Circumference, Area, and Volume

? **ESSENTIAL QUESTION**

How can you apply geometry concepts to solve real-world problems?

Real-World Video

A 16-inch pizza has a diameter of 16 inches. You can use the diameter to find circumference and area of the pizza. You can also determine how much pizza in one slice of different sizes of pizzas.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Multiply with Fractions and Decimals

EXAMPLE

$$\begin{array}{r} 7.3 \\ \times\ 2.4 \\ \hline 2\ 9\ 2 \\ +1\ 4\ 6\ \\ \hline 1\ 7.5\ 2 \end{array}$$

Multiply as you would with whole numbers.
Count the total number of decimal places in the two factors.

Place the decimal point in the product so that there are the same number of digits after the decimal point.

Multiply.

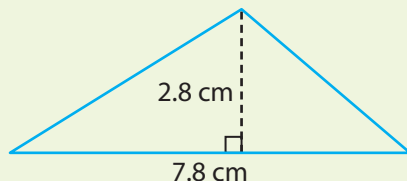**1.** $\begin{array}{r} 4.16 \\ \times\ 13 \\ \hline \end{array}$

2. $\begin{array}{r} 6.47 \\ \times\ 0.4 \\ \hline \end{array}$

3. $\begin{array}{r} 7.05 \\ \times\ 9.4 \\ \hline \end{array}$

4. $\begin{array}{r} 25.6 \\ \times\ 0.49 \\ \hline \end{array}$

Area of Squares, Rectangles, and Triangles

EXAMPLE

2.8 cm

7.8 cm

$A = \frac{1}{2}bh$ Use the formula for area of a triangle.

$= \frac{1}{2}(7.8)(2.8)$ Substitute for each variable.

$= 10.92 \text{ cm}^2$ Multiply.

Find the area of each figure.

5. triangle with base 14 in. and height 10 in. _____

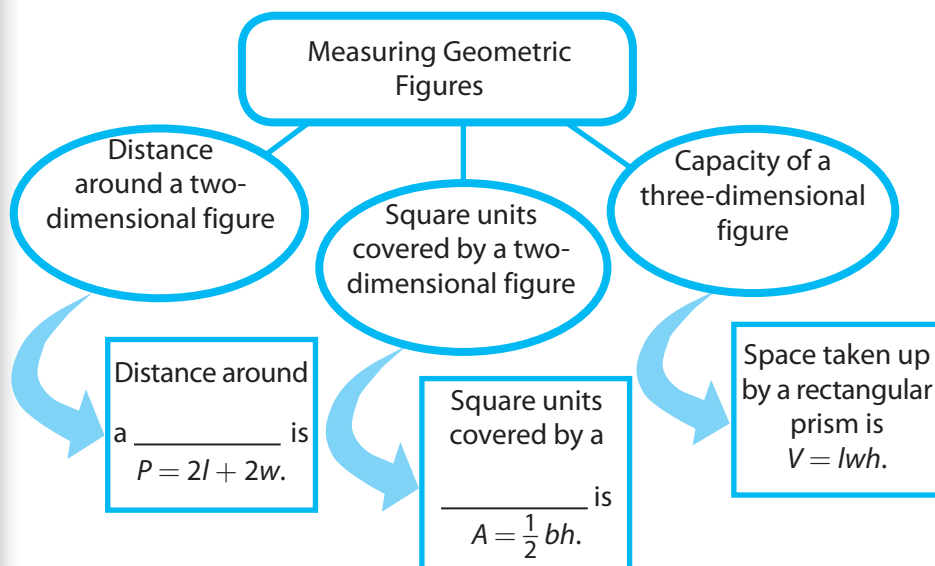**6.** square with sides of 3.5 ft _____

7. rectangle with length $8\frac{1}{2}$ in. and width 6 in. _____

8. triangle with base 12.5 m and height 2.4 m _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You will put one word in each oval. Then write examples of formulas in each rectangle.

```
                    Measuring Geometric
                          Figures

  Distance                                    Capacity of a
  around a two-         Square units          three-dimensional
  dimensional figure    covered by a two-     figure
                        dimensional figure
```

Distance around a _____ is $P = 2l + 2w$.

Square units covered by a _____ is $A = \frac{1}{2} bh$.

Space taken up by a rectangular prism is $V = lwh$.

Vocabulary

Review Words
✔ area *(área)*
  parallelogram *(paralelogramo)*
✔ perimeter *(perímetro)*
  prism *(prisma)*
  rectangle *(rectángulo)*
  square *(cuadrado)*
  trapezoid *(trapecio)*
  triangle *(triángulo)*
✔ volume *(volumen)*

Preview Words
  circumference *(circunferencia)*
  composite figure *(figura compuesta)*
  diameter *(diámetro)*
  radius *(radio)*

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. _____ circumference

2. _____ diameter

3. _____ radius

A. A line segment that passes through the center of a circle and has endpoints on the circle, or the length of that segment.

B. A line segment with one endpoint at the center of the circle and the other on the circle, or the length of that segment.

C. The distance around a circle.

Active Reading

Four-Corner Fold Before beginning the module, create a four-corner fold to help you organize what you learn. As you study this module, note important ideas, such as vocabulary, properties, and formulas, on the flaps. Use one flap each for circumference, area, surface area, and volume. You can use your FoldNote later to study for tests and complete assignments.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 7.G.6

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Key Vocabulary

circumference *(circunferencia)*
The distance around a circle.

What It Means to You

You will use formulas to solve problems involving the area and circumference of circles.

UNPACKING EXAMPLE 7.G.6

Lily is drawing plans for a circular fountain. The diameter of the fountain is 20 feet. What is the approximate circumference?

$$C = \pi d$$

$$C \approx 3.14 \cdot 20 \quad \text{Substitute.}$$

$$C \approx 62.8$$

The circumference of the fountain is about 62.8 feet.

COMMON CORE 7.G.4

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Key Vocabulary

volume *(volumen)*
The number of cubic units inside a three-dimensional solid.

surface area *(área total)*
The sum of the areas of all the surfaces of a three-dimensional solid.

What It Means to You

You will find area, volume and surface area of real-world objects.

UNPACKING EXAMPLE 7.G.4

Find the volume and the surface area of a tissue box before the hole is cut in the top.

The tissue box is a right rectangular prism. The base is $4\frac{3}{8}$ in. by $4\frac{3}{8}$ in. and the height is 5 in.

Use the volume and surface area formulas:

B is the area of the base, h is the height of the box, and P is the perimeter of the base.

$$V = Bh$$
$$= \left(4\frac{3}{8} \cdot 4\frac{3}{8}\right)5$$
$$= 95\frac{45}{64} \text{ in}^3$$

$$S = 2B + Ph$$
$$= 2\left(4\frac{3}{8} \cdot 4\frac{3}{8}\right) + \left(4 \cdot 4\frac{3}{8}\right)5$$
$$= 125\frac{25}{32} \text{ in}^2$$

The volume is $95\frac{45}{64}$ in³ and the surface area is $125\frac{25}{32}$ in².

Visit **my.hrw.com** to see all the **Common Core Standards** unpacked.

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? ESSENTIAL QUESTION

How do you find and use the circumference of a circle?

**EXPLORE ACTIVITY** COMMON CORE 7.G.4

Exploring Circumference

A circle is a set of points in a plane that are a fixed distance from the center.

A **radius** is a line segment with one endpoint at the center of the circle and the other endpoint on the circle. The length of a radius is called the radius of the circle.

A **diameter** of a circle is a line segment that passes through the center of the circle and whose endpoints lie on the circle. The length of the diameter is twice the length of the radius. The length of a diameter is called the diameter of the circle.

The **circumference** of a circle is the distance around the circle.
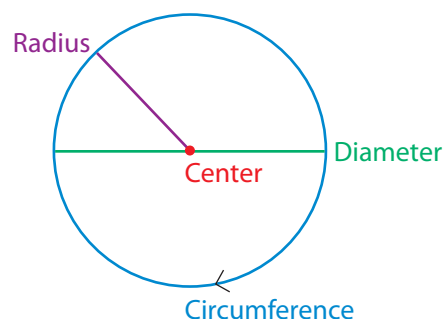
A Use a measuring tape to find the circumference of five circular objects. Then measure the distance across each item to find its diameter. Record the measurements of each object in the table below.

Object	Circumference C	Diameter d	$\frac{C}{d}$

B Divide the circumference of each object by its diameter. Record your answer, rounded to the nearest hundredth, in the table above.

Reflect

1. Make a Conjecture Describe what you notice about the ratio $\frac{C}{d}$ in your table.

_____

Finding Circumference

The ratio of the circumference to the diameter $\frac{C}{d}$ is the same for all circles. This ratio is called π or *pi*, and you can approximate it as 3.14 or as $\frac{22}{7}$. You can use π to find a formula for circumference.

For any circle, $\frac{C}{d} = \pi$. Solve the equation for C to give an equation for the circumference of a circle in terms of the diameter.

$$\frac{C}{d} = \pi \qquad \text{The ratio of the circumference to the diameter is } \pi.$$

$$\frac{C}{d} \times d = \pi \times d \qquad \text{Multiply both sides by } d.$$

$$C = \pi d \qquad \text{Simplify.}$$

The diameter of a circle is twice the radius. You can use the equation $C = \pi d$ to find a formula for the circumference C in terms of the radius r.

$$C = \pi d = \pi(2r) = 2\pi r$$

The two equivalent formulas for circumference are $C = \pi d$ and $C = 2\pi r$.

EXAMPLE 1 — Real World

COMMON CORE 7.G.4

An irrigation sprinkler waters a circular region with a radius of 14 feet. Find the circumference of the region watered by the sprinkler. Use $\frac{22}{7}$ for π.

Use the formula.

$$C = 2\pi r \qquad \text{The radius is 14 feet.}$$

$$C = 2\pi(14) \qquad \text{Substitute 14 for } r.$$

$$C \approx 2\left(\frac{22}{7}\right)(14) \qquad \text{Substitute } \frac{22}{7} \text{ for } \pi.$$

$$C \approx 88 \qquad \text{Multiply.}$$

14 ft
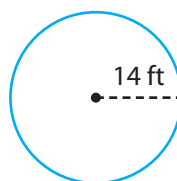
The circumference of the region watered by the sprinkler is about 88 feet.

Reflect

2. Analyze Relationships When is it logical to use $\frac{22}{7}$ instead of 3.14 for π?

_____

YOUR TURN

3. Find the circumference of the circle to the nearest hundredth.
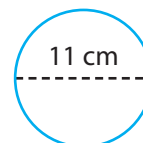
11 cm

_____

Using Circumference

Given the circumference of a circle, you can use the appropriate circumference formula to find the radius or the diameter of the circle. You can use that information to solve problems.

EXAMPLE 2   *Real World*   COMMON CORE 7.G.4

A circular pond has a circumference of 628 feet. A model boat is moving directly across the pond, along a radius, at a rate of 5 feet per second. How long does it take the boat to get from the edge of the pond to the center?

STEP 1   Find the radius of the pond.

$C = 2\pi r$ *Use the circumference formula.*

$628 \approx 2(3.14)r$ *Substitute for the circumference and for π.*

$\dfrac{628}{6.28} \approx \dfrac{6.28r}{6.28}$ *Divide both sides by 6.28.*

$100 \approx r$ *Simplify.*

$C = 628$ ft

$r = ?$ ft
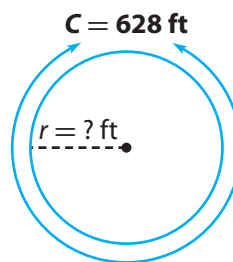
The radius is about 100 feet.

STEP 2   Find the time it takes the boat to get from the edge of the pond to the center along the radius. Divide the radius of the pond by the speed of the model boat.

$$100 \div 5 = 20$$

It takes the boat about 20 seconds to get to the center of the pond.

Reflect

4. Analyze Relationships Dante checks the answer to Step 1 by multiplying it by 6 and comparing it with the given circumference. Explain why Dante's estimation method works. Use it to check Step 1.

_____

5. What If? Suppose the model boat were traveling at a rate of 4 feet per second. How long would it take the model boat to get from the edge of the pond to the center? _____

Math Talk
Mathematical Practices

Would it be reasonable to solve Example 2 using $\frac{22}{7}$ for π? Explain.

YOUR TURN

6.   A circular garden has a circumference of 44 yards. Lars is digging a straight line along a diameter of the garden at a rate of 7 yards per hour. How many hours will it take him to dig across the garden?
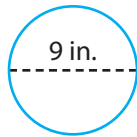
_____

My Notes

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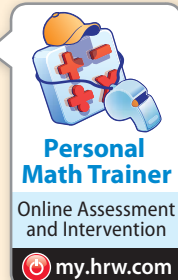
Find the circumference of each circle. (Example 1)

1. $C = \pi d$
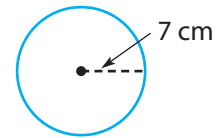
$C \approx$ _____
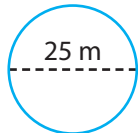
$C \approx$ _____ inches

9 in.

2. $C = 2\pi r$

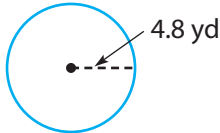$C \approx 2\left(\frac{22}{7}\right)$ (_____)

$C \approx$ _____ cm

7 cm

Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π. Round to the nearest hundredth, if necessary. (Example 1)
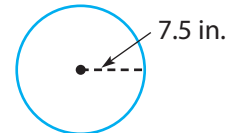
3.

25 m

_____

4.

4.8 yd

_____

5.

7.5 in.

_____

6. A round swimming pool has a circumference of 66 feet. Carlos wants to buy a rope to put across the diameter of the pool. The rope costs $0.45 per foot, and Carlos needs 4 feet more than the diameter of the pool. How much will Carlos pay for the rope? (Example 2)

Find the diameter.

$C = \pi d$

_____ $\approx 3.14d$

$\dfrac{\boxed{}}{3.14} \approx \dfrac{3.14d}{3.14}$

_____ $\approx d$

Find the cost.

Carlos needs _____ feet of rope.

_____ $\times \$0.45 =$ _____

Carlos will pay _____ for the rope.

Find each missing measurement to the nearest hundredth. Use 3.14 for π. (Examples 1 and 2)

7. $r =$ _____

$d =$ _____

$C = \pi$ yd

8. $r \approx$ _____

$d \approx$ _____

$C = 78.8$ ft

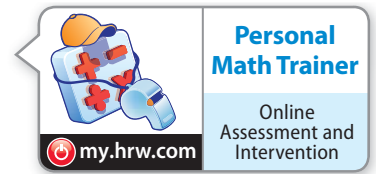**9.** $r \approx$ _____

$d \approx 3.4$ in.

$C =$ _____

? **ESSENTIAL QUESTION CHECK-IN**

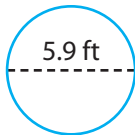**10.** Norah knows that the diameter of a circle is 13 meters. How would you tell her to find the circumference?

_____

_____

9.1 Independent Practice

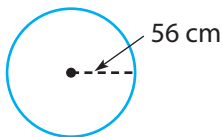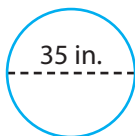**Personal Math Trainer**
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For 11–13, find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π. Round to the nearest hundredth, if necessary.

11.

5.9 ft

_____

12.

56 cm

_____

13.

35 in.

_____

14. In Exercises 11–13, for which problems did you use $\frac{22}{7}$ for π? Explain your choice.

_____

_____

_____

15. A circular fountain has a radius of 9.4 feet. Find its diameter and circumference to the nearest tenth.

_____

16. Find the radius and circumference of a CD with a diameter of 4.75 inches.

_____

17. A dartboard has a diameter of 18 inches. What are its radius and circumference?

_____

18. **Multistep** Randy's circular garden has a radius of 1.5 feet. He wants to enclose the garden with edging that costs $0.75 per foot. About how much will the edging cost? Explain.

_____

_____

19. **Represent Real-World Problems** The Ferris wheel shown makes 12 revolutions per ride. How far would someone travel during one ride?

diameter 63 feet

_____
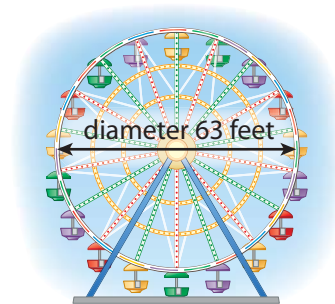
20. The diameter of a bicycle wheel is 2 feet. About how many revolutions does the wheel make to travel 2 kilometers? Explain. Hint: 1 km ≈ 3,280 ft

_____

_____

_____

21. **Multistep** A map of a public park shows a circular pond. There is a bridge along a diameter of the pond that is 0.25 mi long. You walk across the bridge, while your friend walks halfway around the pond to meet you at the other side of the bridge. How much farther does your friend walk?

_____

22. Architecture The Capitol Rotunda connects the House and the Senate sides of the U.S. Capitol. Complete the table. Round your answers to the nearest foot.

Capitol Rotunda Dimensions	
Height	180 ft
Circumference	301.5 ft
Radius	
Diameter	

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

23. Multistep A museum groundskeeper is creating a semicircular statuary garden with a diameter of 30 feet. There will be a fence around the garden. The fencing costs $9.25 per linear foot. About how much will the fencing cost altogether?

_____

24. Critical Thinking Sam is placing rope lights around the edge of a circular patio with a diameter of 18 feet. The lights come in lengths of 54 inches. How many strands of lights does he need to surround the patio edge?

_____

25. Represent Real-World Problems A circular path 2 feet wide has an inner diameter of 150 feet. How much farther is it around the outer edge of the path than around the inner edge?

_____

26. Critique Reasoning A gear on a bicycle has the shape of a circle. One gear has a diameter of 4 inches, and a smaller one has a diameter of 2 inches. Justin says that the circumference of the larger gear is 2 inches more than the circumference of the smaller gear. Do you agree? Explain your answer.

_____

_____

_____

27. Persevere in Problem Solving Consider two circular swimming pools. Pool A has a radius of 12 feet, and Pool B has a diameter of 7.5 meters. Which pool has a greater circumference? How much greater? Justify your answers.

_____

_____

_____

Work Area

LESSON

9.2 Area of Circles

COMMON CORE 7.G.4

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

? ESSENTIAL QUESTION

How do you find the area of a circle?

## EXPLORE ACTIVITY 1   COMMON CORE 7.G.4

Exploring Area of Circles

You can use what you know about circles and π to help find the formula for the area of a circle.

STEP 1 Use a compass to draw a circle and cut it out.

STEP 2 Fold the circle three times as shown to get equal wedges.

STEP 3 Unfold and shade one-half of the circle.

STEP 4 Cut out the wedges, and fit the pieces together to form a figure that looks like a parallelogram.
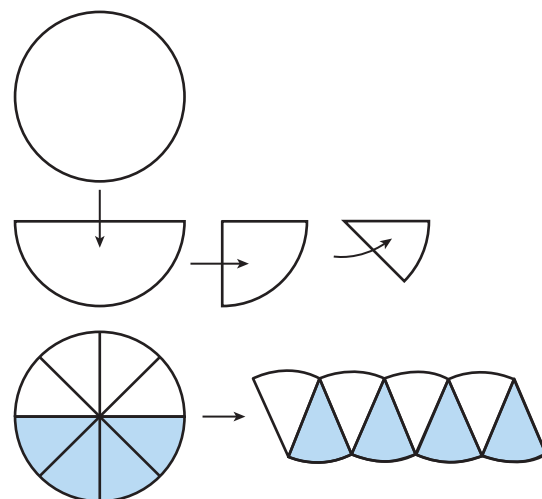
The base and height of the parallelogram relate to the parts of the circle.

base $b = \dfrac{\Box}{\Box}$ the circumference of the circle, or _____

Radius

Half the circumference

height $h =$ the _____ of the circle, or _____
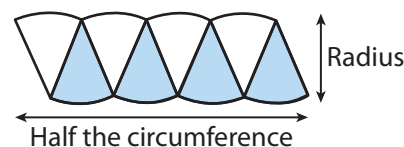
To find the area of a parallelogram, the equation is $A =$ _____.

To find the area of the circle, substitute for b and h in the area formula.

$A = bh$

$A = \boxed{}\, h$ **Substitute for b.**

$A = \pi r\boxed{}$ **Substitute for h.**

$A = \pi \boxed{}$ **Write using an exponent.**

Reflect

1. How can you make the wedges look more like a parallelogram?

_____

_____

_____

Finding the Area of a Circle

Area of a Circle

The area of a circle is equal to π times the radius squared.
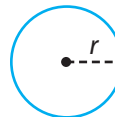
$$A = \pi r^2$$

Remember that area is given in square units.

A biscuit recipe calls for the dough to be rolled out and circles to be cut from the dough. The biscuit cutter has a radius of 4 cm. Find the area of the top of the biscuit once it is cut. Use 3.14 for π.

$A = \pi r^2$	Use the formula.
$A = \pi(4)^2$	Substitute. Use 4 for r.
$A \approx 3.14 \times 4^2$	Substitute. Use 3.14 for π.
$A \approx 3.14 \times 16$	Evaluate the power.
$A \approx 50.24$	Multiply.

Math Talk

Mathematical Practices

If the radius increases by 1 centimeter, how does the area of the top of the biscuit change?

The area of the biscuit is about 50.24 cm².

Reflect

2. Compare finding the area of a circle when given the radius with finding the area when given the diameter.

_____

_____

_____

3. Why do you evaluate the power in the equation before multiplying by pi?

_____

_____

YOUR TURN

4. A circular pool has a radius of 10 feet. What is the area of the *surface* of the water in the pool? Use 3.14 for π. _____

Finding the Relationship between Circumference and Area

You can use what you know about circumference and area of circles to find a relationship between them.
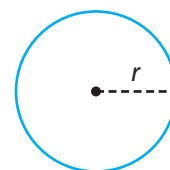
Find the relationship between the circumference and area of a circle.

Start with a circle that has radius r.

Solve the equation $C = 2\pi r$ for r.

$$r = \frac{\boxed{}}{\boxed{}}$$

Substitute your expression for r in the formula for area of a circle.

$$A = \pi\left(\frac{\boxed{}}{\boxed{}}\right)^2$$

> Remember: Because the exponent is outside the parentheses, you must apply it to the numerator and to each factor of the denominator.

Square the term in the parentheses.

$$A = \pi\left(\frac{\boxed{}^2}{\boxed{}^2 \cdot \boxed{}^2}\right)$$

Evaluate the power.

$$A = \frac{\boxed{} \cdot \boxed{}^2}{\boxed{} \cdot \boxed{}^2}$$

Simplify.

$$A = \frac{\boxed{}^2}{\boxed{} \cdot \boxed{}}$$

Solve for C^2.

$$C^2 = 4\boxed{}\boxed{}$$

The circumference of the circle squared is equal to

_____.

Reflect

5. Does this formula work for a circle with a radius of 3 inches? Show your work.

Guided Practice

Find the area of each circle. Round to the nearest tenth if necessary. Use 3.14 for π.
(Explore Activity 1)

1.

14 m _____

2.

12 mm _____

3.

20 yd _____
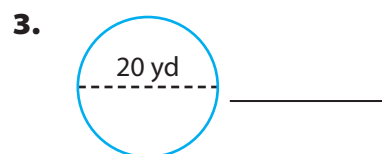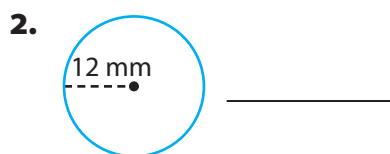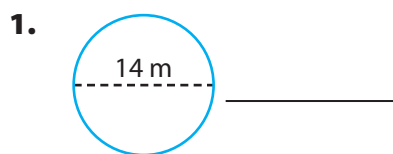
Solve. Use 3.14 for π. (Example 1)

4. A clock face has a radius of 8 inches. What is the area of the clock face? Round your answer to the nearest hundredth.   _____

5. A DVD has a diameter of 12 centimeters. What is the area of the DVD? Round your answer to the nearest hundredth.   _____

6. A company makes steel lids that have a diameter of 13 inches. What is the area of each lid? Round your answer to the nearest hundredth.   _____

Find the area of each circle. Give your answers in terms of π.
(Explore Activity 2)

7. $C = 4\pi$

    $A =$ _____

8. $C = 12\pi$

    $A =$ _____

9. $C = \frac{\pi}{2}$

    $A =$ _____

10. A circular pen has an area of 64π square yards. What is the circumference of the pen? Give your answer in terms of π.
     (Explore Activity 2)   _____

? **ESSENTIAL QUESTION CHECK-IN**

11. What is the formula for the area A of a circle in terms of the radius r?   _____

9.2 Independent Practice

COMMON CORE **7.G.4**
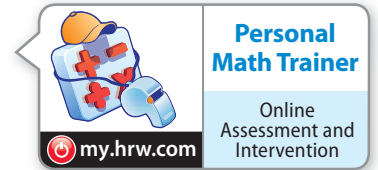
12. The most popular pizza at Pavone's Pizza is the 10-inch personal pizza with one topping. What is the area of a pizza with a diameter of 10 inches? Round your answer to the nearest hundredth.

_____

13. A hubcap has a radius of 16 centimeters. What is the area of the hubcap? Round your answer to the nearest hundredth.

16 cm

_____

14. A stained glass window is shaped like a semicircle. The bottom edge of the window is 36 inches long. What is the area of the stained glass window? Round your answer to the nearest hundredth.

_____

15. Analyze Relationships The point (3, 0) lies on a circle with the center at the origin. What is the area of the circle to the nearest hundredth?

_____

16. Multistep A radio station broadcasts a signal over an area with a radius of 50 miles. The station can relay the signal and broadcast over an area with a radius of 75 miles. How much greater is the area of the broadcast region when the signal is relayed? Round your answer to the nearest square mile.

_____

17. Multistep The sides of a square field are 12 meters. A sprinkler in the center of the field sprays a circular area with a diameter that corresponds to a side of the field. How much of the field is **not** reached by the sprinkler? Round your answer to the nearest hundredth.

_____

18. Justify Reasoning A small silver dollar pancake served at a restaurant has a circumference of 2π inches. A regular pancake has a circumference of 4π inches. Is the area of the regular pancake twice the area of the silver dollar pancake? Explain.

_____

_____

_____

_____

_____

_____

19. Analyze Relationships A bakery offers a small circular cake with a diameter of 8 inches. It also offers a large circular cake with a diameter of 24 inches. Does the top of the large cake have three times the area of that of the small cake? If not, how much greater is its area? Explain.

_____

_____

_____

_____

_____

20. **Communicate Mathematical Ideas** You can use the formula $A = \frac{C^2}{4\pi}$ to find the area of a circle given the circumference. Describe another way to find the area of a circle when given the circumference.

_____

_____

_____

21. **Draw Conclusions** Mark wants to order a pizza. Which is the better deal? Explain.

Donnie's Pizza Palace		
Diameter (in.)	12	18
Cost ($)	10	20

_____

_____

_____

22. **Multistep** A bear was seen near a campground. Searchers were dispatched to the region to find the bear.

a. Assume the bear can walk in any direction at a rate of 2 miles per hour. Suppose the bear was last seen 4 hours ago. How large an area must the searchers cover? Use 3.14 for π. Round your answer to the

nearest square mile. _____

b. **What If?** How much additional area would the searchers have to

cover if the bear were last seen 5 hours ago? _____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

23. **Analyze Relationships** Two circles have the same radius. Is the combined area of the two circles the same as the area of a circle with twice the radius? Explain.

_____

_____

24. **Look for a Pattern** How does the area of a circle change if the radius is multiplied by a factor of n, where n is a whole number?

_____

25. **Represent Real World Problems** The bull's-eye on a target has a diameter of 3 inches. The whole target has a diameter of 15 inches. What part of the whole target is the bull's-eye? Explain.

_____

LESSON
9.3

Area of Composite Figures

COMMON CORE 7.G.6
Solve real-world and mathematical problems involving area, . . . of . . . objects composed of triangles, quadrilaterals, polygons, ...

? ESSENTIAL QUESTION

How do you find the area of composite figures?

EXPLORE ACTIVITY Real World   COMMON CORE 7.G.6

Exploring Areas of Composite Figures

Aaron was plotting the shape of his garden on grid paper. While it was an irregular shape, it was perfect for his yard. Each square on the grid represents 1 square meter.
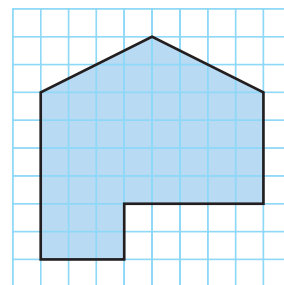
A Describe one way you can find the area of this garden.

_____

_____

_____

B The area of the garden is _____ square meters.

C Compare your results with other students. What other methods were used to find the area?

_____

_____

_____

D How does the area you found compare with the area found using different methods?

_____

Reflect

1. Use dotted lines to show two different ways Aaron's garden could be divided up into simple geometric figures.
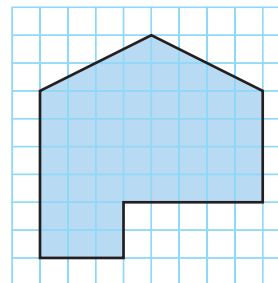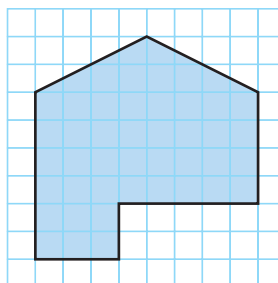
Finding the Area of a Composite Figure

A composite figure is made up of simple geometric shapes. To find the area of a composite figure or other irregular-shaped figure, divide it into simple, nonoverlapping figures. Find the area of each simpler figure, and then add the areas together to find the total area of the composite figure.

Use the chart below to review some common area formulas.

Shape	Area Formula
triangle	$A = \frac{1}{2}bh$
square	$A = s^2$
rectangle	$A = \ell w$
parallelogram	$A = bh$
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$

EXAMPLE 1

COMMON CORE 7.G.6

Find the area of the figure.

STEP 1 Separate the figure into smaller, familiar figures: a parallelogram and a trapezoid.

STEP 2 Find the area of each shape.

Area of the Parallelogram
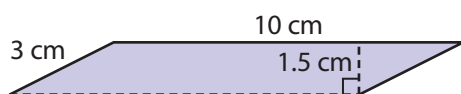
base = 10 cm

height = 1.5 cm

Use the formula.

$A = bh$

$A = 10 \cdot 1.5$

$A = 15$

The area of the parallelogram is 15 cm².
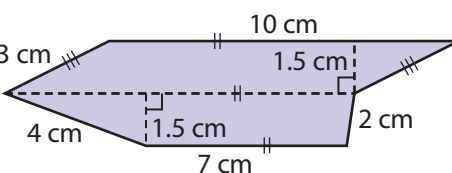
Area of the Trapezoid
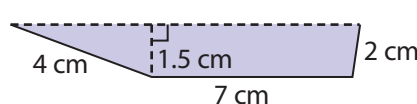
base$_1$ = 7 cm   base$_2$ = 10 cm

height 1.5 cm

Use the formula.

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(1.5)(7 + 10)$

$A = \frac{1}{2}(1.5)(17) = 12.75$

The area of the trapezoid is 12.75 cm².

> The top base of the trapezoid is 10 cm since it is the same length as the base of the parallelogram.

STEP 3 Add the areas to find the total area.

$A = 15 + 12.75 = 27.75$ cm²

The area of the figure is 27.75 cm².

Find the area of each figure. Use 3.14 for π.

2.

2 ft · 8 ft
3 ft
4 ft
3 ft
3 ft · 8 ft

3.

10 m
10 m
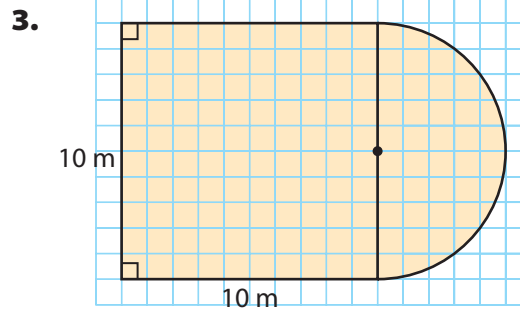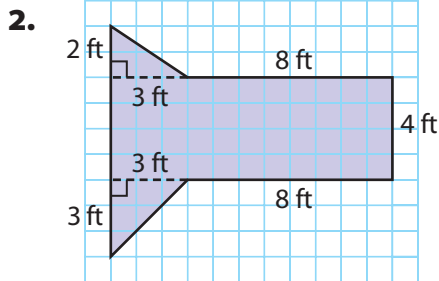
Using Area to Solve Problems

EXAMPLE 2 *Real World*

A banquet room is being carpeted. A floor plan of the room is shown at right. Each unit represents 1 yard. The carpet costs $23.50 per square yard. How much will it cost to carpet the room?
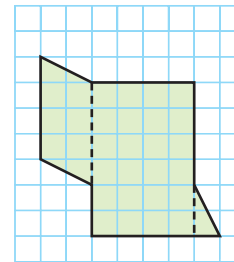
STEP 1 Separate the composite figure into simpler shapes as shown by the dashed lines: a parallelogram, a rectangle, and a triangle.

STEP 2 Find the area of the simpler figures. Count units to find the dimensions.

Parallelogram	**Rectangle**	**Triangle**
$A = bh$	$A = \ell w$	$A = \frac{1}{2}bh$
$A = 4 \cdot 2$	$A = 6 \cdot 4$	$A = \frac{1}{2}(1)(2)$
$A = 8 \text{ yd}^2$	$A = 24 \text{ yd}^2$	$A = 1 \text{ yd}^2$

STEP 3 Find the area of the composite figure.

$A = 8 + 24 + 1 = 33$ square yards

STEP 4 Calculate the cost to carpet the room.

Area · Cost per yard = Total cost

33 · $23.50 = $775.50

The cost to carpet the banquet room is $775.50.
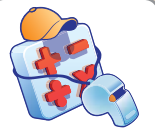
Math Talk
Mathematical Practices

Describe how you can estimate the cost to carpet the room.

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YOUR TURN

4. A window is being replaced with tinted glass. The plan at the right shows the design of the window. Each unit length represents 1 foot. The glass costs $28 per square foot. How much will it cost to replace the glass? Use 3.14 for π.

_____
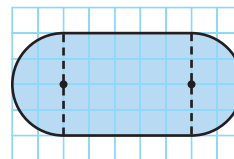
Guided Practice

1. A tile installer plots an irregular shape on grid paper. Each square on the grid represents 1 square centimeter. What is the area of the irregular shape? (Explore Activity, Example 2)

   STEP 1 Separate the figure into a triangle, a _____, and a parallelogram.

   STEP 2 Find the area of each figure.

   triangle: _____ cm²; rectangle: _____ cm²; parallelogram: _____ cm²

   STEP 3 Find the area of the composite figure: ____ + ____ + ____ = ____ cm²

   The area of the irregular shape is _____ cm².
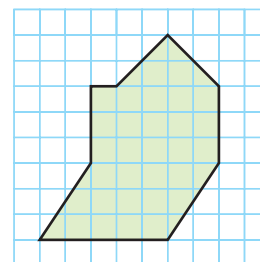
2. Show two different ways to divide the composite figure. Find the area both ways. Show your work below. (Example 1)

   12 cm
   9 cm
   8 cm
   18 cm
   9 cm
   20 cm
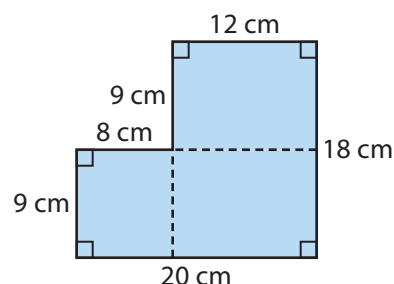
3. Sal is tiling his entryway. The floor plan is drawn on a unit grid. Each unit length represents 1 foot. Tile costs $2.25 per square foot. How much will Sal pay to tile his entryway? (Example 2)

   _____
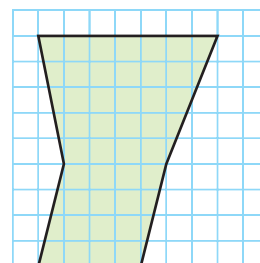
? **ESSENTIAL QUESTION CHECK-IN**

4. What is the first step in finding the area of a composite figure?

   _____

9.3 Independent Practice

COMMON CORE 7.G.6
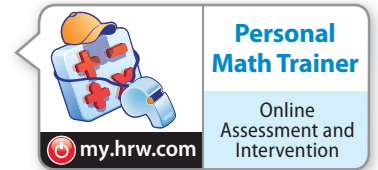
5. A banner is made of a square and a semicircle. The square has side lengths of 26 inches. One side of the square is also the diameter of the semicircle. What is the total area of the banner? Use 3.14 for π.

_____

6. Multistep Erin wants to carpet the floor of her closet. A floor plan of the closet is shown.

10 ft
4 ft
3 ft
6 ft
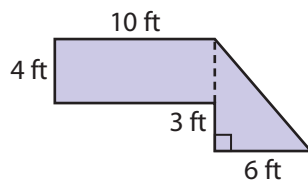
a. How much carpet does Erin need?

_____

b. The carpet Erin has chosen costs $2.50 per square foot. How much will it cost her to carpet the floor?

_____

7. Multiple Representations Hexagon *ABCDEF* has vertices $A(-2, 4)$, $B(0, 4)$, $C(2, 1)$, $D(5, 1)$, $E(5, -2)$, and $F(-2, -2)$. Sketch the figure on a coordinate plane. What is the area of the hexagon?
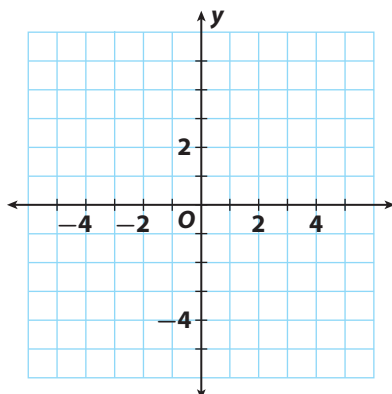
8. A field is shaped like the figure shown. What is the area of the field? Use 3.14 for π.

8 m
8 m
8 m
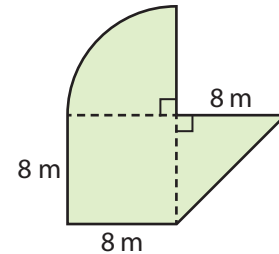
9. A bookmark is shaped like a rectangle with a semicircle attached at both ends. The rectangle is 12 cm long and 4 cm wide. The diameter of each semicircle is the width of the rectangle. What is the area of the bookmark? Use 3.14 for π.

10. Multistep Alex is making 12 pennants for the school fair. The pattern he is using to make the pennants is shown in the figure. The fabric for the pennants costs $1.25 per square foot. How much will it cost Alex to make 12 pennants?

1 ft
3 ft
1 ft
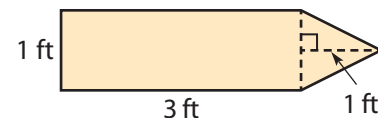
11. Reasoning A composite figure is formed by combining a square and a triangle. Its total area is 32.5 ft². The area of the triangle is 7.5 ft². What is the length of each side of the square? Explain.

_____

_____

_____

_____

Work Area

12. Represent Real-World Problems Christina plotted the shape of her garden on graph paper. She estimates that she will get about 15 carrots from each square unit. She plans to use the entire garden for carrots. About how many carrots can she expect to grow? Explain.

_____

_____

_____
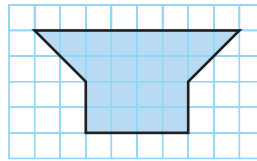
13. Analyze Relationships The figure shown is made up of a triangle and a square. The perimeter of the figure is 56 inches. What is the area of the figure? Explain.

10 in.   8 in.

10 in.

_____

_____

_____

_____
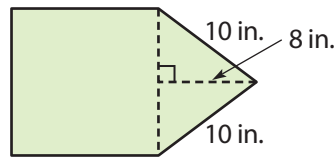
14. Critical Thinking The pattern for a scarf is shown at right. What is the area of the scarf? Use 3.14 for π.

28 in.

15 in.
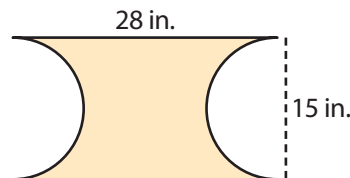
15. Persevere in Problem Solving The design for the palladium window shown includes a semicircular shape at the top. The bottom is formed by squares of equal size. A shade for the window will extend 4 inches beyond the perimeter of the window, shown by the dashed line around the window. Each square in the window has an area of 100 in².
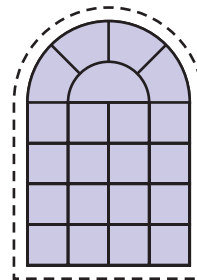
a. What is the area of the window? Use 3.14 for π.

_____

b. What is the area of the shade? Round your answer to the nearest whole number.

_____

Solving Surface Area Problems

? ESSENTIAL QUESTION

How can you find the surface area of a figure made up of cubes and prisms?

EXPLORE ACTIVITY COMMON CORE 7.G.6

Modeling Surface Area of a Prism

The surface area of a three-dimensional figure is the sum of the areas of all its surfaces. You know how to use the net of a figure to find its surface area. Now you will discover a formula that you can use.

8 mm

base 15 mm

20 mm

8 mm

15 mm

20 mm side | front | side | back

8 mm 15 mm

base 15 mm
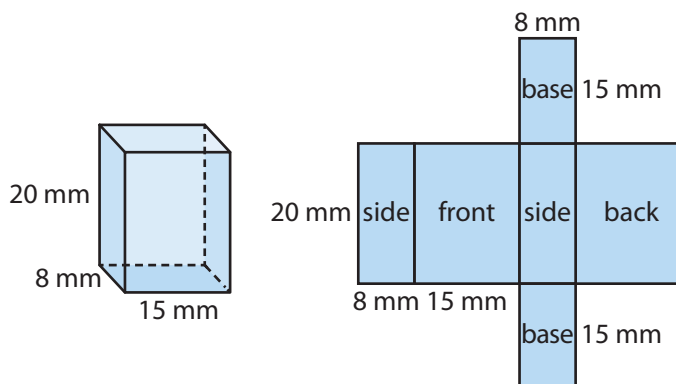
A The lateral area L of a prism is the area of all faces except the bases.

$L = 2(\underline{\hspace{2cm}}) + 2(\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$.

B The area B of each base is $\underline{\hspace{2cm}}$.

C The surface area S of the prism is the sum of the lateral area L and the

total area of the bases, or $\underline{\hspace{2cm}}$.

Reflect

1. Analyze Relationships Use the net above to answer this question: How does the product of the perimeter P of the base of the prism and the height h of the prism compare to the lateral area L? _____

2. Critical Thinking How can you express the surface area S of the prism in terms of P, h, and B? Use your answer to Question 1. _____

Finding the Surface Area of a Prism

Given a prism's dimensions, you can use a formula to find the surface area.

Surface Area of a Prism

The surface area S of a prism with base perimeter P, height h, and base area B is $S = Ph + 2B$.

EXAMPLE 1 🌎 Real World   COMMON CORE   7.G.6

Erin is making a jewelry box of wood in the shape of a rectangular prism. The jewelry box will have the dimensions shown. She plans to spray paint the exterior of the box. How many square inches will she have to paint?
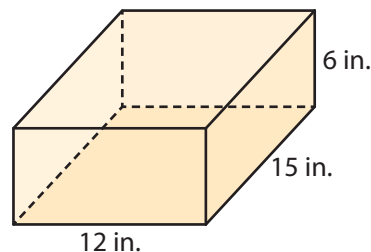
STEP 1 Make a sketch of the box. Drawing a diagram helps you understand and solve the problem.

STEP 2 Identify a base, and find its area and perimeter.

Any pair of opposite faces can be the bases. For example, you can choose the bottom and top of the box as the bases.

$$B = \ell \times w \qquad\qquad P = 2(12) + 2(15)$$
$$= 12 \times 15 \qquad\qquad = 24 + 30$$
$$= 180 \text{ square inches} \qquad = 54 \text{ inches}$$

STEP 3 Identify the height, and find the surface area.

The height h of the prism is 6 inches. Use the formula to find the surface area.

$$S = Ph + 2B$$
$$S = 54(6) + 2(180) = 684 \text{ square inches}$$

Erin will have to spray paint 684 square inches of wood.

Math Talk

Mathematical Practices

How can you express the formula for the surface area S of a rectangular prism in terms of its dimensions ℓ, w, and h?

YOUR TURN

3. A brand of uncooked spaghetti comes in a box that is a rectangular prism with a length of 9 inches, a width of 2 inches, and a height of $1\frac{1}{2}$ inches.

   What is the surface area of the box? _____

Finding the Surface Area of a Composite Solid

A composite solid is made up of two or more solid figures. To find the surface area of a composite solid, find the surface area of each figure. Subtract any area not on the surface.

EXAMPLE 2 · *Problem Solving* · COMMON CORE · 7.G.6

Daniel built the birdhouse shown. What was the surface area of the birdhouse before the hole was drilled?

17 cm
8 cm
18 cm
24 cm
30 cm
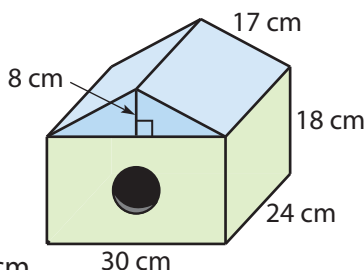
Analyze Information

Identify the important information.
- The top is a triangular prism with $h = 24$ cm. The base is a triangle with height 8 cm and base 30 cm.
- The bottom is a rectangular prism with $h = 18$ cm. The base is a 30 cm by 24 cm rectangle.
- One face of each prism is not on the surface of the figure.

Formulate a Plan

Find the surface area of each prism.

Add the areas. Subtract the areas of the parts not on the surface.

Solve

Find the area of the triangular prism.

Perimeter $= 17 + 17 + 30 = 64$ cm; Base area $= \frac{1}{2}(30)(8) = 120$ cm²

Surface area $= Ph + 2B$

$= 64(24) + 2(120) = 1{,}776$ cm²

Find the area of the rectangular prism.

Perimeter $= 2(30) + 2(24) = 108$ cm; Base area $= 30(24) = 720$ cm²

Surface area $= Ph + 2B$

$= 108(18) + 2(720) = 3{,}384$ cm²

Add. Then subtract twice the areas of the parts not on the surface.

Surface area $= 1{,}776 + 3{,}384 - 2(720) = 3{,}720$ cm²

The surface area before the hole was drilled was 3,720 cm².

Justify and Evaluate

You can check your work by using a net to find the surface areas.

Math Talk
Mathematical Practices

How could you find the surface area by letting the front and back of the prism be the bases?

YOUR TURN

4. Dara is building a plant stand. She wants to stain the plant stand, except for the bottom of the larger prism. Find the surface area of the part of the plant stand she will stain. _____

8 in.

8 in.

8 in.

12 in.

12 in.

12 in.
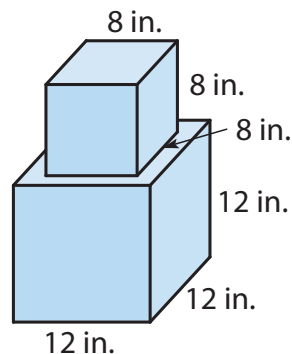
Guided Practice

Find the surface area of each solid figure. (Examples 1 and 2)

1.

5 ft   5 ft

3 ft   7 ft

8 ft
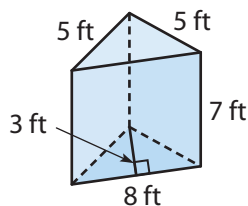
Perimeter of base = _____

Height = _____

Base area = _____

Surface area:

S = (_____)(_____) + 2(_____)

= _____

2.

2.5 m   2.5 m

2.5 m

7 m   9 m

11 m
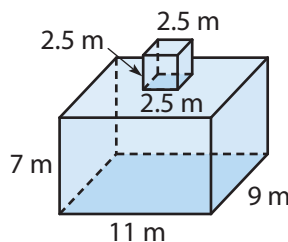
Surface area of cube:

S = _____

Surface area of rectangular prism:

S = _____

Overlapping area: A = _____

Surface area of composite figure:

= _____ + _____ − 2 (_____) =

_____ m²

? ESSENTIAL QUESTION CHECK-IN

3. How can you find the surface area of a composite solid made up of prisms?

_____

_____

_____

9.4 Independent Practice

COMMON CORE 7.G.6
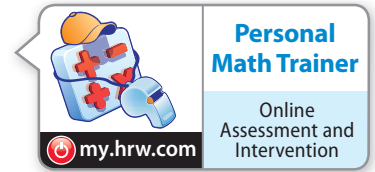
4. Carla is wrapping a present in the box shown. How much wrapping paper does she need, not including overlap?

4 in.
10 in.
3 in.

_____
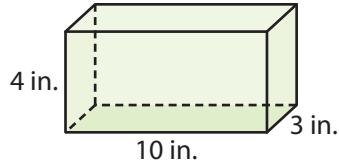
5. Dmitri wants to cover the top and sides of the box shown with glass tiles that are 5 mm square. How many tiles does he need?

9 cm
20 cm
15 cm

_____
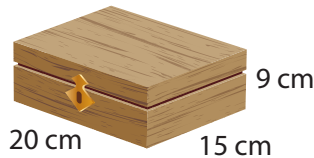
6. Shera is building a cabinet. She is making wooden braces for the corners of the cabinet. Find the surface area of each brace.

1 in.
3 in.
1 in.
3 in.
3 in.

_____
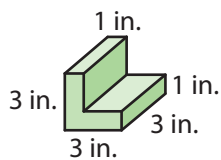
7. The doghouse shown has a floor, but no windows. Find the total surface area of the doghouse, including the door.

2 ft
2.5 ft
2.5 ft
2 ft
4 ft
3 ft
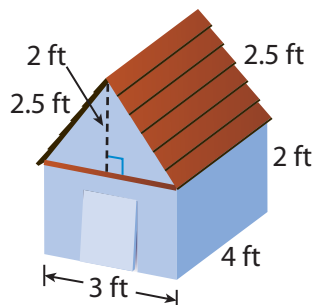
Eddie built the ramp shown to train his puppy to do tricks. Use the figure for 8–9.

12 in.
20 in.
20 in.
24 in.
16 in. 16 in. 16 in.
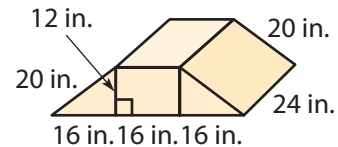
8. Analyze Relationships Describe two ways to find the surface area of the ramp.

_____

_____

_____

_____

9. What is the surface area of the ramp?

_____

Marco and Elaine are building a stand like the one shown to display trophies. Use the figure for 10–11.

1 ft
3 ft
3 ft
3 ft
2 ft
7 ft
1 ft
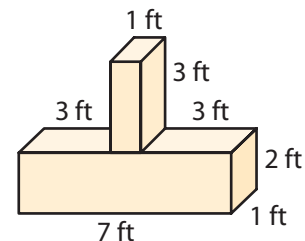
10. What is the surface area of the stand?

_____

11. Critique Reasoning Marco and Elaine want to paint the entire stand silver. A can of paint covers 25 square feet and costs $6.79. They set aside $15 for paint. Is that enough? Explain.

_____

_____

12. Henry wants to cover the box shown with paper without any overlap. How many square centimeters will be covered with paper?

10 cm   24 cm   27 cm

_____
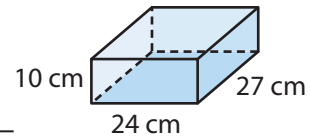
13. What If? Suppose the length and width of the box in Exercise 12 double. Does the surface area *S* double? Explain.

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

14. Persevere in Problem Solving Enya is building a storage cupboard in the shape of a rectangular prism. The rectangular prism has a square base with side lengths of 2.5 feet and a height of 3.5 feet. Compare the amount of paint she would use to paint all but the bottom surface of the prism to the amount she would use to paint the entire prism.

_____

_____

_____

15. Interpret the Answer The oatmeal box shown is shaped like a cylinder. Use a net to find the surface area *S* of the oatmeal box to the nearest tenth. Then find the number of square feet of cardboard needed for 1,500 oatmeal boxes. Round your answer to the nearest whole number.

2 in.

Oatmeal   9 in.

_____

_____
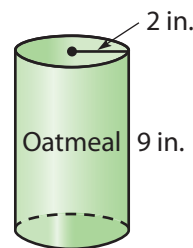
16. Analyze Relationships A prism is made of centimeter cubes. How can you find the surface area of the prism in Figure 1 without using a net or a formula? How does the surface area change in Figures 2, 3, and 4? Explain.
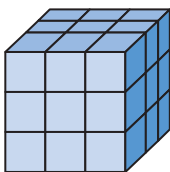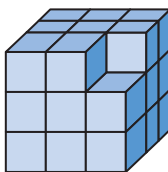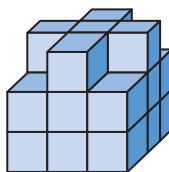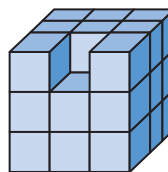
Figure 1   Figure 2   Figure 3   Figure 4

_____

_____

_____

_____

_____

LESSON
9.5

Solving Volume Problems

COMMON CORE **7.G.6**

Solve real-world and mathematical problems involving . . . volume of . . . three-dimensional objects composed of . . . cubes, and right prisms.

? ESSENTIAL QUESTION

How do you find the volume of a figure made of cubes and prisms?

Volume of a Triangular Prism

The formula for the volume of a rectangular prism can be used for *any* prism.

> **Volume of a Prism**
>
> The volume V of a prism is the area of its base B times its height h.
> $$V = Bh$$

EXAMPLE 1 — Real World

COMMON CORE **7.G.6**

Bradley's tent is in the shape of a triangular prism. How many cubic feet of space are in his tent?

4 ft
5 ft
9 ft
6 ft
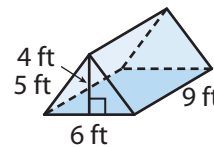
STEP 1 Find the base area B of the triangular prism.

$$B = \frac{1}{2}bh \qquad \text{Area of a triangle with base length } b \text{ and height } h$$

$$= \frac{1}{2}(6)(4) \qquad \text{Substitute 6 for } b \text{ and 4 for } h.$$

$$= 12 \text{ ft}^2$$

STEP 2 Find the volume of the prism.

$$V = Bh \qquad \text{Volume of a prism with base area } B \text{ and height } h$$

$$= (12)(9) \qquad \text{Substitute 12 for } B \text{ and 9 for } h.$$
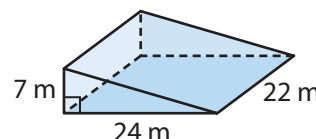
$$= 108 \text{ ft}^3$$

The volume of Bradley's tent is 108 ft³.

Reflect

1. Analyze Relationships For a prism that is **not** a rectangular prism, how do you determine which sides are the bases?

_____

YOUR TURN

2. Find the volume of the prism.

_____

7 m
22 m
24 m

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Volume of a Trapezoidal Prism

Prisms are named for the polygons that form their bases. In this lesson, you will focus on prisms whose bases are either triangles or quadrilaterals other than squares and rectangles.
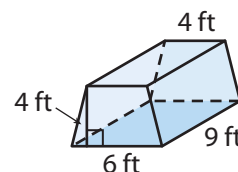
EXAMPLE 2   Real World   COMMON CORE   7.G.6

Cherise is setting up her tent. Her tent is in the shape of a trapezoidal prism. How many cubic feet of space are in her tent?

4 ft
4 ft
9 ft
6 ft

STEP 1   Find the base area B of the trapezoidal prism.

$B = \frac{1}{2}(b_1 + b_2)\, h$ Area of a trapezoid with bases of lengths b_1 and b_2 and height h

$\quad = \frac{1}{2}(6 + 4)4$ Substitute 6 for b_1, 4 for b_2, and 4 for h.

$\quad = \frac{1}{2}(10)4 = 20$ ft

STEP 2   Find the volume of the prism.

$V = Bh$ Volume of a prism with base area B and height h

$\quad = (20)(9)$ Substitute 20 for B and 9 for h.

$\quad = 180$ ft^3

The volume of Cherise's tent is 180 ft^3.

Reflect

3. **Look for a Pattern** How could you double the volume of the tent by doubling just one of its dimensions?

_____

_____

4. **What If?** How would doubling *all* the dimensions of the prism affect the volume of the tent?

_____

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YOUR TURN

5. Find the volume of the prism.

_____

12 cm
10 cm
8 cm
22 cm

Volume of a Composite Solid

You can use the formula for the volume of a prism to find the volume of a composite figure that is made up of prisms.
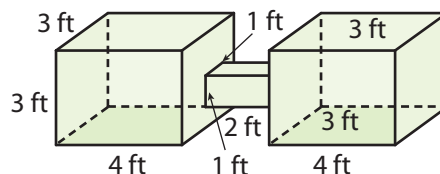

EXAMPLE 3 Real World

COMMON CORE 7.G.6

Allie has two aquariums connected by a small square prism. Find the volume of the double aquarium.

STEP 1 Find the volume of each of the larger aquariums.

$V = Bh$ Volume of a prism

$= (12)(3)$ Substitute $3 \times 4 = 12$ for B and 3 for h.

$= 36 \text{ ft}^3$

STEP 2 Find the volume of the connecting prism.

$V = Bh$ Volume of a prism

$= (1)(2)$ Substitute $1 \times 1 = 1$ for B and 2 for h.

$= 2 \text{ ft}^3$

STEP 3 Add the volumes of the three parts of the aquarium.

$V = 36 + 36 + 2 = 74 \text{ ft}^3$

The volume of the aquarium is 74 ft³.

Reflect

6. **What If?** Find the volume of one of the large aquariums on either end using another pair of opposite sides as the bases. Do you still get the same volume? Explain.

_____

_____

YOUR TURN

7. The figure is composed of a rectangular prism and a triangular prism. Find the volume of the figure.
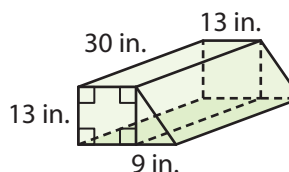
_____

1. Find the volume of the triangular prism. (Example 1)

$B = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12 \text{ ft}^{\boxed{}}$

$V = Bh = \left(\boxed{} \times \boxed{}\right) \text{ ft}^{\boxed{}} = \boxed{} \text{ ft}^3$

(figure: triangular prism, 5 ft, 5 ft, 7 ft, 3 ft, 8 ft)
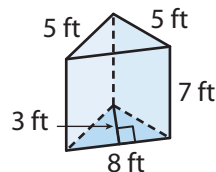
2. Find the volume of the trapezoidal prism. (Example 2)

$B = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(15 + 5)(3) = 30 \text{ m}^{\boxed{}}$

$V = Bh = \left(\boxed{} \times \boxed{}\right) \text{ m}^{\boxed{}} = \boxed{} \text{ m}^3$

(figure: trapezoidal prism, 3 m, 15 m, 11 m, 5 m)
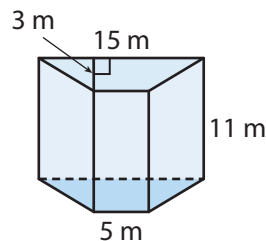
3. Find the volume of the composite figure. (Example 3)

Volume of rectangular prism = _____

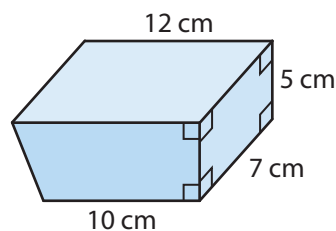Volume of triangular prism = _____

Volume of composite figure = _____

(figure: composite figure, 6 ft, 4 ft, 4 ft, 12 ft, 6 ft)
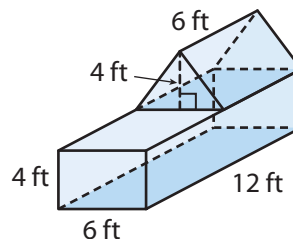
Find the volume of each figure. (Examples 2 and 3)

4. The figure shows a barn that Mr. Fowler is building for his farm.

(figure: barn, 10 ft, 15 ft, 40 ft, 50 ft)

_____
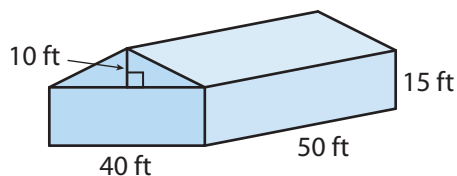
5. The figure shows a container, in the shape of a trapezoidal prism, that Pete filled with sand.

(figure: trapezoidal prism container, 12 cm, 5 cm, 7 cm, 10 cm)
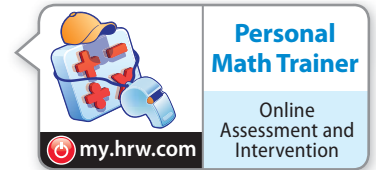
_____

? ESSENTIAL QUESTION CHECK-IN

6. How do you find the volume of a composite solid formed by two or more prisms?

_____

_____

9.5 Independent Practice

COMMON CORE 7.G.6

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7. A trap for insects is in the shape of a triangular prism. The area of the base is 3.5 in^2 and the height of the prism is 5 in. What is the volume of this trap?

_____

8. Arletta built a cardboard ramp for her little brothers' toy cars. Identify the shape of the ramp. Then find its volume.

6 in.
25 in.
7 in.

_____
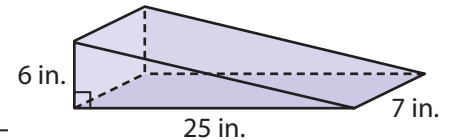
9. Alex made a sketch for a homemade soccer goal he plans to build. The goal will be in the shape of a triangular prism. The legs of the right triangles at the sides of his goal measure 4 ft and 8 ft, and the opening along the front is 24 ft. How much space is contained within this goal?

_____

10. A gift box is in the shape of a trapezoidal prism with base lengths of 7 inches and 5 inches and a height of 4 inches. The height of the gift box is 8 inches. What is the volume of the gift box?
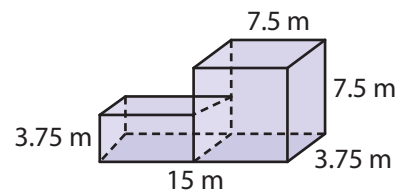
_____

11. Explain the Error A student wrote this statement: "A triangular prism has a height of 15 inches and a base area of 20 square inches. The volume of the prism is 300 square inches." Identify and correct the error.

_____

Find the volume of each figure. Round to the nearest hundredth if necessary.

12. B ≈ 23.4 in^2

3 in.
3 in.
3 in. 3 in.

13.

7.5 m
7.5 m
3.75 m
3.75 m
15 m

_____   _____
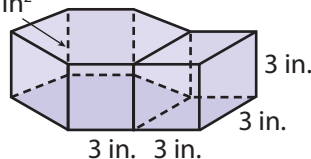
14. Multi-Step Josie has 260 cubic centimeters of candle wax. She wants to make a hexagonal prism candle with a base area of 21 square centimeters and a height of 8 centimeters. She also wants to make a triangular prism candle with a height of 14 centimeters. Can the base area of the triangular prism candle be 7 square centimeters? Explain.

_____

15. A movie theater offers popcorn in two different containers for the same price. One container is a trapezoidal prism with a base area of 36 square inches and a height of 5 inches. The other container is a triangular prism with a base area of 32 square inches and a height of 6 inches. Which container is the better deal? Explain.

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

16. Critical Thinking The wading pool shown is a trapezoidal prism with a total volume of 286 cubic feet. What is the missing dimension?

13 ft
2 ft
?
8 ft

_____
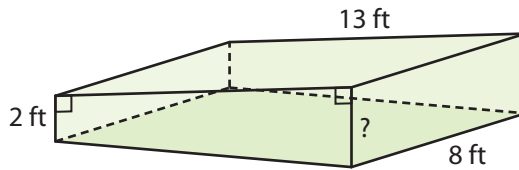
17. Persevere in Problem Solving Lynette has a metal doorstop with the dimensions shown. Each cubic centimeter of the metal in the doorstop has a mass of about 8.6 grams. Find the volume of the metal in the doorstop. Then find the mass of the doorstop.

10 cm
2.5 cm
8 cm
6 cm

_____

_____
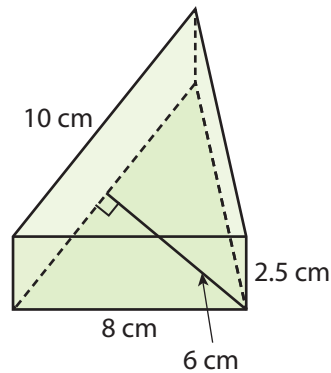
18. Analyze Relationships What effect would tripling all the dimensions of a triangular prism have on the volume of the prism? Explain your reasoning.

_____

_____

_____

_____

19. Persevere in Problem Solving Each of two trapezoidal prisms has a volume of 120 cubic centimeters. The prisms have no dimensions in common. Give possible dimensions for each prism.
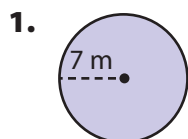
_____

_____

_____

_____

Ready to Go On?

9.1, 9.2 Circumference and Area of Circles

Find the circumference and area of each circle. Use 3.14 for π. Round to the nearest hundredth if necessary.
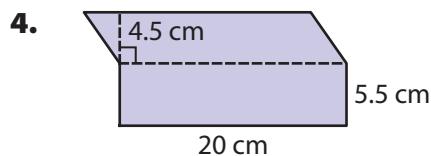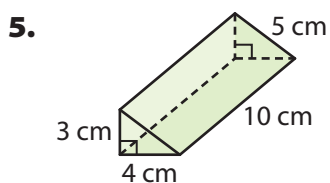
1.

7 m

2.

12 ft

_____   _____
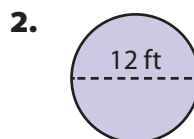
9.3 Area of Composite Figures

Find the area of each figure. Use 3.14 for π.

3.

10 m

16 m

4.

4.5 cm

5.5 cm

20 cm

_____   _____
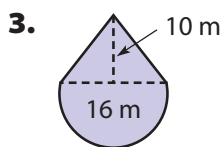
9.4, 9.5 Solving Surface Area and Volume Problems

Find the surface area and volume of each figure.

5.

5 cm

3 cm

10 cm

4 cm

6.

2.5 yd

1.5 yd

2 yd

2.5 yd

4 yd

_____   _____
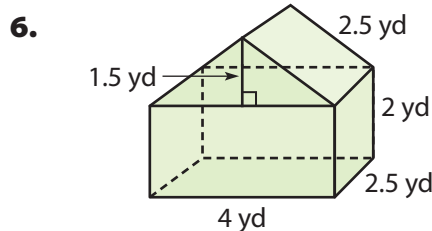
? ESSENTIAL QUESTION

7. How can you use geometry figures to solve real-world problems?

_____

_____

Assessment Readiness
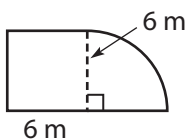
Selected Response

1. What is the circumference of the circle?

11 m
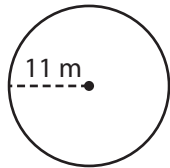
- Ⓐ 34.54 m
- Ⓑ 69.08 m
- Ⓒ 379.94 m
- Ⓓ 1,519.76 m

2. What is the area of the circle?

15 m
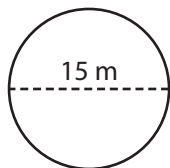
- Ⓐ 23.55 m²
- Ⓒ 176.625 m²
- Ⓑ 47.1 m²
- Ⓓ 706.5 m²

3. What is the area of the figure?

6 m

6 m

- Ⓐ 28.26 m²
- Ⓒ 64.26 m²
- Ⓑ 36 m²
- Ⓓ 92.52 m²

4. A one-year membership to a health club costs $480. This includes a $150 fee for new members that is paid when joining. Which equation represents the monthly cost x in dollars for a new member?

- Ⓐ $12x + 150 = 480$
- Ⓑ $\frac{x}{12} + 150 = 480$
- Ⓒ $12x + 480 = 150$
- Ⓓ $\frac{x}{12} + 480 = 150$

5. What is the volume of the prism?

2 ft

12 ft

8 ft
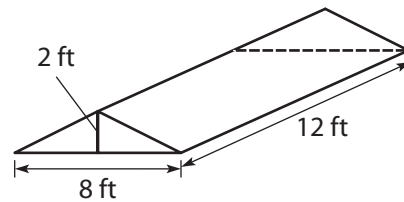
- Ⓐ 192 ft³
- Ⓒ 69 ft³
- Ⓑ 48 ft³
- Ⓓ 96 ft³

6. A school snack bar sells a mix of granola and raisins. The mix includes 2 pounds of granola for every 3 pounds of raisins. How many pounds of granola are needed for a mix that includes 24 pounds of raisins?

- Ⓐ 16 pounds
- Ⓒ 48 pounds
- Ⓑ 36 pounds
- Ⓓ 120 pounds

7. Find the percent change from $20 to $25.

- Ⓐ 25% decrease
- Ⓒ 20% decrease
- Ⓑ 25% increase
- Ⓓ 20% increase

Mini-Task

8. Each dimension of the smaller prism is half the corresponding dimension of the larger prism.

8 in.

10 in.

16 in.
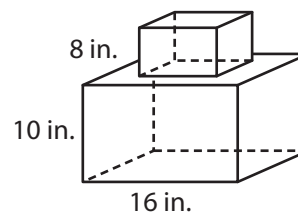
a. What is the surface area of the figure?

_____

b. What is the volume of the figure?

_____