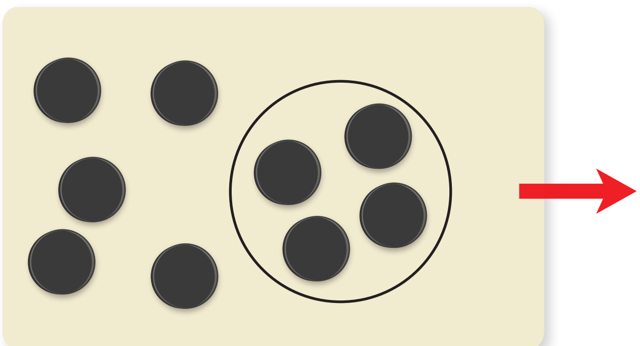
These properties will be useful as you explore subtraction problems with rational numbers.

One way to think about subtraction problems is to take away objects from a set, as in this example:

> Kim had 9 DVDs. She sold 4 at a yard sale. She now has $9 - 4 = 5$ of those DVDs left.

One way to represent this situation is to use a chip board

9 − 4 = 5



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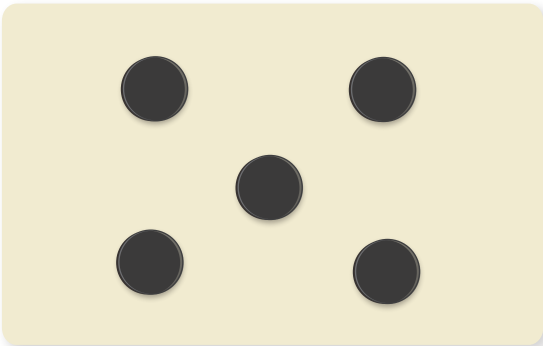
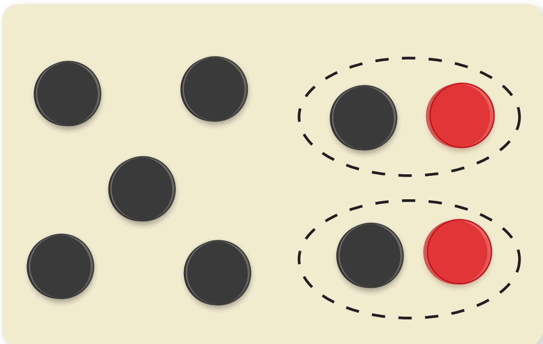
Here is another example:

> Otis earned \$5 raking leaves. He wants to buy a used bike that costs \$7. His older sister puts 5 black chips on the table to represent the money Otis has.



- What is the value of Otis's board?

Otis's sister asks, "How much more money do you need?" Otis replies, "I could find out by taking away \$7. But I can't take away \$7 because there aren't seven black chips on the board!" His sister adds two black chips and two red chips.



- Is the value of the board the same with the new chips added? Explain.
- How does this help Otis find how much more he needs?

You can also use subtraction to find the distance between two points:

> The Arroyo family just passed mile 25 on the highway. They need to get to the exit at mile 80.

- How many more miles do they have to drive?

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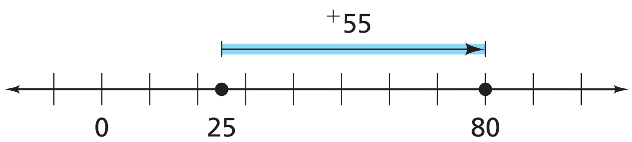
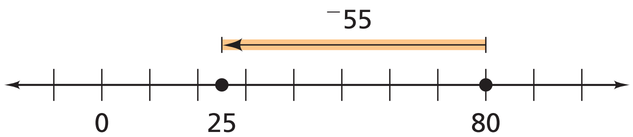
You can use a number line to show this difference.



The number line above shows that they have to travel $80 - 25 = 55$ more miles. The arrow on the number line points in the direction that the Arroyos are traveling. They are traveling in a positive direction, from lesser values to greater values.

Suppose the Arroyos drive back from mile 80 to mile 25. They would travel the same distance as before. However, they would travel in the opposite direction.



The number line above represents the Arroyos' distance as $25 - 80 = {}^{-}55$ miles. In this case, the arrow on the number line points to the left and has a label of $^{-}55$. Their distance is 55, but their direction is negative.

In some situations, such as driving, it makes more sense to describe an overall distance without including the direction. You can find the Arroyos' overall distance by taking the **absolute value** of the distance between the two points on the number line.

You can write two absolute value expressions to represent the distance between 25 and 80:

$$|25 - 80| \text{ and } |80 - 25|$$

You can evaluate these two expressions to show that the distance between the points 25 and 80 on a number line is 55.

$$|25 - 80| = |{}^{-}55| = 55 \text{ and } |80 - 25| = |55| = 55$$

How can you predict whether the difference of two integers is 0, positive, or negative?

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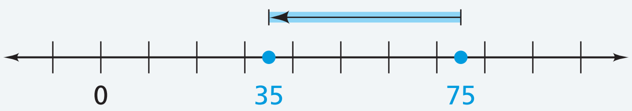
Problem 2.2

A Benjamin takes $75 from his savings. He goes shopping for school supplies and has $35 left when he is done. To figure out how much he has spent, he draws the following number line:



1. How much has he spent?
2. How should Benjamin label what he spent to show that this is money that he no longer has?

B During a game of Math Fever the Super Brains have a score of $^{-}500$ points. Earlier in the game, they incorrectly answered a question for $^{-}150$ points. However, the moderator later determined that the question was unfair. So $^{-}150$ points are taken away from their score.

1. Will subtracting $^{-}150$ points increase or decrease the Super Brains' score? Explain your reasoning.
2. What is the Super Brains' score after $^{-}150$ points are removed?
3. Write a number sentence to represent this situation, and show it on a number line.

C Use chip models or number line models to help solve the following.

1. Find the differences in each group given below.

Group 1
$^{+}12 - {}^{+}8$
$^{-}5 - {}^{-}7$
$^{-}4 - {}^{-}2$
$^{+}2 - {}^{+}4$

Group 2
$^{+}12 - {}^{-}8$
$^{-}5 - {}^{+}7$
$^{-}4 - {}^{+}2$
$^{+}2 - {}^{-}4$

2. What do the examples in each group have in common?
3. Write two new problems that belong to each group.
4. Describe an algorithm for subtracting integers in each group.

continued on the next page >

Problem 2.2 continued

D Apply the algorithm you developed on these rational number problems.

1. $^{-}1 - {}^{+}3$

2. $^{-}1 - {}^{+}\frac{3}{4}$

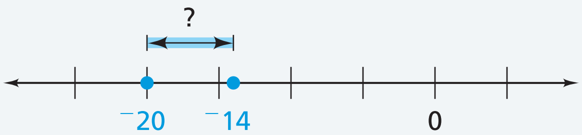
3. $^{-}1\frac{1}{2} - {}^{-}2$

4. $^{-}1\frac{1}{2} - {}^{-}\frac{3}{4}$

E **1.** Consider the points $^{-}10$ and 5 on a number line.

- **a.** Write two absolute value expressions to represent the distance between these two points.
- **b.** Evaluate both of your expressions. What is the distance between the points $^{-}10$ and 5 on a number line?
- **c.** Draw a number line to represent the distance you found in part (b).

2. Write two absolute value expressions for the distance between the two points on the number line below. Then evaluate your expressions.



F For parts (1)–(4), decide whether or not the expressions are equal.

1. $^{-}2 - {}^{+}3$ and $^{+}3 - {}^{-}2$

2. $^{+}12 - {}^{-}4$ and $^{-}4 - {}^{+}12$

3. $^{-}15 - {}^{-}20$ and $^{-}20 - {}^{-}15$

4. $^{+}45 - {}^{+}21$ and $^{+}21 - {}^{+}45$

5. Is there a Commutative Property of subtraction? Explain your answer.

ACE Homework starts on page 44.

2.3 The "+/ –" Connection

Addition and subtraction are related to each other in ways that can help you solve problems.

- If you know that $5 + {}^{-}8 = {}^{-}3$, how can this help you find the answer to $5 - 8$?

Examine these two expressions and think about how they are alike and how they are different.

$$A + {}^{-}B \text{ and } A - B$$

Substitute numbers for A and B and carry out the computations.

- What do your computations tell you about the two expressions: $A + {}^{-}B$ and $A - B$?

Think about points in a game like Math Fever. Write a story problem that could be represented by either expression.

As you work on Problem 2.3, look for ways that addition and subtraction are related.

Problem 2.3

Use your ideas about addition and subtraction of integers to explore the relationship between these two operations.

A The chip board in the picture below shows a value of $^{+}5$.

1. There are two possible moves, one addition and one subtraction, that would change the value on the board to $^{+}2$.
   a. How would you complete the number sentences to represent each move?

   $$^{+}5 + \blacksquare = {}^{+}2 \text{ and } {}^{+}5 - \blacksquare = {}^{+}2$$

   b. Describe how these moves are different on the chip board.

2. a. How would you complete the number sentences below to change the value on the board to $^{+}8$?

   $$^{+}5 + \blacksquare = {}^{+}8 \text{ and } {}^{+}5 - \blacksquare = {}^{+}8$$

   b. Describe how these moves are different on the chip board.

B 1. Complete each number sentence.

a. $^{+}5 + {}^{-}4 = {}^{+}5 - \blacksquare$

b. $^{+}5 + {}^{+}4 = {}^{+}5 - \blacksquare$

c. $^{-}7 + {}^{-}2 = {}^{-}7 - \blacksquare$

d. $^{-}7 + {}^{+}2 = {}^{-}7 - \blacksquare$

2. What patterns do you see from part (1) that can help you restate any addition problem as an equivalent subtraction problem?

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Problem 2.3 continued

C **1.** Think about how you can restate a subtraction problem as an addition problem. For example, how can you complete the number sentences below so that each subtraction problem is restated as an addition problem?

a. $^{+}8 - {}^{+}5 = {}^{+}8 + \blacksquare$

b. $^{+}8 - {}^{-}5 = {}^{+}8 + \blacksquare$

c. $^{-}4 - {}^{+}6 = {}^{-}4 + \blacksquare$

d. $^{-}4 - {}^{-}6 = {}^{-}4 + \blacksquare$

2. What patterns do you see from part (1) that can help you restate any subtraction problem as an equivalent addition problem?

D For parts (1)–(8), write an equivalent expression. Then choose one expression from each part, evaluate it, and explain why you chose to use that expression for the calculation.

1. $^{-}5 + {}^{-}5$

2. $^{-}5 - {}^{-}5$

3. $^{+}396 - {}^{-}400$

4. $^{-}75.8 - {}^{-}35.2$

5. $^{-}25.6 + {}^{-}4.4$

6. $^{+}\frac{3}{2} - {}^{+}\frac{1}{4}$

7. $^{+}\frac{5}{8} + {}^{-}\frac{3}{4}$

8. $^{-}3\frac{1}{2} - {}^{+}5$

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Note on Notation You have been writing rational numbers with raised signs to avoid confusion with the symbols for addition and subtraction. However, most computer software and most writing in mathematics do not use raised signs. Positive numbers are usually written without a sign.

$$^{+}3 = 3 \text{ and } ^{+}7.5 = 7.5$$

Negative numbers are usually written with a dash like a subtraction sign.

$$^{-}3 = -3 \text{ and } ^{-}7.5 = -7.5$$

From now on, we will use this notation to indicate a negative number. This can be confusing if you don't read carefully. Parentheses can help.

$$^{-}5 - {^{-}8} = -5 - -8 = -5 - (-8)$$

The subtraction symbol also indicates the opposite of a number. For example, -8 represents the opposite of 8. The expression $-(-8)$ represents the opposite of -8.

$$-(-8) = 8$$

2.4 Fact Families

You have written fact families for whole numbers:

$$3 + 2 = 5$$

$$2 + 3 = 5$$

$$5 - 3 = 2$$

$$5 - 2 = 3$$

Do the relationships below work for positive and negative numbers?

$$a + b = c \qquad a = c - b \qquad b = c - a$$

Problem 2.4

A For each part, choose values for a and b. Substitute those values into the three relationships below.

$$a + b = c \qquad a = c - b \qquad b = c - a$$

Then find the value of c.

1. a and b are positive rational numbers.
2. a and b are negative rational numbers.
3. a is a positive rational number, and b is a negative rational number.
4. a is a negative rational number, and b is a positive rational number.

For Parts B–F, use fact families to answer each question.

B Write a related subtraction sentence for each.

1. $-3 + (-2) = -5$
2. $25 + (-32) = -7$

C Write a related addition sentence for each.

1. $8 - (-2) = 10$
2. $-14 - (-20) = 6$

D
1. Write a related sentence for each.

   a. $n - 5 = 35$ **b.** $n - (-5) = 35$ **c.** $n + 5 = 35$

2. Do your related sentences make it easier to find the value of n? Why or why not?

E
1. Write a related sentence for each.

   a. $4 + n = 43$ **b.** $-4 + n = 43$ **c.** $-4 + n = -43$

2. Do your related sentences make it easier to find the value of n? Why or why not?

ACE Homework starts on page 44.

ACE Applications | Connections | Extensions

Applications

For Exercises 1–12, use your algorithms to find each sum without using a calculator.

1. $^{+}12 + ^{+}4$

2. $^{+}12 + ^{-}4$

3. $^{-}12 + ^{+}4$

4. $^{-}7 + ^{-}8$

5. $^{+}4.5 + ^{-}3.8$

6. $^{-}4.5 + ^{+}3.8$

7. $^{-}250 + ^{-}750$

8. $^{-}6200 + ^{+}1200$

9. $^{+}0.75 + ^{-}0.25$

10. $^{+}\frac{2}{3} + ^{-}\frac{1}{6}$

11. $^{-}\frac{5}{12} + ^{+}\frac{2}{3}$

12. $^{-}\frac{8}{5} + ^{-}\frac{3}{5}$

13. Find each sum.

a. $^{+}3.8 + ^{+}2.7$

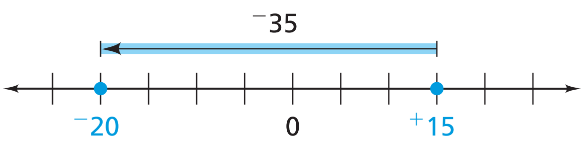
b. $^{-}3.8 + ^{-}2.7$

c. $^{-}3.8 + ^{+}2.7$

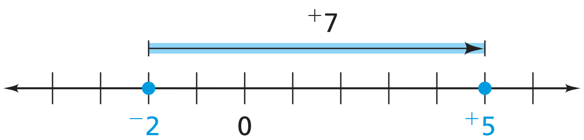
d. $^{+}3.8 + ^{-}2.7$

14. Write an addition number sentence that matches each diagram.

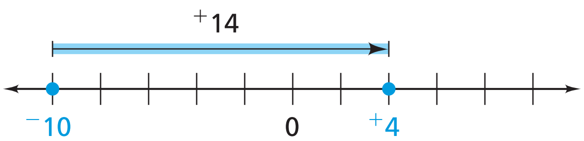
a.



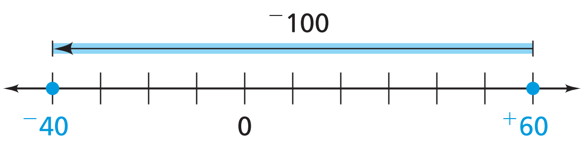
b.



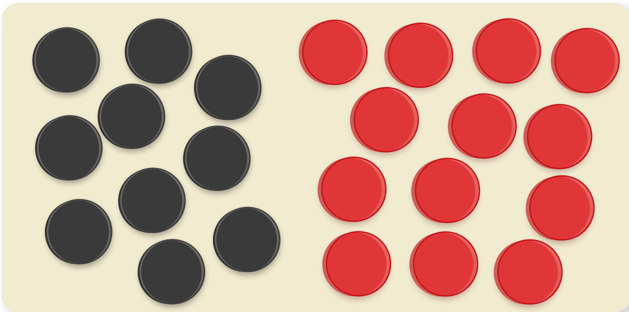
c.



d.



For Exercises 15 and 16, use the chip board below. The chip board has 10 black chips and 13 red chips.



15. What is the value shown on the board?

16. Write a number sentence to represent each situation. Then find the new value of the chip board.

a. Remove 5 red chips from the original board.

b. Then add 5 black chips.

c. Then add 4 black chips and 4 red chips.

17. Use properties of addition to find each value.

a. $^{+}43 + {}^{-}47 + {}^{-}43$ **b.** $^{+}5.2 + {}^{-}5.2 + {}^{-}\frac{4}{7}$ **c.** $^{+}5\frac{2}{5} + {}^{+}\frac{3}{7} + {}^{-}5\frac{2}{5}$

For Exercises 18–29, use your algorithms to find each difference without using a calculator. Show your work.

18. $^{+}12 - {}^{+}4$ **19.** $^{+}12 - {}^{+}12$ **20.** $^{-}12 - {}^{+}12$

21. $^{-}7 - {}^{+}8$ **22.** $^{+}45 - {}^{-}40$ **23.** $^{+}45 - {}^{-}50$

24. $^{-}25 - {}^{-}75$ **25.** $^{-}62 - {}^{-}12$ **26.** $^{+}0.8 - {}^{-}0.5$

27. $^{+}\frac{1}{2} - {}^{+}\frac{3}{4}$ **28.** $^{-}\frac{2}{5} - {}^{+}\frac{1}{5}$ **29.** $^{-}\frac{7}{10} - {}^{+}\frac{4}{5}$

30. Find each value without using a calculator.

a. $^{+}12 + {}^{-}12$ **b.** $^{+}4 - {}^{+}12$ **c.** $^{-}12 - {}^{+}4$

d. $^{-}12 - {}^{-}12$ **e.** $^{-}12 + {}^{-}12$ **f.** $^{-}12 + {}^{+}12$

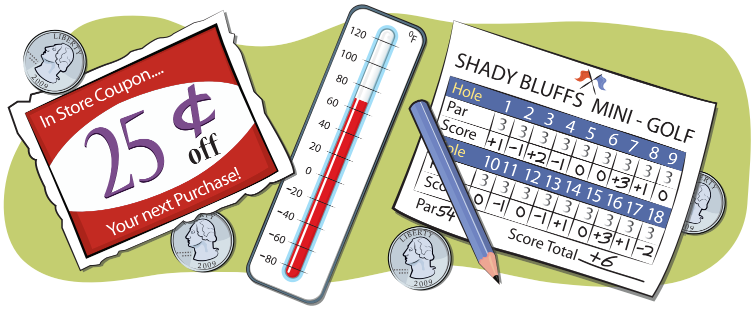
For Exercises 31–36, find each value.

31. $^{+}50 + {}^{-}35$ **32.** $^{+}50 - {}^{-}20$ **33.** $^{-}19 - {}^{+}11$

34. $^{-}30 - {}^{+}50$ **35.** $^{-}35 + {}^{-}15$ **36.** $^{+}12 + {}^{-}18$

37. For each part below, write a problem about temperature, money, or game scores that can be represented by the number sentence.

a. $^{+}7 - {}^{-}4 = {}^{+}11$ **b.** $^{-}20 + n = {}^{+}30$ **c.** $^{-}n + {}^{-}150 = {}^{-}450$



38. Without doing any calculations, decide which expression is greater. Explain your reasoning.

a. $5{,}280 + {}^{-}768$ or $5{,}280 - {}^{-}768$

b. $1{,}760 - {}^{-}880$ or $1{,}760 - 880$

c. $1{,}500 + 3{,}141$ or $1{,}500 - {}^{-}3{,}141$

39. Without doing any calculations, determine which of the following results are positive and which are negative. Explain your reasoning.

a. $^{-}23 + 19$ **b.** $3.5 - {}^{-}2.7$

c. $^{-}3.5 - {}^{-}2.04$ **d.** $3.1 + {}^{-}6.2$

40. Find each missing part.

	Start With	Rule	End With
a.	🔴🔴	■	⚫⚫⚫⚫⚫⚫⚫
b.	⚫⚫⚫	■	🔴🔴🔴
c.	■	Add 5 🔴	🔴🔴🔴
d.	■	Subtract 5 ⚫	🔴🔴

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For Exercises 41–46, find each sum or difference. Show your work.

41. $15 + {}^{-}10$

42. ${}^{-}20 - 14$

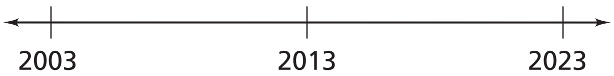
43. $200 - {}^{-}125$

44. ${}^{-}20 - {}^{-}14$

45. ${}^{-}200 + 125$

46. $7 - 12$

47. Below is part of a time line with three years marked.



a. Write two sentences in words that refer to the year 2013. One should relate 2013 to 2003, and the other should relate 2013 to 2023.

b. Write two number sentences that refer to the year 2013. One should relate 2013 to 2003, and the other should relate 2013 to 2023.

c. Describe how these two number sentences are alike and different.

48. Compute each of the following.

a. $3 + {}^{-}3 + {}^{-}7$

b. $3 - 3 - 7$

c. ${}^{-}10 + {}^{-}7 + {}^{-}28$

d. ${}^{-}10 - 7 - 28$

e. $7 - 8 + {}^{-}5$

f. $7 + {}^{-}8 - 5$

g. ${}^{-}97 + {}^{-}35 - 10$

h. ${}^{-}97 - 35 + {}^{-}10$

i. What can you conclude about the relationship between subtracting a positive number and adding a negative number with the same absolute value? In other words, what is the relationship between a $(- \, {}^{+})$ situation and a $(+ \, {}^{-})$ situation?

49. Compute each of the following.

a. $3 - {}^{-}3 - {}^{-}7$

b. $3 + 3 + 7$

c. ${}^{-}10 - {}^{-}7 - {}^{-}28$

d. ${}^{-}10 + 7 + 28$

e. $7 + 8 + 5$

f. $7 - {}^{-}8 - {}^{-}5$

g. ${}^{-}97 - {}^{-}35 - 10$

h. ${}^{-}97 + 35 + {}^{-}10$

i. What can you conclude about the relationship between subtracting a negative number and adding a positive number with the same absolute value? In other words, what is the relationship between a $(- \, {}^{-})$ situation and a $(+ \, {}^{+})$ situation?

Multiple Choice **In each set of calculations, one result is different from the others. Find the different result without doing any calculations.**

50. **A.** $54 + {}^{-}25$ **B.** $54 - 25$
C. $25 - 54$ **D.** ${}^{-}25 + 54$

51. **F.** ${}^{-}6.28 - {}^{-}3.14$ **G.** ${}^{-}6.28 + 3.14$
H. $3.14 + {}^{-}6.28$ **J.** ${}^{-}3.14 - {}^{-}6.28$

52. **A.** $534 - 275$ **B.** $275 - 534$
C. ${}^{-}534 + 275$ **D.** $275 + {}^{-}534$

53. **F.** $175 + {}^{-}225$ **G.** $225 - 175$
H. $175 - 225$ **J.** ${}^{-}225 + 175$

54. Fill in the missing information for each problem.

a. $5 + \frac{3}{4} = \blacksquare$ **b.** $\frac{4}{8} + (-6) = \blacksquare$ **c.** $-3\frac{3}{4} - \left(-\frac{3}{4}\right) = \blacksquare$
d. $2\frac{2}{3} - \frac{1}{3} = \blacksquare$ **e.** $-2 + \blacksquare = -2\frac{1}{2}$ **f.** $-4.5 + \blacksquare = -5$

55. **Multiple Choice** Which is the correct addition and subtraction fact family for $-2 + 3 = 1$?

A. $-2 + 3 = 1$, $-2 + 1 = 3$, $3 - 1 = 2$
B. $-2 + 3 = 1$, $3 - 2 = 1$, $3 - 1 = 2$
C. $-2 + 3 = 1$, $1 - 3 = -2$, $1 - (-2) = 3$
D. $1 - 3 = -2$, $1 - (-2) = 3$, $3 - 1 = 2$

56. For each of the following, write a related equation. Then find the value of n.

a. $n - 7 = 10$ **b.** $-\frac{1}{2} + n = -\frac{5}{8}$ **c.** $\frac{2}{3} - n = -\frac{7}{9}$

57. Are ${}^{+}8 - {}^{+}8$ and $8 - 8$ equal? Explain.

58. Are ${}^{+}100 - {}^{+}99$ and $100 - 99$ equal? Explain.

59. Are the expressions in each group below equivalent? If so, which form makes the computation easiest?

a. $8 + {}^{-}10$
$8 - {}^{+}10$
$8 - 10$

b. $3 + {}^{-}8$
$3 - {}^{+}8$
$3 - 8$

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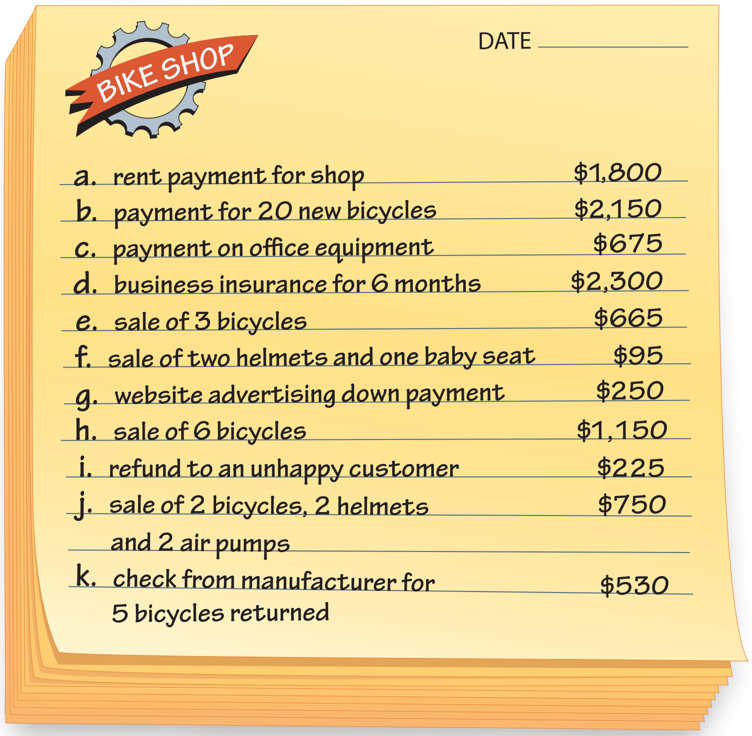
Connections

60. The Spartan Bike Shop keeps a record of their business transactions. They start their account at zero dollars. Write a number sentence to represent each transaction. Then find the new balance.



For Exercises 61 and 62, write a number sentence for each situation. Then answer the question.

61. The air temperature drops from 94°F to 72°F in 15 minutes. What is the change in temperature?

62. The Teacher's Pets team has 50 points in Math Fever. They miss a question worth 200 points. What is their new score?

63. Find four different numbers, in order from least to greatest, that lie between the two given numbers.

a. $^-4.5$ and $^-3.5$

b. $^-0.5$ and 0.5

Extensions

64. Which numbers, when added to $^-15$, give a sum

a. greater than 0?

b. less than 0?

c. equal to 0?

65. Use a number line to find the distance between each pair of numbers.

a. $^+8$, $^+4$

b. $^-8$, $^+4$

c. $^+8$, $^-4$

d. $^-8$, $^-4$

e. $^-3\frac{1}{2}$, $^+\frac{3}{4}$

f. $^+5.4$, $^-1.6$

66. Find each absolute value.

a. $|^+8 - {}^+4|$

b. $|^-8 - {}^+4|$

c. $|^+8 - {}^-4|$

d. $|^-8 - {}^-4|$

e. $|^-3\frac{1}{2} + {}^+\frac{3}{4}|$

f. $|^+5.4 - {}^-1.6|$

g. Compare the results of parts (a)–(f) with the distances found in Exercise 65. What do you notice? Why do you think this is so?

67. Replace n with a number to make each statement true.

a. $n + {}^-18 = 6$

b. $^-24 - n = 12$

c. $43 + n = {}^-12$

d. $^-20 - n = {}^-50$

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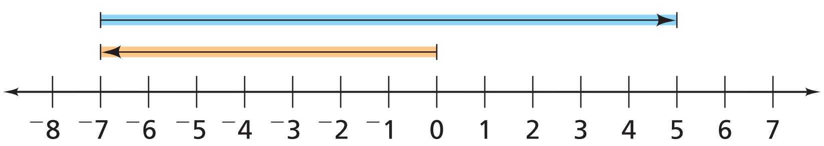
68. The table shows the profits or losses (in millions of dollars) earned by three companies from 2004 to 2013. Find the range of the annual results and the overall profit (or loss) for each company over that time period.

Top 3 Market Competitors

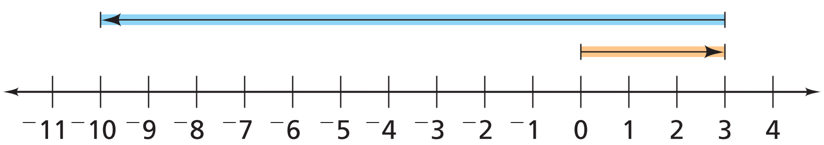
	Company	'04	'05	'06	'07	'08	'09	'10	'11	'12	'13
a.	Sands Motor	−5.3	−4.8	−7.2	−2.1	1.4	6.5	3.2	−3.5	10.2	2.4
b.	Daily Trans	6.0	3.4	−5.8	−12.3	−20.3	−1.5	2.5	9.8	19.4	32.1
c.	Sell to You	120	98	−20	−40	−5	85	130	76	5	−30

69. Starting from 0, write an addition sentence for the diagrams below.

a.



b.



Mathematical Reflections 2

In this Investigation you applied your informal ideas about rational numbers to develop algorithms for calculating any sums and differences. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. **What** algorithm(s) will produce the correct result for the sum "$a + b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

   b. **What** algorithm(s) will produce the correct result for the difference "$a - b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

2. **How** can any difference "$a - b$" be restated as an equivalent addition statement, where a and b each represent any rational number?

3. a. **What** does it mean to say that an operation is *commutative*?

   b. **Describe** some ways that the additive inverse of a number is important.

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Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Sophie described her thoughts in the following way:

We noticed there was a relationship between addition and subtraction in Problem 2.4. If you know that $7 + {}^{-}8 = {}^{-}1$, then you can write two equivalent subtraction problems: ${}^{-}1 - 7 = {}^{-}8$ and ${}^{-}1 - {}^{-}8 = 7$. You can also write subtraction number sentences as addition problems. These patterns enabled us to rewrite number sentences to make it easier to find the missing number and complete the sentence.

Common Core Standards for Mathematical Practice

MP7 Look for and make use of structure.

- What other Mathematical Practices can you identify in Sophie's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 3

Multiplying and Dividing Rational Numbers

In this Investigation, you will use time, distance, speed, and direction to think about multiplication and division of integers. You will also explore number patterns and develop algorithms for multiplying and dividing rational numbers.

You can use the times symbol, $\times$, the multiplication dot, $\bullet$, or parentheses, ( ), to show multiplication.

$$3 \times 5 = 3 \bullet 5 = 3(5)$$

Did You Know?

Florence Griffith Joyner set an Olympic record when she ran 100 meters in 10.62 seconds in 1988.

How long would it take her to run 1,000 meters at her record speed?

Common Core State Standards

7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$ . . .

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Also 7.NS.A.2, 7.NS.A.2a, 7.NS.A.3, 7.EE.B.3

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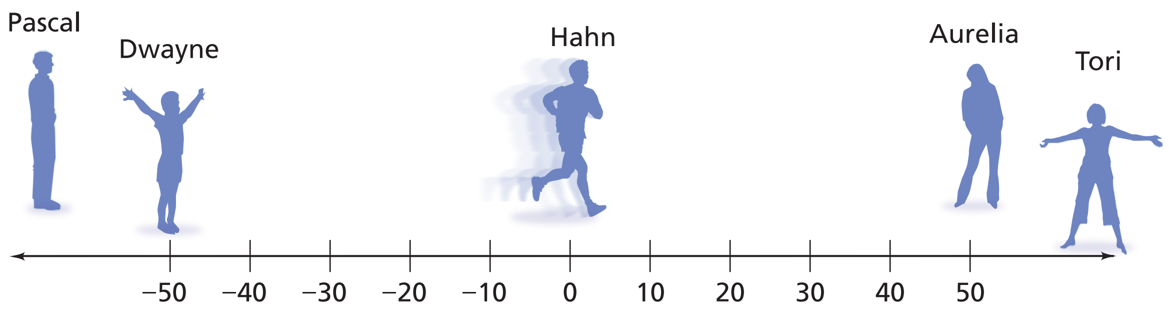
3.1 Multiplication Patterns With Integers

The math department at Everett Middle School sponsors a contest called the Number Relay Race. A number line measured in meters is drawn on the school field. Each team has five runners. Runners 1, 3, and 5 stand at the −50-meter line. Runners 2 and 4 stand at the 50-meter line.

Team 1



For Team 1:

- Hahn starts and runs from −50 to 50. He tags Aurelia.
- Aurelia runs back from 50 to −50. She tags Dwayne.
- Dwayne runs from −50 to 50. He tags Tori.
- Tori runs from 50 to −50. She tags Pascal.
- Pascal runs from −50 to the finish line at position 0.

Team 2 does the same with its 5 runners. Both teams line up on the same number line and start the race at the same time.

The team whose final runner reaches the 0 point first wins.

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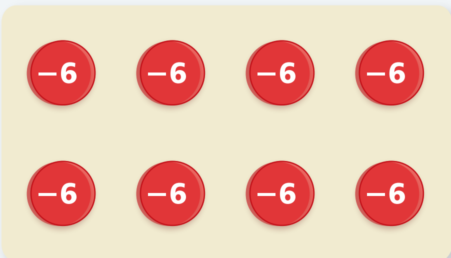
Problem 3.1

A **1.** Write and solve a number sentence for each situation. Use positive numbers for running speeds to the right, and use negative numbers for running speeds to the left. Use positive numbers for time in the future, and use negative numbers for time in the past. (Note: Each runner runs at a constant speed.)

- **a.** Hahn passes the 0 point running 5 meters per second to the right. Where will he be 6 seconds later?
- **b.** Aurelia passes the 0 point running to the left at 6 meters per second. Where will she be 8 seconds later?
- **c.** Dwayne passes the 0 point running 4 meters per second to the right. Where was he 6 seconds earlier?
- **d.** Tori passes the 0 point running to the left at 5 meters per second. Where was she 7 seconds earlier?

2. Determine whether the answer to each situation in part (1) is to the left or right of zero.

B Kalman wants to use red and black chips to model the relay race. He draws the following chip board to represent Aurelia's part of the race. How does this chip board relate to your work on Aurelia's part of the relay race? Explain.



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Problem 3.1 continued

C **1.** After studying the relay race problem, some students started looking for number patterns to see if what they found in the race made sense. How do the products change as the numbers multiplied by 5 decrease?

$$5 \cdot 3 = 15$$

$$5 \cdot 2 = 10$$

$$5 \cdot 1 = 5$$

$$5 \cdot 0 = 0$$

2. Predict the following products. Explain your reasoning.

$$5 \cdot (-1) = \blacksquare$$

$$5 \cdot (-2) = \blacksquare$$

$$5 \cdot (-3) = \blacksquare$$

D **1.** Find each product. How do the products change as the numbers multiplied by -4 decrease?

$$-4 \cdot 3 = \blacksquare$$

$$-4 \cdot 2 = \blacksquare$$

$$-4 \cdot 1 = \blacksquare$$

$$-4 \cdot 0 = \blacksquare$$

2. Predict the following products. Explain your reasoning.

$$-4 \cdot (-1) = \blacksquare$$

$$-4 \cdot (-2) = \blacksquare$$

$$-4 \cdot (-3) = \blacksquare$$

E **1.** The product $-4(-12)$ represents the location of a runner in the Number Relay. What question does the product answer? What location does it specify?

2. The product $4(-12)$represents the location of a runner in the Number Relay. What question does the product answer? What location does it specify?

3. How do the locations in parts (1) and (2) relate to each other?

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3.2 Multiplication of Rational Numbers

You have already examined patterns in multiplication of rational numbers that are integers. Now you will use patterns to develop algorithms for multiplication of rational numbers that include fractions and decimals.

Which of the following products will have the same value?

$4 \cdot 5$ $\quad 4 \cdot (-5)$ $\quad -4 \cdot 5$ $\quad -4 \cdot (-5)$

Problem 3.2

A **1.** What do the examples in each group below have in common?

Group 1	Group 2	Group 3
$4 \cdot 3$	$4 \cdot (-3)$	$-4 \cdot (-3)$
$5.1 \cdot 1$	$-5.1 \cdot 1$	$-5.1 \cdot (-1)$
$3 \cdot 4\frac{1}{2}$	$3 \cdot \left(-4\frac{1}{2}\right)$	$-3 \cdot \left(-4\frac{1}{2}\right)$

2. Find the products in each group above.

3. Write and solve two additional problems for each group.

B Find the products in each group below. Is multiplication commutative?

$$2 \times 3 \text{ and } 3 \times 2$$

$$-2 \times (-3) \text{ and } -3 \times (-2)$$

$$-2 \times 3 \text{ and } 3 \times (-2)$$

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Problem 3.2 continued

C **1.** Describe an algorithm for multiplying two positive rational numbers.

2. Describe an algorithm for multiplying a positive rational number and a negative rational number.

3. Describe an algorithm for multiplying a negative rational number and a negative rational number.

D **1.** For each product, predict the sign. Then find the product.

a. $7 \cdot (-8) \cdot (-3)$

b. $-12 \cdot (-5) \cdot (-4)$

c. $\frac{1}{2} \cdot \left(-\frac{2}{3}\right) \cdot 3$

2. Explain how you used what you know about multiplying two rational numbers to multiply three rational numbers.

E **1.** Predict whether the sign of each product is positive or negative. Explain your reasoning.

a. $2 \cdot 3 \cdot 4 \cdot 5$

b. $2 \cdot (-3) \cdot 4 \cdot 5$

c. $2 \cdot (-3) \cdot 4 \cdot (-5)$

d. $-2 \cdot (-3) \cdot 4 \cdot (-5)$

e. $-2 \cdot (-3) \cdot (-4) \cdot (-5)$

2. Find each product in part (1). Check whether your predictions are correct.

3. Explain how to determine whether a product will be positive or negative.

ACE Homework starts on page 66.

3.3 Division of Rational Numbers

You know that there is a relationship between addition and subtraction facts. A similar relationship exists between multiplication and division. For any multiplication fact, you can write another multiplication fact and two different related division facts. Here are three examples of rational-number fact families.

Example 1	Example 2	Example 3
$5 \cdot 3 = 15$	$6 \cdot (-3) = -18$	$4.5 \cdot (-2) = -9$
$3 \cdot 5 = 15$	$-3 \cdot 6 = -18$	$-2 \cdot 4.5 = -9$
$15 \div 3 = 5$ or $\frac{15}{3} = 5$	$-18 \div (-3) = 6$ or $\frac{-18}{-3} = 6$	$-9 \div (-2) = 4.5$ or $\frac{-9}{-2} = 4.5$
$15 \div 5 = 3$ or $\frac{15}{5} = 3$	$-18 \div 6 = -3$ or $\frac{-18}{6} = -3$	$-9 \div 4.5 = -2$ or $\frac{-9}{4.5} = -2$

Recall that a rational number can be written as $\frac{a}{b}$ where a and b are integers and b is not zero. Fact families help to clarify why division by zero is impossible. If $\frac{15}{0} = a$, then $a \cdot 0 = 15$ and $\frac{15}{0} = a$ are in the same fact family.

- How does a fact family show that $\frac{15}{0} = a$ cannot be a true statement for any value of a?

Problem 3.3

A Use what you know about fact families and multiplication to rewrite, if necessary, and find the missing value. Then find the missing value.

1. $-6 \times (-13) = \blacksquare$
2. $6 \times (-13) = \blacksquare$
3. $\blacksquare \times (-9) = 108$
4. $8 \times \blacksquare = -48$

B The team in Problem 3.1 runs another relay. Write division sentences that express your answers to the questions below.

1. Dwayne goes from 0 to 15 meters in 5 seconds. At what rate (meters per second) does he run?
2. Pascal reaches -12 meters only 3 seconds after passing 0. At what rate does he run?
3. Aurelia passes 0 running to the right at a rate of 5 meters per second. When did she leave the point -50? When did she leave the point -24?
4. Tori wants to reach the point -40, running to the left at 8 meters per second. How long will it take her from the time she passes 0?

C 1. What do the examples in each group have in common?

Group 1
$12 \div 3$
$4.5 \div 9$
$2\frac{1}{4} \div \frac{1}{2}$

Group 2
$12 \div (-3)$
$-4.5 \div 9$
$2\frac{1}{4} \div \left(-\frac{1}{2}\right)$

Group 3
$-12 \div (-3)$
$-4.5 \div (-9)$
$-2\frac{1}{4} \div \left(-\frac{1}{2}\right)$

2. Find the quotients in each group above.
3. Write and solve two additional problems for each group.
4. Describe an algorithm for dividing rational numbers.

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Problem 3.3 continued

D 1. Find the quotients in each group below. Is division commutative?

$$-2 \div 3 \text{ and } 3 \div (-2)$$

$$-12 \div (-4) \text{ and } -4 \div (-12)$$

$$16 \div 8 \text{ and } 8 \div 16$$

2. Give two other examples to support your answer to part (1).

E 1. Zero is the additive identity for addition. For example, $0 + a = a$, where a is a rational number. Explain in words what this means. Provide an example.

2. Is there a *multiplicative identity* n such that $a \cdot n = a$ for any rational number a? Explain.

F 1. Each rational number has an additive inverse. For example, $a + (-a) = 0$, where a is a rational number. Explain in words what this means. Provide an example.

2. Is there a *multiplicative inverse* b, such that $a \times b = 1$, for each rational number a? Explain.

G Use properties of multiplication and division to find each value. State which properties you use.

1. $\dfrac{\frac{5}{4} \times 7}{\frac{5}{4}}$

2. $\frac{3}{5}\left(\frac{5}{3}\right)$

3. $0.2 \times 3 \times \frac{1}{0.2}$

4. $\frac{1.3 \times 8.2}{1.3}$

5. $\frac{4}{5}\left(\frac{10}{3} + \frac{15}{6}\right)$

6. $-\frac{3}{5}\left(-\frac{8}{21}\right)\left(-\frac{5}{3}\right)$

7. $1.6 \times \frac{5}{8} - 2.4 \times \frac{5}{8}$

8. $\dfrac{-2.4 \times \frac{4}{7}}{\frac{4}{7}}$

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Problem 3.3 *continued*

H Recall that some fractions have decimals that terminate. For example, $\frac{3}{4} = 0.75$. Other fractions have decimals that repeat. For example, $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$. The 3 repeats.

1. State whether each fraction will *terminate* or *repeat*. Then write each fraction as a decimal.

   a. $\frac{2}{5}$ **b.** $\frac{3}{8}$ **c.** $\frac{-5}{6}$

   d. $\frac{35}{10}$ **e.** $\frac{8}{-9}$ **f.** $\frac{-3}{-11}$

2. List two other fractions that will terminate and two that will repeat. Give their decimal representations.

ACE Homework starts on page 66.

Note on Notation You know that a rational number is any number that you can write in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. When a rational number is negative, the negative sign can be associated with the numerator, the denominator, or the entire fraction. For positive integers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

For example, suppose $a = 6$ and $b = 2$.

$$\frac{-6}{2} = \frac{6}{-2} = -\frac{6}{2} = -3$$

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3.4 Playing the Integer Product Game

Applying Multiplication and Division of Integers

You have developed algorithms for adding, subtracting, multiplying, and dividing integers. You will apply your multiplication and division algorithms in the Integer Product Game.

The game board consists of a list of factors and a grid of products. To play, you need a game board, two paper clips, and colored markers or chips.

Integer Product Game

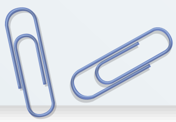
Rules

1. Player A puts a paper clip on a number in the factor list.
2. Player B puts the other paper clip on any number in the factor list, including the number chosen by Player A. Player B then marks the product of the two factors on the product grid.
3. Player A moves *either one* of the paper clips to another number. He or she then marks the new product with a different color than Player B.
4. Each player takes turns moving a paper clip and marking a product. A product canonly be marked by one player.
5. The winner is the first player to mark four squares in a row (up and down, across, or diagonally).

−36	−30	−25	−24	−20	−18
−16	−15	−12	−10	−9	−8
−6	−5	−4	−3	−2	−1
1	2	3	4	5	6
8	9	10	12	15	16
18	20	24	25	30	36

Factors:

−6 −5 −4 −3 −2 −1 1 2 3 4 5 6



- What product would give the least number? What product would give the greatest number?

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Problem 3.4

Play the Integer Product Game with positive and negative factors.
Look for strategies for picking the factors and products.

A What strategies did you find useful in playing the game? Explain.

B What pair(s) of numbers from the factor list will give each product?

1. 5
2. -12
3. 12
4. -25

C Your opponent puts a paper clip on -4. List five products that you can form, assuming they are not marked. Tell where you would need to put your paper clip in each case.

D Describe the moves to make in each case.

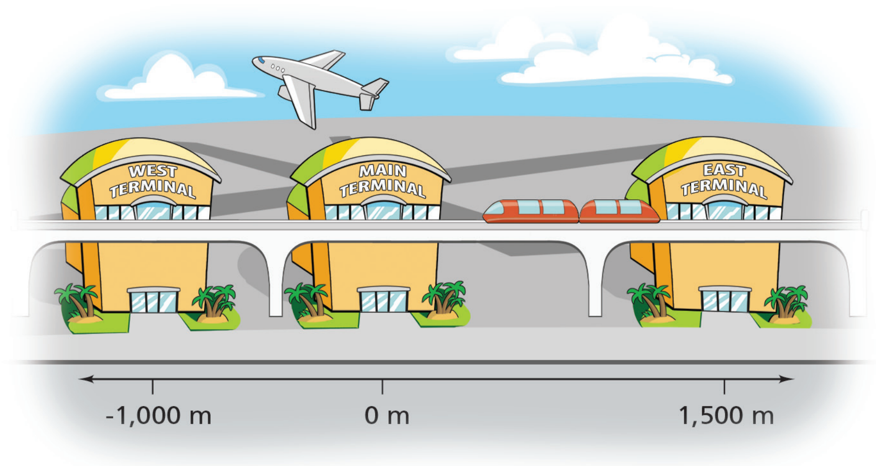
1. The paper clips are on -5 and -2. You want a product of -15.
2. The paper clips are on -3 and -2. You want a product of -6.
3. Your opponent will win with 24. What numbers should you avoid with your paper clip moves?

E Mia thinks the game could also be called the Division Game. Explain why Mia might think this.

ACE Homework starts on page 66.

Applications

1. At some international airports, trains carry passengers between the separate terminal buildings. Suppose that one such train system moves along a track like the one below.



a. A train leaves the main terminal going east at 10 meters per second. Where will it be in 10 seconds? When will it reach the east terminal?

b. A train passes the main terminal going east at 10 meters per second. Where was that train 15 seconds ago? When was it at the west terminal?

c. A train leaves the main terminal going west at 10 meters per second. Where will it be in 20 seconds? When will it reach the west terminal?

d. A train passes the main terminal going west at 10 meters per second. When was it at the east terminal? Where was it 20 seconds ago?

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2. Julia thinks a bit more about how to use red and black chips to model operations with integers. She draws the following chip board. She decides it represents $8 \times (-5) = -40$ and $-40 \div 8 = -5$. Explain why Julia's reasoning makes sense.

Use Julia's reasoning from Exercise 2 to find each value.

3. $10 \times (-5)$
4. $4 \times (-15)$
5. $3 \times (-5)$
6. $-14 \div 2$
7. $-14 \div 7$
8. $-35 \div 7$

9. Find each product.

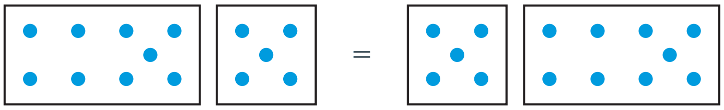
   a. $7 \cdot 2$
   b. $-7 \cdot (-2)$
   c. $7 \cdot (-2)$
   d. $-7 \cdot 2$
   e. $8 \cdot 2.5$
   f. $-9 \cdot (-4)$
   g. $12 \cdot (-3)$
   h. $-1.5 \cdot 4$
   i. $3.5 \cdot 7$
   j. $-8.1 \cdot (-1)$
   k. $1 \cdot (-6)$
   l. $-2\frac{1}{2} \cdot 1$

10. Tell whether each product is greater than or less than zero.

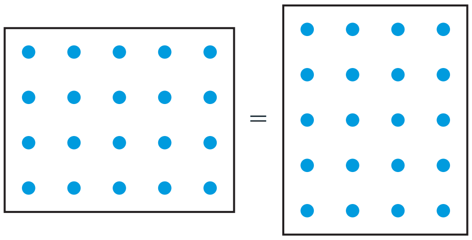
   a. $5 \cdot (-7)$
   b. $-3.2 \cdot 1.5$
   c. $10.5 \cdot (-4)$
   d. $-2 \cdot (-3) \cdot (-1)$
   e. $-\frac{2}{3} \cdot 2\frac{3}{4}$
   f. $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \left(-\frac{7}{4}\right)$
   g. $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \frac{7}{4}$
   h. $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \left(-\frac{7}{4}\right) \cdot \left(-2\frac{3}{8}\right)$
   i. $\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \frac{7}{4} \cdot \left(-2\frac{3}{8}\right)$
   j. $\frac{3}{4} \cdot 1\frac{5}{6} \cdot \frac{7}{4} \cdot \left(-2\frac{3}{8}\right)$

The dot patterns illustrate commutative properties for operations on whole numbers. Write a number sentence for each case.

11.



12.



13. Find the values for each pair.

a. $4 \cdot (-3)$ and $-3 \cdot 4$

b. $2 \cdot (-4)$ and $-4 \cdot 2$

c. $-2 \cdot (-3)$ and $-3 \cdot (-2)$

d. $\frac{1}{5} \cdot \left(-\frac{4}{9}\right)$ and $-\frac{4}{9} \cdot \frac{1}{5}$

e. What can you conclude about multiplication with negative numbers?

14. You have located fractions such as $-\frac{5}{7}$ on a number line. You have also used fractions to show division: $\frac{-5}{7} = -5 \div 7$ and $\frac{5}{-7} = 5 \div (-7)$. Tell whether each statement is *true* or *false*. Explain.

a. $\frac{-1}{2} = \frac{1}{-2}$

b. $-\frac{1}{2} = \frac{-1}{-2}$

15. For each number sentence, find a value for n that makes the sentence true.

a. $24 \div 2 = n$

b. $-24 \div (-2) = n$

c. $24 \div n = -12$

d. $n \div 2 = -12$

e. $5 \div 2.5 = n$

f. $-12 \div n = 3$

g. $n \div (-3) = -4$

h. $(-16) \div \frac{1}{4} = n$

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For Exercises 16–18, write four related multiplication and division facts for each set of integers.

Sample 27, 9, 3

$9 \cdot 3 = 27$

$3 \cdot 9 = 27$

$27 \div 9 = 3$

$27 \div 3 = 9$

16. $7, -3, -21$ **17.** $-4, -5, 20$ **18.** $1.5, -3, -4.5$

For Exercises 19–24, determine whether the product of or quotient of each expression is greater than, less than, or equal to 0 without doing any calculations. Explain your reasoning.

19. $-1{,}105.62 \div 24.3$

20. $0 \cdot (-67)$

21. $-27.5 \cdot (-63)$

22. $0 \div 89$

23. $-54.9 \div (-3)$

24. $-2{,}943 \cdot 1.06$

25. Use the multiplication and division algorithms you developed to find each value.

a. $12 \cdot 9$ **b.** $5 \cdot (-25)$ **c.** $-220 \div (-50)$

d. $48 \div (-6)$ **e.** $-63 \div 9$ **f.** $\frac{2}{-3} \cdot \left(\frac{-4}{5}\right)$

g. $\frac{-99}{33}$ **h.** $-2.7 \div (-0.3)$ **i.** $-36 \cdot 5$

j. $52.5 \div (-7)$ **k.** $-2\frac{1}{2} \cdot \left(-\frac{2}{3}\right)$ **l.** $9 \div 5$

m. $-9 \cdot (-50)$ **n.** $\frac{-96}{24}$ **o.** $6 \cdot 1\frac{1}{2}$

p. $-\frac{5}{8} \cdot \frac{8}{5}$ **q.** $4 \cdot \left(-1\frac{1}{4}\right)$ **r.** $-2.5 \cdot 2\frac{1}{5}$

Multiple Choice **For Exercises 26 and 27, find each value.**

26. $-24 \div 4$

A. -96 **B.** -6 **C.** 6 **D.** 96

27. $-10 \cdot (-5)$

F. -50 **G.** -2 **H.** 2 **J.** 50

Use properties of multiplication and division to find each value. State which properties you use.

28. $\dfrac{\frac{2}{7} \cdot 9}{\frac{2}{7}}$

29. $\frac{3}{8} \cdot (-4) \cdot 3.5 \cdot \frac{8}{3}$

30. $-1.3 \cdot 5 \cdot (-6) \cdot 0.2$

31. $\dfrac{4.9 \cdot 5.8}{4.9}$

For Exercises 32–35, state whether each fraction will *terminate* or *repeat*. Write each fraction as a decimal.

32. $\frac{-5}{9}$ **33.** $\frac{7}{-8}$ **34.** $\frac{-4}{-11}$ **35.** $\frac{5}{16}$

36. Chris and Elizabeth are making a version of the Integer Product Game in which players need three products in a row to win.

Chris and Elizabeth's Product Game

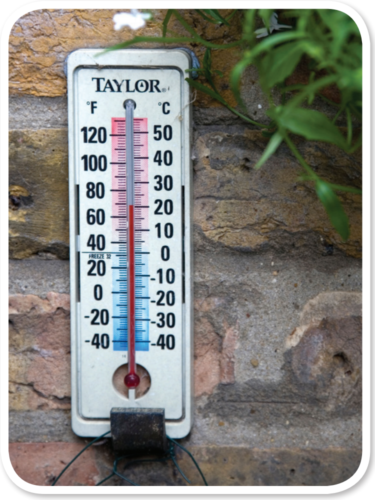
4	–4	6	–6
9	–9	10	–10
15	–15	25	–25

Factors:

What six factors do they need for their game? Explain your reasoning.

Connections

37. The temperature changed $-2°$F per hour from noon on Tuesday until 10:00 A.M. the next morning. The temperature at noon on Tuesday is shown.



- **a.** What was the temperature at 4:00 P.M. on Tuesday?
- **b.** What was the temperature at 9:00 A.M. on Wednesday?

Write a number sentence to represent each situation. Then answer the question.

38. The Extraterrestrials have a score of -300. They answer four 50-point questions incorrectly. What is their new score?

39. The Super Computers answered three 100-point questions incorrectly. They now have 200 points. What was their score before answering the three questions incorrectly?

40. A football team is at its own 25-yard line. In the next three plays, it loses an average of 4 yards per play. Where is the team after the three plays?

41. A new convenience store wishes to attract customers. For a one-day special, the store sells gasoline for \$.25 per gallon below its regular cost per gallon. Suppose the store sells 5,750 gallons of gas that day. What is the store's profit or loss in comparison to the amount the store would have made without the special?

42. Multiply or divide. Show your work.

- **a.** $52 \cdot 75$
- **b.** $52 \cdot (-75)$
- **c.** $-2{,}262 \div (-58)$
- **d.** $\frac{2}{3} \cdot \frac{4}{5}$
- **e.** $-9{,}908 \div 89$
- **f.** $-7.77 \div (-0.37)$
- **g.** $-34 \cdot 15$
- **h.** $53.2 \div (-7)$
- **i.** $\frac{-2}{3} \cdot \frac{6}{8}$
- **j.** $90 \div 50$
- **k.** $-90 \cdot (-50)$
- **l.** $-108 \div 24$
- **m.** $19.5 \div (-3)$
- **n.** $-8.4 \cdot 6$
- **o.** $6 \cdot 2\frac{1}{2}$
- **p.** $-3\frac{2}{3} \cdot (-9)$
- **q.** $4 \cdot \left(-1\frac{1}{4}\right)$
- **r.** $-2.5 \cdot 2\frac{1}{5}$

43. The list below gives average temperatures (in °C) for a city for each month of the year, from January through December.

$$-25, -20, -13, -2, 9, 15, 17, 14, 7, -4, -16, -23$$

a. What is the median?

b. What is the range?

c. What is the mean?

d. Number the months from 1 (for January) through 12 (for December). Plot a graph of the (month, temperature) data.

44. Find the sum, difference, product, or quotient.

a. $-5 - 18$ **b.** $-23 + 48$ **c.** $\frac{3}{4} \cdot \left(-\frac{5}{9}\right)$

d. $119 + (-19.3)$ **e.** $-1.5 - (-32.8)$ **f.** $12 \div 15$

g. $-169 \div (-1.3)$ **h.** $0.47 - 1.56$ **i.** $6 \cdot (-3.5)$

j. $\frac{2}{-3} \div \frac{5}{6}$ **k.** $\frac{7}{12} - \left(-\frac{2}{3}\right)$ **l.** $-\frac{4}{5} \div \left(-\frac{1}{4}\right)$

45. Estimate the sum, difference, product, or quotient.

a. $-52 - 5$ **b.** $-43 + (-108)$ **c.** $2\frac{3}{4} \cdot \left(-\frac{5}{9}\right)$

d. $79 + (-25.3)$ **e.** $-12.5 - (-37.3)$ **f.** $89 \div 15$

g. $-169 \div (-13)$ **h.** $6.3 - 1.86$ **i.** $61 \cdot (-3.9)$

j. $-\frac{2}{3} \div \left(1\frac{5}{6}\right)$ **k.** $5\frac{7}{12} - \left(-\frac{2}{3}\right)$ **l.** $-\frac{4}{5} \div \left(-\frac{1}{4}\right)$

Find integers to make each sentence true.

46. $\blacksquare \cdot \blacksquare = 30$

47. $\blacksquare \cdot \blacksquare = -30$

48. $-24 \div \blacksquare = \blacksquare$

Extensions

Determine whether each statement is *always*, *sometimes*, or *never* *true*. Explain.

49. If m and n are positive rational numbers, then $m + n$ is positive.

50. If m and n are negative rational numbers, then $m + n$ is negative.

51. If m is a positive rational number and n is a negative rational number, then $m + n$ is negative.

52. If m and n are positive rational numbers, then $m \times n$ is positive.

53. If m and n are negative rational numbers, then $m \times n$ is negative.

54. To add $5 + 3 + 2$, you might think that it is easier to add the $3 + 2$ and then add the answer to the 5. The mathematical property that allows you to change the grouping of addends (or factors) is called the *Associative Property*.

Test the Associative Property for addition and multiplication of integers by simplifying the expressions below. Find the values within the parentheses first. When you need a grouping symbol like parentheses inside another set of parentheses, you can use brackets to make it easier to read. For example, $(4 - (-6))$ can be written as $[4 - (-6)]$.

a. $[3 \cdot (-3)] \cdot 4$ and $3 \cdot (-3 \cdot 4)$

b. $(-5 \cdot 4) \cdot (-3)$ and $-5 \cdot [4 \cdot (-3)]$

c. $[-2 \cdot (-3)] \cdot (-5)$ and $-2 \cdot [-3 \cdot (-5)]$

d. $(3 \cdot 4) \cdot (-5)$ and $3 \cdot [4 \cdot (-5)]$

e. $[3 + (-3)] + 4$ and $3 + (-3 + 4)$

f. $(-5 + 4) + (-3)$ and $-5 + [4 + (-3)]$

g. $[-2 + (-3)] + (-5)$ and $-2 + [-3 + (-5)]$

h. $(3 + 4) + (-5)$ and $3 + [4 + (-5)]$

i. Does the Associative Property work for addition and multiplication of integers?

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Tell whether each statement is *true* or *false*. Explain.

55. $-1 = -1 + 0$

56. $-3\frac{3}{8} = -\frac{21}{8}$

57. $-6.75 = -6 + \left(-\frac{3}{4}\right)$

For Exercises 58 and 59, write a story for a problem that is answered by finding the value of *n*.

58. $-4n = -24$

59. $\frac{n}{2} = 16$

60. Find a set of addends to make a Sum Game. Each sum on the board below should be the sum of two numbers (possibly a single number added to itself). Each pair of numbers should add to a sum on the board.

Hint: You need 11 numbers, all with different absolute values.

Sum Game

–24	–22	–20	–18	–16	–14
–12	–11	–10	–9	–8	–7
–6	–5	–4	–3	–2	–1
0	1	2	3	4	5
6	7	8	9	10	11
12	14	16	18	20	22

Factors:

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Mathematical Reflections 3

In this Investigation, you studied ways to use multiplication and division of rational numbers to answer questions about speed, time, distance, and direction of motion. You used the results of those calculations to develop algorithms for multiplying and dividing any two rational numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Give** an example of a multiplication problem, involving two integers, in which the product is

   a. less than 0. **b.** greater than 0. **c.** equal to 0.

   d. In general, describe the signs of the factors for each product in parts (a)–(c).

2. **Give** an example of a division problem, involving two integers, in which the quotient is

   a. less than 0. **b.** greater than 0. **c.** equal to 0.

   d. In general, describe the signs of the dividend and divisor for each quotient in parts (a)–(c).

3. **a.** Suppose three numbers are related by an equation of the form $a \cdot b = c$, where a, b, and c are not equal to 0. Write two related number sentences using division.

   b. Suppose three numbers are related by an equation of the form, $a \div b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication.

4. **Which** operations on integers are commutative? Give numerical examples to support your answer.

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Hector described his thoughts in the following way:

Jake and I knew that a negative number times a negative number is positive. This helped us predict the sign of the product of a set of numbers in Problem 3.2. If there is an even number of numbers that are negative, we can group these numbers in pairs. The product of each pair is positive. Then we are left with positive numbers, so the final product is positive. If there are an uneven number of negative numbers, the product is negative because there will always be a negative number that is not paired with another negative number. And a negative times a positive is negative.

Common Core Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

- What other Mathematical Practices can you identify in Hector's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 4

Properties of Operations

When you study new types of numbers, you need to know what properties apply to them. You know that both addition and multiplication of rational numbers are commutative.

$$-\frac{2}{3} + \frac{1}{6} = \frac{1}{6} + \left(-\frac{2}{3}\right) \text{ and } -\frac{2}{3} \cdot \frac{1}{6} = \frac{1}{6} \cdot \left(-\frac{2}{3}\right)$$

You also used a set of conventions called the *Order of Operations* to help you decide how to carry out a computation.

1. Compute all expressions within parentheses or brackets first.
   Note: To avoid confusion, you use brackets in sentences that contain many parentheses.
2. Compute all numbers with exponents.
3. Then compute all multiplications and divisions in order from left to right.
4. Then compute all additions and subtractions in order from left to right.

Common Core State Standards

7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically . . .

Also 7.NS.A.1, 7.NS.A.2, 7.NS.A.2d

Using the Order of Operations, you get results like this:

$$
\begin{aligned}
7\tfrac{1}{2} + \left(6 \cdot 4\tfrac{1}{2} - 9\right) \div 3 &= 7\tfrac{1}{2} + (27 - 9) \div 3 \\
&= 7\tfrac{1}{2} + 18 \div 3 \\
&= 7\tfrac{1}{2} + 6 \\
&= 13\tfrac{1}{2}
\end{aligned}
$$

The following example shows why these conventions are necessary.

The soccer team orders 20 new jerseys from Custom Jersey Designs. The total cost is represented by the equation $C = 100 + 15n$, where C is the cost in dollars and n is the number of jerseys ordered. Pedro and David calculate the total cost.

Pedro's calculation:
$$
\begin{aligned}
C &= 100 + 15 \cdot 20 \\
&= 100 + 300 \\
&= \$400
\end{aligned}
$$

David's calculation:
$$
\begin{aligned}
C &= 100 + 15 \cdot 20 \\
&= 115 \cdot 20 \\
&= \$2{,}300
\end{aligned}
$$

- Who did the calculation correctly?

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4.1 Order of Operations

The Order of Operations applies to calculations involving positive and negative numbers. The following questions provide practice in using the Order of Operations.

In a game called Dealing Up, a player draws four cards. The player uses all four cards to write a number sentence that gives the greatest possible result.

What is the greatest result you can make from two of the following numbers? Three? Four?

$-25 \quad +2 \quad -3 \quad +3$

Problem 4.1

A Jamar and Elena are playing Dealing Up. Jamar draws the following four cards:

1. Jamar writes $5 - (-6) \cdot 4 + (-3) = 41$. Elena says the result should be 26. Who is correct and why?
2. Elena starts by writing $-3 - (-6) + 5^4$. What is her result?
3. Insert parentheses into $-3 - (-6) + 5^4$ to give a greater result than in part (2).

continued on the next page >

Problem 4.1 continued

B Find each value.

1. $-7 \cdot 4 + 8 \div 2$
2. $(3 + 2)^2 \cdot 6 - 1$
3. $2\frac{2}{5} \cdot 4\frac{1}{2} - 5^3 + 3$
4. $8 \cdot (4 - 5)^3 + 3$
5. $-8 \cdot [4 - (-5 + 3)]$
6. $-16 \div 8 \cdot 2^3 + (-7)$

C Use parentheses, where needed, to make the greatest and least possible values.

1. $7 - 2 + 3^2$
2. $46 + 2.8 \cdot 7 - 2$
3. $25 \cdot (-3.12) + 21.3 \div 3$
4. $5.67 + 35.4 - 178 - 181$

D Rodrigo performs the following computation:

$$3 + 2 \cdot 7 - 6$$

His answer is 29.

1. Explain how Rodrigo obtained his answer.
2. Is Rodrigo's answer correct? If not, what is the correct answer? Explain.

E Use the Order of Operations to find the value. Show your work.

$$3 + 4 \cdot 5 \div 2 \cdot 3 - 7^2 + 6 \div 3 = \blacksquare$$

ACE Homework starts on page 86.

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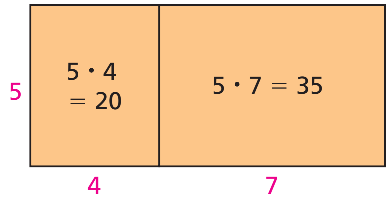
4.2 The Distributive Property

Recall that you can use the Distributive Property to rewrite an expression. The rewritten expression may be easier to calculate or may give new information.

An expression written as a sum of terms is in *expanded form*. If the terms have a common factor, then you can use the Distributive Property to write an equivalent expression. You can write the expression as a product of the common factor and the sum of the other two factors. This is called *factored form*.

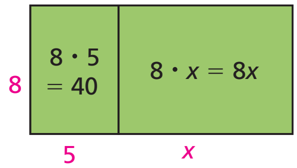
With integers: $20 + 35 = 55$

$$5 \cdot 4 + 5 \cdot 7 = 5 \cdot (4 + 7)$$



With a variable: $40 + 8x = 8 \cdot 5 + 8 \cdot x$

$$8 \cdot 5 + 8 \cdot x = 8 \cdot (5 + x)$$



You can also use the Distributive Property to rewrite expressions with negative numbers. Use the Distributive Property to multiply the first factor by each number in the second factor and add the two resulting products.

With integers: $-3 \cdot (4 + 8) = -3 \cdot 4 + (-3) \cdot 8$

With a variable: $-2 \cdot (x + 6) = -2 \cdot x + (-2) \cdot 6$

When you apply the Distributive Property to rewrite $5 \cdot x + 5 \cdot (-2.5)$ as $5 \cdot [x + (-2.5)]$, you are factoring out the common factor 5 from the two parts of the sum. When you write the equivalent expression $5 \cdot [x + (-2.5)]$, you can say you have factored the expression into the product of two terms, 5 and $[x + (-2.5)]$.

- Will the values of these expressions be the same or different?

$$-2 \cdot (-3 - 7) \qquad 2 \cdot (3 + 7)$$

Problem 4.2

A The checkbook shows Juan's bank account balance at the beginning of the week. During the week he withdraws \$19 and \$30. How much money does he have left at the end of the week?

1. Show two different ways to solve this problem.
2. Describe how the two ways are different.

B Use the Distributive Property to write each expression in expanded form.

1. $5 \cdot (3 + 2)$
2. $5 \cdot [3 + (-2)]$
3. $5 \cdot (3 - 2)$
4. $5 \cdot [3 - (-2)]$
5. For parts (1)–(4), find the value of each expression.

C Use the Distributive Property to write each expression in factored form.

1. $6 \cdot 2 + 6 \cdot 3$
2. $6 \cdot 2 - 6 \cdot 3$
3. $-6 \cdot 2 + (-6) \cdot 3$
4. $-6 \cdot 2 - (-6) \cdot 3$
5. $5x - 8x$
6. $-3x - 4x$
7. Explain how to factor an expression with subtraction.

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Problem 4.2 continued

D 1. If you apply the Order of Operations to $3\frac{1}{2} \cdot 15 - 3\frac{1}{2} \cdot 5$, you get $52\frac{1}{2} - 17\frac{1}{2} = 35$. What value do you get if you use the Distributive Property first to factor $3\frac{1}{2} \cdot 15 - 3\frac{1}{2} \cdot 5$?

2. If you apply the Order of Operations to $17(8.5 - 3.5)$, you get $17 \cdot 5 = 85$. What value do you get if you apply the Distributive Property first to $17(8.5 - 3.5)$?

3. What would you say to someone who is wondering whether to apply the Distributive Property or the Order of Operations first?

E Ling claims she has another way to show that $-1(-1) = 1$. Ling wrote her reasoning:

> By the Distributive Property, I know that -1[1 + (-1)] equals -1(1) + (-1)(-1).
> Because 1 + (-1) equals 0, I also know that -1[1 + (-1)] equals -1(0), or 0.
> So -1(1) + (-1)(-1) must equal 0. I know -1(1) equals -1,
> so -1 + (-1)(-1) must equal 0.
> Therefore, -1(-1) must equal 1.

Do you agree with Ling's reasoning? Does $(^{-}1)(^{-}1) = {}^{+}1$?

F In parts (1) and (2), use the Order of Operations and properties of operations to compute each expression. Give your answers as decimals.

1. **a.** $(1 + 5 + 3) \div 4$ **b.** $\frac{1}{4} + \frac{5}{4} + \frac{3}{4}$

2. **a.** $(1 + 5 - 2) \div 3$ **b.** $\frac{1}{3} + \frac{5}{3} - \frac{2}{3}$

3. What can you say about the expressions in parts (1) and (2)?

4. How would you describe the relationship between the Distributive Property and division?

ACE Homework starts on page 86.

4.3 What Operations Are Needed?

In the questions below, you will use what you have learned about operations on rational numbers to solve problems. Always ask yourself the following question:

- What operation(s) do you need to solve the problem, and how do you know?

Problem 4.3

A Three friends are going hiking with Latisha. For each of the four hikers, she buys two bottles of water and three packs of trail mix. The bottles of water cost $1.50 each, and the packs of trail mix cost $3.75 each.

1. **a.** Can Latisha go through the express checkout lane for customers with 15 or fewer individual items?

b. Write a number sentence to show how you found the total number of items Latisha bought.

c. Write a different number sentence that shows a different way to find the total number of items.

d. Explain how you know which operation(s) to use.

2. Latisha has $60. Does she have enough money to pay for the items?

B Mr. Chan buys a roll of paper towels for $2.19 and a bottle of window cleaner for $2.69. In his state, there is a 4% sales tax on these items. Mr. Chan also buys a gallon of milk for $3.95. There is no sales tax on milk. Mr. Chan has a $5 coupon to use at the store.

1. Write a number sentence to find Mr. Chan's total bill. What is his total bill?

2. Is there more than one way to compute this total?

3. Explain how you know which operation(s) to use.

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Problem 4.3 continued

C Eli's class held a fund raising event last month. The class's expenses were \$5.75, \$4.75, and \$3.75. The amounts of money the class raised were \$13.50, \$24.70, \$13.15, and \$19.50.

1. Write a number sentence to find how much money the class has after the fundraiser. Did the class make any money? If so, how much?
2. Can you use two different orders of computation?
3. Explain how you know which operation(s) to use.

D The following sequence of scoring occurred during a Math Fever game between the Super Brains and the Rocket Scientists.

Super Brains: −50, +150, −100, +250, −150
Rocket Scientists: −150, +250, −50, −50, +100

1. Write a number sentence to find each team's score. Who is ahead at this stage of the game? By how many points?
2. Can you use a different number sentence to find each team's score?

E The table shows the hourly amount of water flowing into and out of a water tower for given time periods. For example, for the first 4 hours, 5,000 gallons flowed into the tower each hour.

Water Tower Water Flow

Water Flow In (gallons per hour)	Water Flow Out (gallons per hour)	Time (hours)
5,000	0	4
4,000	0	7
0	7,500	3
5,000	3,000	6.5

1. **a.** If there are 5,000 gallons of water at the start, how much is there at the end of the entire time period?
   b. What number sentence shows your reasoning?
2. **a.** What was the average rate of water flow per hour in the first 11 hours?
   b. What was the average rate of flow of water per hour in the last 9.5 hours?
   c. At the end of the entire time period, what was the average rate of flow (in or out) per hour?

ACE Homework starts on page 86.

Applications

1. Find the values of each pair of expressions.

a. $-12 + (-4 + 9)$ $\quad$ $[-12 + (-4)] + 9$

b. $(14 - 20) - 8$ $\quad$ $14 - (20 - 8)$

c. $[14 + (-20)] + (-8)$ $\quad$ $14 + [-20 + (-8)]$

d. $-1 - [-1 + (-1)]$ $\quad$ $[-1 - (-1)] + (-1)$

e. Which cases lead to expressions with different results? Explain.

For Exercises 2–7, find the value of each expression.

2. $(5 - 3) \div (-2) \cdot (-1)$

3. $2 + (-3) \cdot 4 - (-5)$

4. $4 \cdot 2 \cdot (-3) + (-10) \div 5$

5. $-3 \cdot [2 + (-10)] - 2^2$

6. $(4 - 20) \div 2^2 - 5 \cdot (-2)$

7. $10 - [50 \div (-2 \cdot 25) - 7] \cdot 2^2$

For Exercises 8–11, rewrite each expression in an equivalent form to show a simpler way to do the arithmetic. Explain how you know the two results are equal without doing any calculations.

8. $(-150 + 270) + 30$

9. $(43 \cdot 120) + [43 \cdot (-20)]$

10. $23 + (-75) + 14 + (-23) - (-75)$

11. $[0.8 \cdot (-23)] + [0.8 \cdot (-7)]$

12. Without doing any calculations, determine whether each number sentence is true. Explain. Then check your answer.

a. $50 \cdot 432 = (50 \cdot 400) + (50 \cdot 32)$

b. $50 \cdot 368 = (50 \cdot 400) - (50 \cdot 32)$

c. $-50 \cdot 998 = [-50 \cdot (-1{,}000)] + [-50 \cdot 2]$

d. $-50 + (400 \cdot 32) = (-50 + 400) \cdot (-50 + 32)$

e. $(-70 \cdot 20) + (-50 \cdot 20) = (-120 \cdot 20)$

f. $6 \cdot 17 = 6 \cdot 20 - 6 \cdot 3$

For each part, use the Distributive Property to write an equivalent expression.

13. $-2 \cdot [5 + (-8)]$

14. $(-3 \cdot 2) - [-3 \cdot (-12)]$

15. $x \cdot (-3 + 5)$

16. $-7x + 4x$

17. $2x \cdot [2 - (-4)]$

18. $x - 3x$

19. A grocery store receipt shows 5% state tax due on laundry detergent and a flower bouquet. Does it matter whether the tax is calculated on each separate item or the total cost? Explain.

Connections

For Exercises 20–37, find the sum, difference, product, or quotient.

20. $3 \cdot 12$

21. $3 \cdot (-12)$

22. $-3 \div (-12)$

23. $-10 \cdot (-11)$

24. $-10 + 11$

25. $10 - 11$

26. $-24 - (-12)$

27. $\frac{-24}{-12}$

28. $-18 \div 6$

29. $50 \cdot 70$

30. $50 \cdot (-70)$

31. $2{,}200 \div (-22)$

32. $-50 \cdot (-120)$

33. $-139 + 899$

34. $5{,}600 - 7{,}800$

35. $-4{,}400 - (-1{,}200)$

36. $\frac{-9{,}900}{-99}$

37. $-580 + (-320)$

38. When using negative numbers and exponents, you sometimes need parentheses to make it clear what you are multiplying.

You can think of -5^4 as "the opposite of 5^4" or
$-(5^4) = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$

You can think of $(-5)^4$ as "negative five to the fourth power" or
$(-5)^4 = -5 \cdot (-5) \cdot (-5) \cdot (-5) = 625$

Indicate whether the following expressions will be negative or positive. Explain your answers.

a. -3^2
b. $(-6)^3$
c. $(-4)^4$
d. -1^6
e. $(-3)^4$
f. -2^3

39. This list shows the yards gained and lost during the first several plays of a football game:

$$8, 4, 3, 7, -15, 20, 5, -12, 32, 1$$

Write an expression that shows how to compute the team's average gain or loss per play. Then compute the average.

40. Complete each number sentence.

a. $-34 + (-15) = \blacksquare$

b. $-12 \cdot (-23) = \blacksquare$

c. $-532 \div \blacksquare = -7$

d. $-777 - \blacksquare = -740$

e. Write a fact family for part (a).

f. Write a fact family for part (b).

For Exercises 41–44, write a related fact. Use it to find the value of *n* that makes the sentence true.

41. $n - (-5) = 35$

42. $4 + n = -43$

43. $-2n = -16$

44. $\frac{n}{4} = -32$

45. Insert parentheses (or brackets) in each expression if needed to make the equation true.

a. $1 + (-3) \cdot (-4) = 8$

b. $1 + (-3) \cdot (-4) = 13$

c. $-6 \div (-2) + (-4) = 1$

d. $-6 \div (-2) + (-4) = -1$

e. $-4 \cdot 2 - 10 = -18$

f. $-4 \cdot 2 - 10 = 32$

46. Multiple Choice Which set of numbers is in order from least to greatest?

A. $31.4, -14.2, -55, 75, -0.05, 0.5, 3.140$

B. $\frac{2}{5}, \frac{-3}{5}, \frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, \frac{5}{3}$

C. $-0.2, -0.5, 0.75, 0.6, -1, 1.5$

D. None of these

47. Find the absolute values of the numbers for each set in Exercise 46. Write them in order from least to greatest.

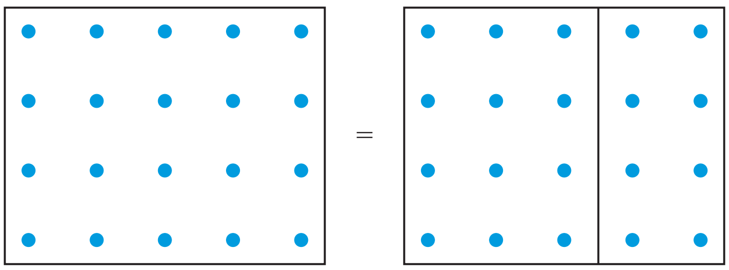
For Exercises 48–50, decide whether each statement is correct, and explain your answer.

48. $|-2 + 3| = |-2| + |3|$

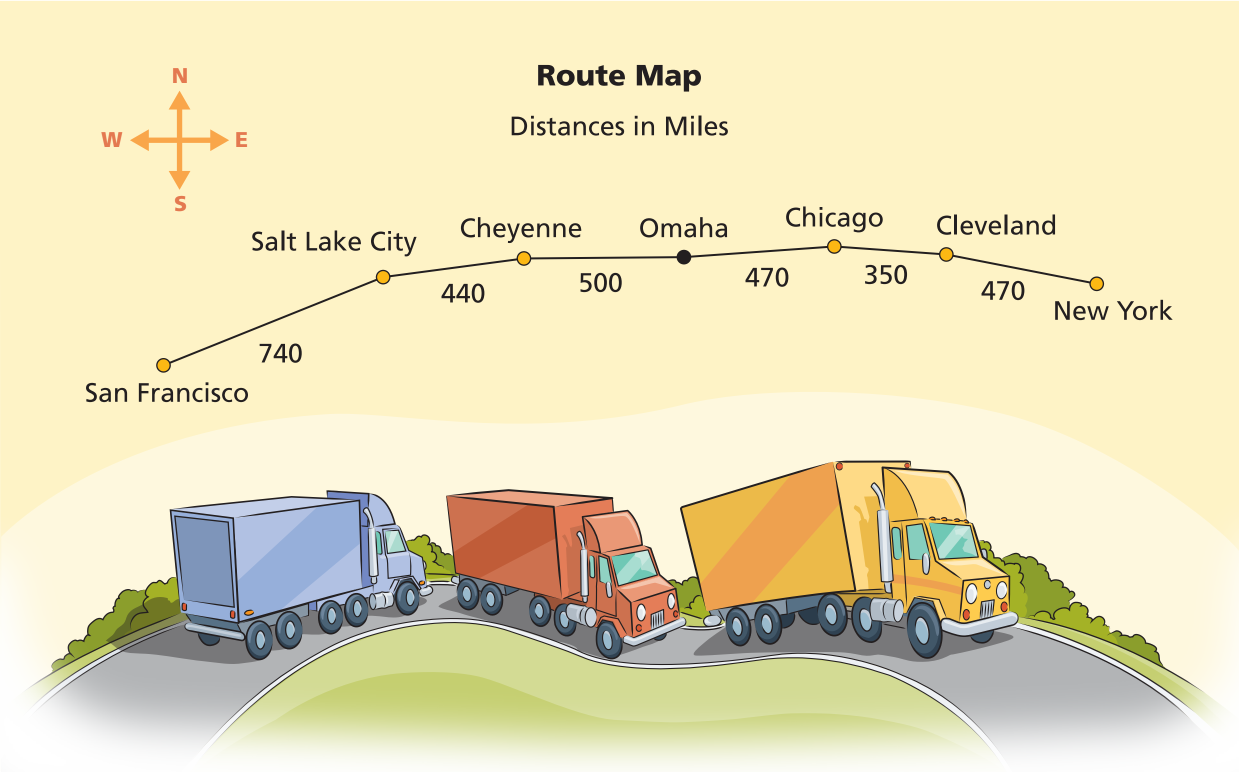
49. $5 - |-2 + 3| = 5 - |-2| + |3|$

50. $|-2 - 3| = |-2| + |-3|$

51. You can use dot patterns to illustrate the distributive properties for operations on whole numbers. Write a number sentence to represent the pair of dot patterns.



52. A trucking company carries freight along a highway from New York City to San Francisco. Its home base is in Omaha, Nebraska, which is about halfway between the two cities. Truckers average about 50 miles per hour on this route.



Make a number line to represent this truck route. Put Omaha at 0. Use positive numbers for cities east of Omaha and negative numbers for cities west of Omaha. Then write number sentences to answer each question.

a. A truck leaves Omaha heading east and travels for 7 hours. About how far does the truck go? Where on the number line does it stop?

b. A truck leaves Omaha heading west and travels for 4.5 hours. About how far does the truck go? Where on the number line does it stop?

c. A truck heading east arrives in Omaha. About where on the number line was the truck 12 hours earlier?

d. A truck heading west arrives in Omaha. About where on the number line was the truck 11 hours earlier?

Extensions

Copy each pair of expressions in Exercises 53–57. Insert $<$ or $>$ to make a true statement.

53. $-23 \blacksquare -45$

54. $-23 + 10 \blacksquare -45 + 10$

55. $-23 - 10 \blacksquare -45 - 10$

56. $-23 \cdot 10 \blacksquare -45 \cdot 10$

57. $-23 \cdot (-10) \blacksquare -45 \cdot (-10)$

For Exercises 58–60, refer to your results in Exercises 53–57. Complete each statement. Test your ideas with other numerical cases, or develop another kind of explanation, perhaps using chip board or number line ideas.

58. If $a > b$, then $a + c \blacksquare b + c$.

59. If $a > b$, then $a - c \blacksquare b - c$.

60. If $a > b$, then $a \cdot c \blacksquare b \cdot c$.

For Exercises 61–63, find the value for *n* that makes the sentence true.

61. $n - (-24) = 12$

62. $2.5n = -10$

63. $2.5n + (-3) = -13$

64. Complete each pair of calculations.

a. $12 \div (-8 + 4) = \blacksquare$ $\quad$ $[12 \div (-8)] + (12 \div 4) = \blacksquare$

b. $-12 \div [-5 - (-3)] = \blacksquare$ $\quad$ $[-12 \div (-5)] - [-12 \div (-3)] = \blacksquare$

c. $(-2 - 6) \div 4 = \blacksquare$ $\quad$ $(-2 \div 4) - (6 \div 4) = \blacksquare$

d. $(5 + 6) \div 3 = \blacksquare$ $\quad$ $(5 \div 3) + (6 \div 3) = \blacksquare$

e. What can you conclude from parts (a)–(d) about the Distributive Property?

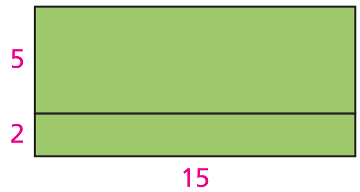
65. When you find the mean (average) of two numbers, you add them together and divide by 2.

a. Does the order in which you do the operations matter? Give examples.

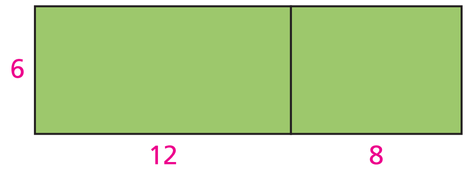
b. Does multiplication distribute over the averaging operation? That is, will a number a times the average of two numbers, x and y, give the same result as the average of ax and ay? Give examples.

For Exercises 66–69, write equivalent expressions to show two different ways to find the area of each rectangle. Use the ideas of the Distributive Property.

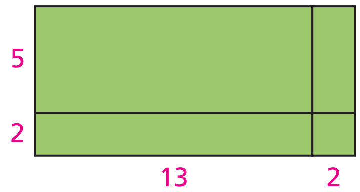
66.



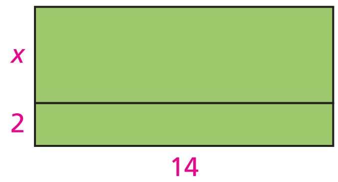
67.



68.



69.



For Exercises 70–73, draw and label the edges and areas of a rectangle to illustrate each pair of equivalent expressions.

70. $(3 + 2) \cdot 12 = 3 \cdot 12 + 2 \cdot 12$

71. $9 \cdot 3 + 9 \cdot 5 = 9 \cdot (3 + 5)$

72. $x \cdot (5 + 9) = 5x + 9x$

73. $2 \cdot (x + 8) = 2x + 16$

Mathematical Reflections 4

In this Investigation, you compared important properties of arithmetic with positive numbers to properties of arithmetic with negative numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. **What** is the Order of Operations? Why is the Order of Operations important?

   b. **Give** an example of a numerical expression in which the use of parentheses changes the result of the computation.

2. **Describe** how the Distributive Property relates addition and multiplication. Give numerical examples.

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Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Elena described her thoughts in the following way:

While working on Problem 4.1, we noticed the importance of where the parentheses are in an expression. We were given an expression and had to put parentheses in to make the greatest and least possible values. To help us with this, we needed a good understanding of the Order of Operations, so we would know where to put the parentheses.

Common Core Standards for Mathematical Practice

MP6 Attend to precision.

- What other Mathematical Practices can you identify in Elena's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Dealing Down

Dealing Down is a mathematics card game that tests your creative skill at writing expressions. Play several rounds. Then write a report on the strategies you used.

How to Play Dealing Down

- Work in small groups.
- Shuffle the 25 cards marked with the following numbers.

  $-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, 0, 0.25, \frac{1}{3}, 0.5, 1, 2, 3, 4, 5, 7, 8, 10$
- Deal four cards to the center of the table.
- All players use the four numbers to write an expression with the least possible quantity. The numbers can be used in any order and with any operation.
- Players compare answers. Each player explains why his or her answer is accurate and the least possible quantity.
- Each player with an expression for the least quantity gets 1 point.
- Record the results in a table such as the one below. Play more rounds and update the table.

Round 1

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

- The player with the most points at the end of the game wins.

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In Your Notebook

Write a report about strategies for writing an expression for the least possible quantity using four numbers. Consider the following ideas as you look at the strategies in Dealing Down.

- Operations with negative and positive numbers
- Order of Operations including the use of parentheses and exponents
- Commutative properties of addition and multiplication
- Distributive Property

Looking Back

In this Unit, you investigated properties, operations, and applications of integers and rational numbers. You learned how to

- Add, subtract, multiply, and divide integers and rational numbers
- Represent operations with integers and rational numbers on a number line, and represent operations with integers on a chip board
- Use integers and rational numbers in real-world problems

Use Your Understanding: Rational Numbers

Test your understanding of rational numbers by solving the following problems.

1. An absent-minded scorekeeper writes the number sentences below. Find the value of n that makes each sentence true. Explain what each sentence tells about the team's performance.

a. BrainyActs: $-250 + (-100) + 200 + n = 50$

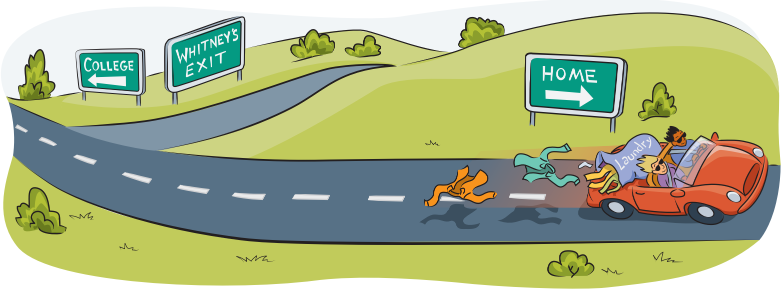
b. Xtremes: $450 + (-250) + n = 0$

c. ExCells: $n + 50 + 200 + (-150) = -250$

d. AmazingM's: $350 + (-300) + n = -150$

2. Irving goes to college 127 miles away from home. When he drives home for vacation, he plans to drop off his friend, Whitney. Her exit is 93 miles before his.

Irving and Whitney are so busy talking that they miss the exit to her house. They are now only 36 miles from Irving's exit! They turn around and drive back to Whitney's exit. How far did Irving and Whitney travel in total from when they left college to when they finally reached Whitney's exit? Model this problem on a number line.



3. For each number sentence:
   - Write a fact family.
   - Identify the member of each fact family that is easiest to use to solve for n. Explain your reasoning.
   - Find the value for n that makes each sentence true.

   a. $-2\frac{1}{2} + n = -3\frac{3}{4}$

   b. $\frac{2}{3}n = 10$

Explain Your Reasoning

Answer the following questions to summarize what you now know.

4. Describe what a number line looks like now that you have learned about negative numbers.

5. Which number is greater? Explain.

   a. $-20, -35$

   b. $-2\frac{3}{4}, -2\frac{1}{3}$

   c. $-12.5, 10.5$

6. Use a number line or chip model to check each calculation. Show your work.

   a. $5 + (-7) = -2$

   b. $-2 + (-9) = -11$

   c. $3 \cdot (-2) = -6$

   d. $-3 \cdot (-2) = 6$

   e. Describe how a number line and a chip model can be used to model an addition or multiplication problem.

7. Suppose you are given two integers. How do you find their

   a. sum?

   b. difference?

   c. product?

   d. quotient?

8. Which operations have the following properties? Give numerical examples.

   a. commutative

   b. distributive

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English / Spanish Glossary

A

absolute value The absolute value of a number is its distance from 0 on a number line. It can be thought of as the value of a number when its sign is ignored. For example, -3 and 3 both have an absolute value of 3.

valor absoluto El valor absoluto de un número es su distancia del 0 en una recta numérica. Se puede interpretar como el valor de un número cuando no importa su signo. Por ejemplo, tanto -3 como 3 tienen un valor absoluto de 3.

additive identity Zero is the additive identity for rational numbers. Adding zero to any rational number results in a sum identical to the original rational number. For any rational number a, $0 + a = a$. For example, $0 + 4.375 = 4.375$.

identidad de suma El cero es la identidad de suma para los números racionales. Sumarle cero a cualquier número racional da como resultado un total idéntico al número racional original. Para cualquier número racional a, $0 + a = a$. Por ejemplo, $0 + 4.375 = 4.375$.

additive inverses Two numbers, a and b, that satisfy the equation $a + b = 0$. For example, 3 and -3 are additive inverses, and $\frac{1}{2}$ and $-\frac{1}{2}$ are additive inverses.

inversos de suma Dos números, a y b, que cumplen con la ecuación $a + b = 0$. Por ejemplo, 3 y -3 son inversos de suma, y $\frac{1}{2}$ y $-\frac{1}{2}$ son inversos de suma.

algorithm A set of rules for performing a procedure. Mathematicians invent algorithms that are useful in many kinds of situations. Some examples of algorithms are the rules for long division or the rules for adding two fractions.

algoritmo Un conjunto de reglas para realizar un procedimiento. Los matemáticos crean algoritmos que son útiles en muchos tipos de situaciones. Las reglas para realizar una división larga o para sumar dos fracciones son algunos ejemplos de algoritmos.

C

Commutative Property The order of the addition or multiplication of two numbers does not change the result. For two numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$. For example, $\frac{3}{7} + 8 = 8 + \frac{3}{7}$ and $\frac{3}{7} \cdot 8 = 8 \cdot \frac{3}{7}$.

Propiedad conmutativa El orden de dos números cuando se los suma o se los multiplica no altera el resultado. Para dos números a y b, $a + b = b + a$ y $a \cdot b = b \cdot a$. Por ejemplo, $\frac{3}{7} + 8 = 8 + \frac{3}{7}$ y $\frac{3}{7} \cdot 8 = 8 \cdot \frac{3}{7}$.

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D

describe Academic Vocabulary
To explain or tell in detail. A written description can contain facts and other information needed to communicate your answer. A diagram or a graph may also be included.

related terms *express, explain*

Sample Given the pair of points (5, 7) and (−4, 7), describe the direction and the distance between the first point and the second point on a coordinate graph.

The direction from $(^+5, ^+7)$ to $(^-4, ^+7)$ is to the left. The distance between the two points is the distance between the *x*-coordinates because the *y*-coordinates are the same. The distance between the *x*-coordinates is the distance from $^+5$ to the *y*-axis plus the distance from the *y*-axis to $^-4$: $5 + 4 = 9$.

describir Vocabulario académico
Explicar o decir con detalle. Una descripción escrita puede contener datos y otro tipo de información necesaria para comunicar tu respuesta. También puede incluir un diagrama o una gráfica.

términos relacionados *expresar, explicar*

Ejemplo Dados los pares de los puntos (+5, +7) y (−4, +7), describe la dirección y la distancia entre el primer punto y el segundo punto en una gráfica de coordenadas.

La dirección desde $(^+5, ^+7)$ a $(^-4, ^+7)$ es hacia la izquierda. La distancia entre los dos puntos es la distancia entre las coordenadas *x*, porque las coordenadas *y* son iguales. La distancia entre las coordenadas *x* es la distancia desde $^+5$ al eje de *y* más la distancia del eje de *y* a $^-4$: $5 + 4 = 9$.

Distributive Property A mathematical property used to rewrite expressions involving addition and multiplication. The Distributive Property states that for any three real numbers a, b, and c, $a(b + c) = ab + ac$. If an expression is written as a factor multiplied by a sum, you can use the Distributive Property to multiply the factor by each term in the sum.

$$4(5 + x) = 4(5) + 4(x) = 20 + 4x$$

If an expression is written as a sum of terms and the terms have a common factor, you can use the Distributive Property to rewrite the expression as the common factor multiplied by a sum. This process is called factoring.

$$20 + 4x = 4(5) + 4(x) = 4(5 + x)$$

Propiedad distributiva Una propiedad matemática que se usa para volver a escribir expresiones que incluyen la suma y la multiplicación. La propiedad distributiva establece que para tres números reales cualesquiera, a, b, y c, $a(b + c) = ab + ac$. Si una expresión se escribe como la multiplicación de un factor por una suma, la propiedad distributiva se puede usar para multiplicar el factor por cada término de esa suma.

$$4(5 + x) = 4(5) + 4(x) = 20 + 4x$$

Si una expresión se escribe como una suma de términos y los términos tienen un factor común, la propiedad distributiva se puede usar para volver a escribir la expresión como la multiplicación del factor común por una suma. Este proceso se llama descomponer en factores.

$$20 + 4x = 4(5) + 4(x) = 4(5 + x)$$

E

expanded form The form of an expression made up of sums or differences of terms rather than products of factors. The expressions $20 + 30$, $5(4) + 5(21)$, $x^2 + 7x + 12$ and $x^2 + 2x$ are in expanded form.

forma desarrollada La forma de una expresión que está compuesta de sumas o diferencias de términos en vez de productos de factores. Las expresiones $20 + 30$, $5(4) + 5(21)$, $x^2 + 7x + 12$ y $x^2 + 2x$ están representadas en forma desarrollada.

explain Academic Vocabulary
To give facts and details that make an idea easier to understand. Explaining can involve a written summary supported by a diagram, chart, table, or any combination of these.

related terms *describe, show, justify, tell, present*

Sample Explain how to multiply two negative numbers.

> To multiply two negative numbers, multiply as if both numbers were positive. The product will always be a positive number.

explicar Vocabulario académico
Dar datos y detalles que hacen que una idea sea más fácil de comprender. Explicar puede incluir un resumen escrito apoyado por un diagrama, una gráfica, una tabla o una combinación de éstos.

términos relacionados *describir, demostrar, justificar, decir, presentar*

Ejemplo Explica cómo se multiplican dos números negativos.

> Para multiplicar dos números negativos, multiplica como si ambos números fueran positivos. El producto siempre será un número positivo.

F

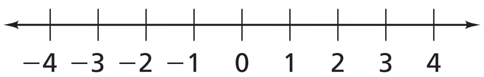
factored form The form of an expression made up of products of factors rather than sums or differences of terms. The expressions $2 \times 2 \times 5$, $3(2 + 7)$, $(x + 3)(x + 4)$ and $x(x-2)$ are in factored form.

forma factorizada La forma de una expresión que está compuesta de productos de factores en lugar de sumas o diferencias de términos. Las expresiones $2 \times 2 \times 5$, $3(2 + 7)$, $(x + 3)(x + 4)$ y $x(x-2)$ están representadas en forma factorizada.

I

integers The whole numbers and their opposites. 0 is an integer, but is neither positive nor negative. The integers from -4 to 4 are shown on the number line below.

enteros Los números enteros y sus opuestos. El 0 es un entero, pero no es positivo ni negativo. En la siguiente recta numérica se muestran los enteros comprendidos entre -4 y 4.



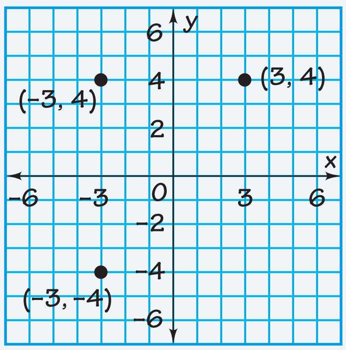
L

locate Academic Vocabulary
To find or identify a value, usually on a number line or coordinate graph.

related terms *find, identify*

Sample Locate and label the points $(-3, 4)$, $(-3, -4)$ and $(3, 4)$on the coordinate graph.

I can draw and label an *x* and *y*-axis on grid paper and locate the points.

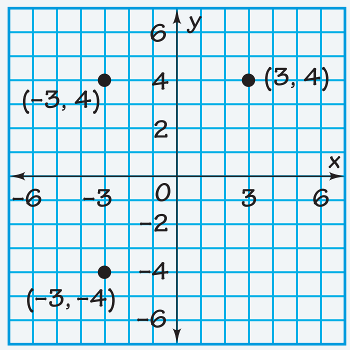


ubicar Vocabulario académico
Hallar o identificar un valor, generalmente en una recta numérica o en una gráfica de coordenadas.

términos relacionados *hallar, identificar*

Ejemplo Ubica y rotula los puntos $(-3, 4)$, $(-3, -4)$ y $(3, 4)$ en una gráfica de coordenadas.

Puedo dibujar y rotular un eje para las *x* y un eje para las *y* en el papel cuadriculado, y ubicar los puntos.



M

multiplicative identity The multiplicative identity for rational numbers is 1 or any rational expression equal to 1. Multiplying any rational number by 1 results in a product identical to the original rational number. For any rational number N, $N \times 1 = N$, or for any rational numbers R and $\frac{a}{b}$, $\frac{a}{b} \times \frac{R}{R} = \frac{a}{b}$. For example, $\frac{4}{9} \times 1 = \frac{4}{9}$, and $\frac{5}{8} \times \frac{3}{3} = \frac{5}{8}$.

identidad multiplicativa La identidad multiplicativa para los números racionales es 1 o cualquier otra expresión racional igual a 1. Multiplicar cualquier número racional por 1 da como resultado un producto idéntico al número racional original. Para cualquier número racional, N, $N \times 1 = N$, o para cualquiera de los números racionales R y $\frac{a}{b}$, $\frac{a}{b} \times \frac{R}{R} = \frac{a}{b}$. Por ejemplo, $\frac{4}{9} \times 1 = \frac{4}{9}$ y $\frac{5}{8} \times \frac{3}{3} = \frac{5}{8}$.

multiplicative inverses Two numbers, a and b, that satisfy the equation $ab = 1$. For example, 3 and $\frac{1}{3}$ are multiplicative inverses, and $-\frac{1}{2}$ and -2 are multiplicative inverses.

inversos multiplicativos Dos números, a y b, que cumplen con la ecuación $ab = 1$. Por ejemplo, 3 y $\frac{1}{3}$ son inversos multiplicativos, y $-\frac{1}{2}$ y -2 son inversos multiplicativos.

N

negative number A number less than 0. On a number line, negative numbers are located to the left of 0 (on a vertical number line, negative numbers are located below 0).

número negativo Un número menor que 0. En una recta numérica, los números negativos están ubicados a la izquierda del 0 (en una recta numérica vertical, los números negativos están ubicados debajo del 0).

number sentence A mathematical statement that gives the relationship between two expressions that are composed of numbers and operation signs. For example, $3 + 2 = 5$ and $6 \times 2 > 10$ are number sentences; $3 + 2$, 5, 6×2, and 10 are expressions.

oración numérica Enunciado matemático que describe la relación entre dos expresiones compuestas por números y signos de operaciones. Por ejemplo, $3 + 2 = 5$ y $6 \times 2 > 10$ son oraciones numéricas. $3 + 2$, 5, 6×2, y 10 son expresiones.

O

opposites Two numbers whose sum is 0. For example, -3 and 3 are opposites. On a number line, opposites are the same distance from 0 but in different directions from 0. The number 0 is its own opposite.

opuestos Dos números cuya suma da 0. Por ejemplo, -3 y 3 son opuestos. En una recta numérica, los opuestos se encuentran a la misma distancia de 0 pero en direcciones opuestas del 0 en la recta numérica. El número 0 es su propio opuesto.

Order of Operations A set of agreements or conventions for carrying out calculations with one or more operations, parentheses, or exponents.

1. Work within parentheses.
2. Write numbers written with exponents in standard form.
3. Do all multiplication and division in order from left to right.
4. Do all addition and subtraction in order from left to right.

Orden de las operaciones Un conjunto de acuerdos o convenciones para llevar a cabo cálculos con más de una operación, paréntesis o exponentes.

1. Resolver lo que está entre paréntesis.
2. Escribir los números con exponentes en forma estándar.
3. Multiplicar y dividir en orden de izquierda a derecha.
4. Sumar y dividir en orden de izquierda a derecha.

P

positive number A number greater than 0. (The number 0 is neither positive nor negative.) On a number line, positive numbers are located to the right of 0 (on a vertical number line, positive numbers are located above 0).

número positivo Un número mayor que 0. (El número 0 no es positivo negativo.) En una recta numérica, los números positivos se ubican a la derecha del 0 (en una recta numérica vertical, los números positivos están arriba del 0).

R

rational numbers Numbers that can be expressed as a quotient of two integers where the divisor is not zero. For example, $\frac{1}{2}$, $\frac{9}{11}$, and $-\frac{7}{5}$ are rational numbers. Also, 0.799 is a rational number, since $0.799 = \frac{799}{1{,}000}$.

números racionales Números que se pueden expresar como el cociente de dos números enteros donde el divisor no es cero. Por ejemplo, $\frac{1}{2}$, $\frac{9}{11}$, y $-\frac{7}{5}$ son números racionales. También 0.799 es un número racional, porque $0.799 = \frac{799}{1{,}000}$.

represent Academic Vocabulary
To stand for or take the place of something else. Symbols, equations, charts, and tables are often used to represent particular situations.

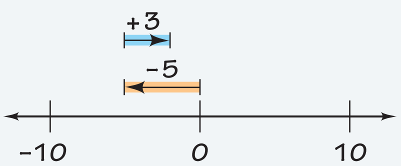
related terms *symbolize, stand for*

Sample Players spin a 0–5 spinner to see how far and in which direction they will move. Sally started at zero, spun a 5, and picked a negative card. She then spun a 3 and picked a positive card. Which of the following expressions represents her distance from zero on the number line?

A. $|3 + 5|$

B. $|-3 - 5|$

C. $|-5 + 3|$



Sally moved five units in a negative direction and then three units in a positive direction. Absolute value signs are used to show distance, so the answer is C.

representar Vocabulario académico
Significar o tomar el lugar de algo. Con frecuencia, se usan símbolos, ecuaciones, gráficas y tablas para representar situaciones determinadas.

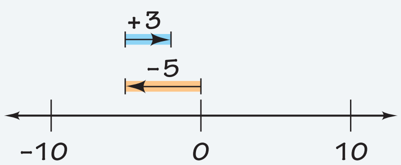
términos relacionados *simbolizar, significar*

Ejemplo Los jugadores hacen girar una rueda giratoria numerada del 0 al 5 para ver cuánto y en qué dirección se tienen que mover. Sally empezó en el cero, le salió un 5 en la rueda giratoria y sacó una tarjeta negativa. Después, le salió un 3 y sacó una tarjeta positiva. ¿Cuál de las siguientes expresiones representa la distancia que ella recorrió desde el cero en una recta numérica?

A. $|3 + 5|$

B. $|-3 - 5|$

C. $|-5 + 3|$



Sally se movió cinco unidades en dirección negativa y después tres unidades en dirección positiva. Para mostrar la distancia se usan signos de valor absoluto, por lo tanto la respuesta es la C.

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Acknowledgments

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Data Sources

Temperature data on page 15 are from Temperatures in Spearfish South Dakota. Source: National Weather Service.

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