

# Englewood Public School District

## Algebra II

### Second Marking Period

#### Unit 2: Polynomial, Radical, Rational, and Logarithmic Functions

**Overview:** During this unit, students will investigate polynomial, radical, rational, and logarithmic expressions and equations including their graphs.

**Time Frame:** 43 to 47 Days

#### Enduring Understandings:

- *A polynomial of degree  $n$  has  $n$  linear factors. The graph of the related function crosses the  $x$ -axis an even or odd number of times depending on whether  $n$  is even or odd.*
- *$(x - a)$  is a linear factor if and only if  $a$  is a zero and a root and if and only if  $(a, 0)$  is an  $x$ -intercept when  $a$  is a real number.*
- *You can simplify the  $n^{\text{th}}$  root of an expression that contains an  $n^{\text{th}}$  power as a factor.*
- *When you square each side of an equation the resulting equation may have more solutions than the original equation*
- *If  $f$  and  $f^{-1}$  are inverse functions and if one maps  $a$  to  $b$  then the other maps  $b$  to  $a$ .*
- *The function  $y = ab^x$  where  $a > 0$  and  $b > 1$  models exponential growth. When  $0 < b < 1$  the function models decay.*
- *Logarithms are exponents.*
- *Exponential functions and logarithms are inverse operations.*
- *Quantities are inversely proportional only if increasing one quantity by a factor means shrinking the other factor by the inverse.*
- *A rational function may have zero or one horizontal or oblique asymptote, and zero or more vertical asymptotes.*

#### Essential Questions:

- *What does the degree of a polynomial tell you about its related polynomial function?*
- *For a polynomial function how are factors, zeros, and  $x$ -intercepts related?*
- *For a polynomial equation how are factors and roots related?*
- *To simplify  $n^{\text{th}}$  root of an expression, what must be true about the expression?*
- *When you square each side of an equation, is the resulting equation equivalent to the original?*
- *How are a function and its inverse function related?*
- *How do you model a quantity that changes regularly over time by the same percentage?*
- *How are exponents and logarithms related?*
- *How are exponential functions and logarithmic functions related?*
- *Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?*
- *What kinds of asymptotes are possible for a rational function?*
- *Are rational expression and its simplified form equivalent?*

Standards	Topics and Objectives	Activities	Resources	Assessments
MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8  <b>N-CN.C.7</b> Solve quadratic equations with real coefficients that have complex solutions <b>N-CN.C.8</b> Extend polynomial identities to the complex numbers <b>N-CN.C.9</b> Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials <b>A-SSE.A.1a, b</b> Interpret expressions that represent a quantity in terms of its context <b>A-SSE.A.2</b> Use the structure of an expression to identify ways to rewrite it. <b>A-SSE.B.3c</b> Use the properties of exponents to transform expressions for exponential functions <b>A-CED.A.1</b> Create equations and inequalities in one variable and use them to solve problems <b>A-CED.A.2</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales <b>A-CED.A.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context	<p><b>Topics</b></p> <p>Polynomials, linear factors, zeros, dividing polynomials, Fundamental Theorem of Algebra, Binomial Theorem, radical expressions, radical exponents, solving radical equations, inverse relations and functions, exponential functions, logarithmic functions, exponential and logarithmic equations, natural logs, rational functions and their graphs, adding and subtracting rational expressions, solving rational equations</p> <p>Twenty-First Century Themes and Skills include:</p> <ul style="list-style-type: none"> <li>The Four C's</li> <li><a href="#">Global awareness</a></li> <li><a href="#">Financial, economic, business and entrepreneurial literacy</a></li> </ul> <p><b>Objectives</b></p> <p>Students will</p> <ul style="list-style-type: none"> <li>Classify polynomials</li> <li>Graph polynomial functions and describe their end behavior</li> <li>Analyze the factored form of a polynomial</li> <li>Write a polynomial function from its zeros</li> <li>Solve polynomials by graphing or factoring</li> <li>Divide polynomials using</li> </ul>	<p>Standards Solution Common Core Algebra Lessons:</p> <ul style="list-style-type: none"> <li>Population Growth and Decay</li> <li>Finding the Greatest Investment</li> <li>Dividing Polynomials</li> </ul> <p>Standards Solution Common Core Function lessons:</p> <ul style="list-style-type: none"> <li>Linear Vs. Exponential</li> </ul> <p>Non Negative Polynomials  <a href="https://www.illustrativemathematics.org/content-standards/HSA/APR/A/1/tasks/1656">https://www.illustrativemathematics.org/content-standards/HSA/APR/A/1/tasks/1656</a></p> <p>Solving a Simple Cubic Equation  <a href="https://www.illustrativemathematics.org/content-standards/HSA/APR/B/3/tasks/2138">https://www.illustrativemathematics.org/content-standards/HSA/APR/B/3/tasks/2138</a></p> <p>Powers of 11  <a href="https://www.illustrativemathematics.org/content-standards/HSA/APR/C/5/tasks/1654">https://www.illustrativemathematics.org/content-standards/HSA/APR/C/5/tasks/1654</a></p> <p>Combined Fuel Efficiency  <a href="https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/825">https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/825</a></p> <p>Egyptian Fractions II  <a href="https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/1346">https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/1346</a></p>	<p>Pearson Realize Chapters 5, 6, 7, and 8</p> <p>Standards Solution Common Core Lessons</p> <p>Illustrative Mathematics  <a href="https://www.illustrativemathematics.org/">https://www.illustrativemathematics.org/</a></p> <p>Alabama Learning Exchange  <a href="http://alex.state.al.us/search.php?fa_submit=ALLPLANS">http://alex.state.al.us/search.php?fa_submit=ALLPLANS</a></p> <p>Arizona Math Flipbook  <a href="http://www.azed.gov/azcommomcore/files/2012/11/high-school-ccss-flip-book-usd-259-2012.pdf">http://www.azed.gov/azcommomcore/files/2012/11/high-school-ccss-flip-book-usd-259-2012.pdf</a></p> <p>NYC Department of Education  <a href="http://schools.nyc.gov/default.htm">http://schools.nyc.gov/default.htm</a></p> <p>Mathematics Assessment Project  <a href="http://map.mathshell.org/">http://map.mathshell.org/</a></p> <p>Texas Instruments  <a href="https://education.ti.com/en/us/home">https://education.ti.com/en/us/home</a></p> <p>Desmos  <a href="https://teacher.desmos.com/">https://teacher.desmos.com/</a></p> <p>Worksheets for every topic:  <a href="http://kutasoftware.com/free.html">http://kutasoftware.com/free.html</a></p> <p>(CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)</p> <p>Algebra assessments,</p>	<p><b>Formative Assessments:</b>            Textbook Pages 311, 347, 353, 389, 427, 428, 461, 491, 492, 526, 557, 558</p> <p>Math journal (NJSLSA.R1, NJSLSA.W2)</p> <p><b>Summative Assessments:</b>            Multiple choice / short answer assessments (CRP2, CRP4, CRP8)</p> <p>Chapter quizzes/tests</p> <ul style="list-style-type: none"> <li>Pearson Realize</li> <li>MathXL</li> </ul> <p>Grade 11 Algebra II Common Core Assessment 2, Standards Solution</p> <p><b>Benchmark Assessment:</b>            Midterm Assessment</p> <p><b>Alternative Assessments:</b>            Learning centers: each learning center focuses on a different type of problem (9.3.ST.2, 9.3.ST-ET.5)</p> <p>Create posters illustrating the main objectives of the unit</p>

<p><b>A-CED.A.4</b> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equation</p> <p><b>A-APR.B.2</b> Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</p> <p><b>A-APR.B.3</b> Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial</p> <p><b>A-APR.C.4</b> Prove polynomial identities and use them to describe numerical relationships</p> <p><b>A-APR.C.5</b> Know and apply the Binomial Theorem for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers, with coefficients determined for example by Pascal's Triangle</p> <p><b>A-APR.D.6</b> Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system</p> <p><b>A-APR.D.7</b> Understand that</p>	<p>long division or synthetic division</p> <ul style="list-style-type: none"> <li>• Solve equations using the Rational Root theorem or Conjugate Root theorem</li> <li>• Use the Fundamental Theorem of algebra to solve polynomials with complex solutions</li> <li>• Expand a binomial using Pascal's Triangle</li> <li>• Use the Binomial theorem</li> <li>• Fit data to linear, quadratic, cubic or quartic models</li> <li>• Apply transformations to graphs of polynomials</li> <li>• Find <math>n^{\text{th}}</math> roots</li> <li>• Add, subtract, multiply and divide radical expressions</li> <li>• Simplify expressions with rational exponents</li> <li>• Solve square root and other radical equations</li> <li>• Add, subtract, multiply, and divide functions</li> <li>• Find the composite of two functions</li> <li>• Find the inverse of a relation or function</li> <li>• Graph square root and other radical functions</li> <li>• Model exponential growth and decay</li> <li>• Explore the properties of functions of the form <math>y = ab^x</math></li> <li>• To graph exponential functions that have base <math>e</math></li> <li>• Write and evaluate logarithmic functions</li> <li>• Graph logarithmic functions</li> </ul>	<p>graphs of Power Functions  <a href="https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/627">https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/627</a></p> <p>Identifying Exponential Functions  <a href="https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/2115">https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/2115</a></p> <p>Exponential Kiss  <a href="https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/1824">https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/1824</a></p> <p>Invertible or Not?  <a href="https://www.illustrativemathematics.org/content-standards/HSF/BF/B/4/tasks/1374">https://www.illustrativemathematics.org/content-standards/HSF/BF/B/4/tasks/1374</a></p> <p>Exponential and Logarithms I  <a href="https://www.illustrativemathematics.org/content-standards/HSF/BF/B/5/tasks/600">https://www.illustrativemathematics.org/content-standards/HSF/BF/B/5/tasks/600</a></p> <p>Exponentials and Logarithms II  <a href="https://www.illustrativemathematics.org/content-standards/HSF/BF/B/4/tasks/615">https://www.illustrativemathematics.org/content-standards/HSF/BF/B/4/tasks/615</a></p> <p>Algae Blooms  <a href="https://www.illustrativemathematics.org/content-">https://www.illustrativemathematics.org/content-</a></p>	<p>interactive, videos, games, lessons, homework:  <a href="https://www.opened.com/search?area=mathematics&amp;grade=9&amp;offset=0&amp;resource_type=interactive-assessment">https://www.opened.com/search?area=mathematics&amp;grade=9&amp;offset=0&amp;resource_type=interactive-assessment</a>  (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5, 8.1.12.A.3)</p> <p>Algebra common core worksheets:  <a href="https://www.ixl.com/math/algebra-1">https://www.ixl.com/math/algebra-1</a>  (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)</p> <p>Khan Academy – videos, lessons, assessments  <a href="http://www.khanacademy.org">www.khanacademy.org</a>  (CRP2, CRP4, CRP8, CRP11, 9.3.ST.2, 9.3.ST-ET.5, 8.1.12.A.3)</p> <p>Worksheets / assessment items for all topics based on standards:  <a href="http://jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI">http://jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI</a>  (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)</p>	<p>(CRP6)</p>
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rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions

**A-REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

**A-REI.D.11** Explain why the  $x$ ---coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions

**F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship  
**F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes

**F-IF.B.6** Calculate and

- Use the properties of logarithms
- Solve exponential and logarithmic equations
- Evaluate and simplify natural logarithmic expressions
- Solve equations using natural logarithms
- Recognize and use inverse, joint, and other variations
- Graph reciprocal functions
- Graph translations of reciprocal functions
- Identify properties and graph rational functions
- Simplify rational expressions
- Add, subtract, multiply and divide rational expressions
- Solve rational equations
- Use rational equations to solve problems

[standards/HSF/LE/A/4/tasks/570](https://www.illustrativemathematics.org/content-standards/HSF/LE/A/4/tasks/570)

Newton's Law of Cooling  
<https://www.illustrativemathematics.org/content-standards/HSF/LE/A/4/tasks/382>

Exponential Growth and Decay  
<http://alex.state.al.us/lessonview.php?id=24092>

Modeling Exponential Growth: Having Kittens  
<http://map.mathshell.org/lessons.php?unit=9100&collection=8&redir=1>

Representing Linear and Exponential Growth  
<http://map.mathshell.org/lessons.php?unit=9240&collection=8>

Representing Polynomials Graphically  
<http://map.mathshell.org/lessons.php?unit=9270&collection=8>

Exponential Reflections  
<https://education.ti.com/en/84activitycentral/us/detail?id=A99AC1F511BB4C97924D7A9B8960F046&t=4630994C160747DFAA66A7E4C9F6A26A>

Exponential Growth  
<https://education.ti.com/en/84activitycentral/us/detail?id=>

interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**F-IF.C.7b, c, e** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

**F-IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

**F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**F-BF.A.1b** Combine standard function types using arithmetic operations

**F-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

**F-BF.B.4a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse

**F-LE.4** Understand the

[FABF80DD572743E89EBE4A8BB2BB9202&t=4630994C160747DFAA66A7E4C9F6A26A](https://education.ti.com/en/84activitycentral/us/detail?id=9841154F449B4F6595FA750ACCF5099C&t=4630994C160747DFAA66A7E4C9F6A26A)

Change of Base

<https://education.ti.com/en/84activitycentral/us/detail?id=9841154F449B4F6595FA750ACCF5099C&t=4630994C160747DFAA66A7E4C9F6A26A>

Properties of Logarithms

<https://education.ti.com/en/84activitycentral/us/detail?id=E71949690CAA4D83844C468080CCE18B&t=4630994C160747DFAA66A7E4C9F6A26A>

Asymptotes and Zeros of Rational functions

<https://education.ti.com/en/84activitycentral/us/detail?id=B13C4236F1214CACBC388DCF123D7288&t=632C9D5E616E4C03ABB301E7B402C773>

Polygraph: Exponential and Logarithmic Functions

<https://teacher.desmos.com/polygraph/custom/560ad29058fd074d156300af>

Logarithmic Functions

<https://teacher.desmos.com/activitybuilder/custom/56d8738eafc5524106002f6d>

Transformations of the Logarithmic Function

inverse relationship between  
exponents and logarithms

<https://teacher.desmos.com/activitybuilder/custom/5609754c686358ae072fff9a>

Polygraph: Rational  
Functions

<https://teacher.desmos.com/polygraph-rationals>

Building Rational Functions

<https://teacher.desmos.com/activitybuilder/custom/5626c9d5f747b47b0d054475>

Polynomials and Roots

<https://teacher.desmos.com/activitybuilder/custom/561ec2e425af637122910fad>

Polygraph: Square Root  
Functions

<https://teacher.desmos.com/polygraph/custom/560ad29158fd074d156300b6>

Everything you need to  
know about math journals:

<https://thecornerstoneforteachers.com/math-journals/>

(NJSLA.R1,  
NJSLA.W2)

Additional texts:

[www.newsela.com](http://www.newsela.com)

[www.readworks.org](http://www.readworks.org)

[www.commonlit.org](http://www.commonlit.org)

**Key Vocabulary:**

End behavior, monomial, multiplicity, Pascal's Triangle, polynomial function, relative maximum, relative minimum, standard form of a polynomial function, synthetic division, turning point, composite function, inverse function, nth root, principal root, radical equation, radicand, rational exponent,



rationalize the denominator, square root equation, square root function, asymptote, Change of Base Formula, common logarithm, exponential equation, exponential function, exponential growth, logarithm, logarithmic equation, logarithmic function, natural logarithmic function

### **Accommodations and Modifications:**

**Students with special needs:** Support staff will be available to aid students related to IEP specifications. 504 accommodations will also be attended to by all instructional leaders. Modifications, alternative assessments, and scaffolding strategies will be used to support this learning. The use of Universal Design for Learning (UDL) will be considered for all students as teaching strategies are considered. Additional staff should be included so all students can fully participate in the standards associated with this curriculum.

**ELL/ESL students:** Students will be supported according to the recommendations for “can do’s” as outlined by WIDA - [https://www.wida.us/standards/CAN\\_DOs/](https://www.wida.us/standards/CAN_DOs/)

**Students at risk of school failure:** Formative and summative data will be used to monitor student success at first signs of failure. Student work will be reviewed to determine support. This may include parent consultation, basic skills review and differentiation strategies. With considerations to UDL, time may be a factor in overcoming developmental considerations. More time will be made available with a certified instructor to aid students in reaching the standards.

**Gifted and Talented Students:** Students excelling in mastery of standards will be challenged with complex, high level challenges.

<b>English Language Learners:</b>	<b>Special Education:</b>	<b>At-Risk:</b>	<b>Gifted and Talented:</b>
<ul style="list-style-type: none"><li>• Teaching modeling</li><li>• Peer modeling</li><li>• Word walls</li><li>• Give directions in small steps and in as few words as possible</li><li>• Provide visual aids</li><li>• Group similar problems together</li><li>• Repeat directions when necessary</li><li>• Provide a vocabulary list with definitions</li><li>• Use of alge-tiles when needed</li><li>• Use of number line when needed</li></ul>	<ul style="list-style-type: none"><li>• Utilize modifications &amp; accommodations delineated in the students’ IEP</li><li>• Work with paraprofessional</li><li>• Work with a partner</li><li>• Shorten assignments to focus on mastery or key concepts</li><li>• Maintain adequate space between desks</li><li>• Keep workspaces clear of unrelated materials</li><li>• Provide fewer problems to attain passing grades</li><li>• Tape a number line to the student’s desk</li><li>• Create a math journal that</li></ul>	<ul style="list-style-type: none"><li>• Use visual demonstrations, illustrations and models</li><li>• Give directions / instructions verbally and in simple written format</li><li>• Peer support</li><li>• Increased one – on – one time</li><li>• Teachers may modify instructions by modeling what the student is expected to do</li><li>• Instructions may be printed out in large print and hung up for the students to see during the time of the lesson</li></ul>	<ul style="list-style-type: none"><li>• Inquiry based instruction</li><li>• Independent study</li><li>• Higher order thinking skills</li><li>• Adjusting the pace of the lessons</li><li>• Real world scenarios</li><li>• Student driven instruction</li><li>• Allow students to complete an independent project as an alternative test</li></ul>

	<p>they can use during class, on assignments and (if teacher allows) on assessments</p> <ul style="list-style-type: none"> <li>• Provide extra time to complete a task when needed</li> <li>• Provide definitions of different graphs / charts with illustrations</li> <li>• Allow tests to be taken in a separate room</li> <li>• Allow students to use a calculator when appropriate</li> <li>• Divide test into small sections of similar questions or problems</li> <li>• Use of alge-tiles when needed</li> <li>• Use of number line when needed</li> </ul>	<ul style="list-style-type: none"> <li>• Review behavior expectations and make adjustments</li> <li>• Create a math journal that they can use during class, on assignments and (if teacher allows) on assessments</li> <li>• Allow students to complete an independent project as an alternative test</li> <li>• Use of alge-tiles when needed</li> <li>• Use of number line when needed</li> </ul>	
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#### **Interdisciplinary Connections: ELA**

**NJSLSA.R1.** Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

**NJSLSA.W2.** Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content

#### **21<sup>st</sup> Century Standards**

**9.2.12.C.1:** Review career goals and determine steps necessary for attainment.

**9.2.12.C.2:** Modify Personalized Student Learning Plans to support declared career goals.

**9.3.ST.2:** Use technology to acquire, manipulate, analyze and report data.

**9.3.ST-ET.5:** Apply the knowledge learned in STEM to solve problems.

#### **Career Ready Practices:**

**CRP2:** Apply appropriate academic and technical skills



**CRP4:** Communicate clearly and effectively and with reason

**CRP6:** Demonstrate creativity and innovation

**CRP8:** Utilize critical thinking to make sense of problems and persevere in solving them

**CRP11:** Use technology to enhance productivity

**Technology Standards:**

**8.1.12.A.3:** Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.

**Major** **Supporting** **Additional** (Identified by PARCC Model Content Frameworks)