Englewood Public School District
Precalculus – Graphical, Numerical, Algebraic
Second Marking Period

Unit 2: Trigonometric functions, graphs, expressions, equations, applications and Vectors

Overview: During this unit, students will review the concepts of trigonometric functions and will investigate their graphs. Students will work with trigonometric equations and use them to solve real-world application problems.

Time Frame: 43 to 47 Days

Enduring Understandings:
- *Combinations of circular functions model natural periodic behavior.*
- *A radian is an alternative unit to measure angles.*
- *Using right triangles, we can determine the approximate (or exact) value of an angle.*
- *Angles can be drawn on the coordinate plane and can be viewed as a rotating ray.*
- *Reference angles allow us to find solutions to trigonometric equations within all quadrants.*
- *Trigonometric graphs are distinctive in that they are continuous and repetitive.*
- *A function is a sinusoid if it can be written in the form f(x) = a sin (bx + c) + d, where a, b, c, and d are constants and neither a nor b is 0.*
- *The amplitude of the sinusoid function f(x) = a sin (bx + c) + d is* $|a|$.
- *The frequency of the sinusoid function f(x) = a sin (bx + c) + d is* $|b|2\pi$.
- *The tangent function can be defined as* $tanx = \frac{sinx}{cosx}$.
- *We can combine trigonometric functions with other functions following the same operations as real numbers.*
- *The interrelationships among the six basic trigonometric functions make it possible to write trigonometric expressions in various equivalent forms.*
- *To solve some trigonometric equations, you can use an inverse function to find one solution. Then you can use periodicity to find all solutions.*
- *Several trigonometric identities involve a single angle, whereas others involve two angles or half angles.*
- *The double angle identities are special cases of angle sum identities.*
- *Identities can be used to simplify trigonometric expressions into an equivalent form.*
- *Identities can be used to solve trigonometric equations.*
- *Identities can involve the sum or difference of angles.*
- *The law of Sines and the law of Cosines can help solve real-world problems involving triangles.*
- *A vector in the coordinate plane is a directed line segment.*
- *The dot product of u =* $\langle u_1, u_2 \rangle$ *and v =* $\langle v_1, v_2 \rangle$ *is* $u \cdot v = u_1 v_1 + \ u_2 v_2$
- *We can model the motion of objects with parametric curves and equations.*
- *Another coordinate system exists, and it's called the polar coordinate system.*
- *We can graph equations on the polar coordinate system, just like we can on the Cartesian coordinate system.*

- *We can plot complex numbers on the complex coordinate plane.*

Essential Questions:
- *How can you model periodic behavior?*
- *What is the relationship between degrees and radian measurement?*
- *How can you write a formula to represent a trigonometric function?*
- *How can you find the value of cos, tan, cosecant, secant and cotangent from the sine value?*
- *How do you verify that an equation involving the variable x is an identity?*
- *How is the inverse of a trigonometric function also a function?*
- *How do trigonometric functions relate to trigonometric ratios for a right angle?*
- *How do you prove a trigonometric equation is a trigonometric identity?*
- *What do the different variables of a trigonometric function mean?*
- *How can you solve trigonometric equations using identities?*
- *When do you use the law of Sines to solve a problem as opposed to the law of Cosines?*
- *Why is having different coordinate planes useful?*

Standards	Topics and Objectives	Activities	Resources	Assessments
MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8 **F-TF.A** Extend the domain of trigonometric functions using the unit circle **F-TF.A1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **G-CO.C.9** Prove theorems about lines and angles **G-CO.C.10** Prove theorems about triangles.	**Topics** Radians and degrees, circular arc length, trigonometric functions, right triangles, trigonometric graphs, inverse trigonometric functions, trigonometric identities and laws, vectors, dot product, polar coordinates, the complex plane. Twenty-First Century Themes and Skills include: • The Four C's • Global awareness • Financial, economic, business and entrepreneurial literacy	Powers of complex numbers https://www.illustrativemathematics.org/content-standards/tasks/1689 Computations with complex numbers https://www.illustrativemathematics.org/content-standards/tasks/617 What exactly is a radian? https://www.illustrativemathematics.org/content-standards/tasks/1874	Pearson Chapters 4, 5, 6 Illustrative Mathematics https://www.illustrativemathematics.org/ Alabama Learning Exchange http://alex.state.al.us/search.php?fa_submit=ALLPLANS Texas Instruments http://education.ti.com Mathematics Assessment Project http://map.mathshell.org/ jmap.org kutasoftware.com	Textbook Pages 399 – 402, 450 – 454, 514 – 517

G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **G-C.B.5** Find arc lengths and areas of sectors of circles. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **G-SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G-SRT.C.7** Explain and use the relationship between the	**Objectives** Students will • Convert between radians and degrees • Solve problems involving angular speed • Define the six trigonometric functions • Solve problems involving trigonometric functions • Utilize the 16-point unit circle • Generate the sine and cosine graphs • Transform the sine and cosine graphs • Explore how the tangent function is related to the sine and cosine functions • Graph the tangent function • Graph the reciprocal trigonometric functions • Graph sums, differences and other combinations of trigonometric functions • Relate the concepts of inverse functions to the trigonometric functions • Solve real world application problems using trigonometric functions	Trigonometric functions for arbitrary angles (radians) https://www.illustrativemathematics.org/content-standards/tasks/1692 Properties of trigonometric functions https://www.illustrativemathematics.org/content-standards/tasks/1704 Trigonometric identities and rigid motions https://www.illustrativemathematics.org/content-standards/tasks/1698 Orbiting satellite https://www.illustrativemathematics.org/content-standards/tasks/1639 Model airplane acrobatics https://www.illustrativemathematics.org/content-standards/tasks/1415 Two wheels and a belt https://www.illustrativemathematics.org/content-standards/tasks/621 Satellite https://www.illustrativemathematics.org/content-standards/tasks/416	khanacademy.org

sine and cosine of complementary angles. **G-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **G-SRT.D.9** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **F-TF.A2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.A3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. **F-TF.A4** Use the unit circle to explain symmetry (odd and	• Use fundamental identities to simplify and solve trigonometric equations • Confirm whether or not an equation is an identity • Apply the identities for the sine, cosine and tangent of a difference or sum • Apply the double angle, half angle and power-reducing identities • Use the Law of Sines to solve a variety of problems • Apply the Law of Cosine to solve problems • Determine the area of a triangle • Apply the arithmetic of vectors and use vectors to solve real world problems • Calculate dot products and projections of vectors • Define parametric equations • Graph curves parametrically • Solve real world application problems using parametric equations • Convert points and equations from polar	The lighthouse problem https://www.illustrativemathematics.org/content-standards/tasks/397 Trigonometric function values https://www.illustrativemathematics.org/content-standards/tasks/1902 Seven circles III https://www.illustrativemathematics.org/content-standards/tasks/710 Radians: Just another way https://alex.state.al.us/lesson_view.php?id=35695 Trigonometric Art https://alex.state.al.us/lesson_view.php?id=32800 Traveling around the unit circle https://alex.state.al.us/lesson_view.php?id=27478 Representing Trigonometric Functions http://map.mathshell.org/lessons.php?unit=9255&collection=8 Basic trig functions

even) and periodicity of trigonometric functions **F-TF.C8** Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship **F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-BF.B.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive	coordinates to rectangular coordinates • Graph polar equations • Determine the maximum |r|-value and symmetry of a graph • Represent complex numbers on the complex plane and write them in polar form • Simplify algebraic operations with complex numbers • Use De Moivre's Theorem	https://education.ti.com/en/timathnspired/us/detail?id=4E9BA7808CA74F6599BD5EA2037C088A&t=C52AEC55A39243D182772F76318B901C Identifying sinusoidal graphs https://education.ti.com/en/timathnspired/us/detail?id=CB75E32D07AE44E286E3F78F3794667B&t=C52AEC55A39243D182772F76318B901C The unit circle https://education.ti.com/en/timathnspired/us/detail?id=873ACBF93E584712A9D24804C9E92F83&t=C52AEC55A39243D182772F76318B901C Trigonometric transformations https://education.ti.com/en/timathnspired/us/detail?id=1B5FA0B4D82144249C77DA5F931F25CE&t=C52AEC55A39243D182772F76318B901C Law of cosines https://education.ti.com/en/timathnspired/us/detail?id=1758B2B300EE408CB0B60A702B507407&t=660	

and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **F-IF.C.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude **F-TF.B5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **F-TF.B6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F-TF.B7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.		02E52F2DB47D3BD9923F6E257BD76 Law of sines https://education.ti.com/en/timathnspired/us/detail?id=E8F17149BCDC472DB81151C215C09C49&t=66002E52F2DB47D3BD9923F6E257BD76 Proof of identities https://education.ti.com/en/timathnspired/us/detail?id=A2E2C2BED0194DB183E7D190EA15A46C&t=66002E52F2DB47D3BD9923F6E257BD76 Complex number addition https://education.ti.com/en/timathnspired/us/detail?id=07EF321269B64BB398EABD1C0E0D9061&t=3270A55B10BB4BF29C7D8883E3990203 Complex number multiplication https://education.ti.com/en/timathnspired/us/detail?id=95F04720818B470A9B87DB4E44A23E44&t=3270A55B10BB4BF29C7D8883E3990203 Parametric projectile motion

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling. **N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **F-TF.C.9** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **G-SRT.D.10** Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.D.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		https://education.ti.com/en/timathnspired/us/detail?id=2EE9CB185F8346B395081717DD86A661&t=3270A55B10BB4BF29C7D8883E3990203 Polar coordinates https://education.ti.com/en/timathnspired/us/detail?id=C397CBF604FE4E29838DFE8174DD7BED&t=3270A55B10BB4BF29C7D8883E3990203 Rose curve https://education.ti.com/en/timathnspired/us/detail?id=0F0EF35E181241869ECA8BE9B7F19D43&t=3270A55B10BB4BF29C7D8883E3990203 Sinusoidal modeling https://education.ti.com/en/timathnspired/us/detail?id=469A5FBBB74F4031882FEC9806D32867&t=3270A55B10BB4BF29C7D8883E3990203

G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **N-VM.A.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). **N-VM.A.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.A.3** Solve problems involving velocity and other quantities that can be represented by vectors **N-VM.B.4** Add and subtract vectors. **N-VM.B.4a**. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	

N-VM.B.4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

N-VM.B.4c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.B.5. Multiply a vector by a scalar.

N-VM.B.5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).

N-VM.B.5b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠0, the direction of cv is either along

v (for c > 0) or against v (for c < 0). **N-CN.A.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.A.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N-CN.B.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN.B.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (-1 + √3i)3 = 8 because (-1+ √3i) has modulus 2 and argument 120°.	

N-CN.B.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoint **N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays **G-CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle	

Modifications:

- New Jersey Department of Education – Instructional Supports and Scaffolds
- Suggested Strategies for English Language Learners
- Secondary activities were created to allow for greater personalized learning to meet the needs of all learners including students with gifts and talents

Key Vocabulary: central angle, degree, radian, arc length, standard position, sine, cosine, tangent, cosecant, secant, cotangent, initial side, terminal side, coterminal angles, reference angle, quadrantal angles, period, frequency, amplitude, circular functions, unit circle, sinusoid, phase shift, midline, damping, arcsine, arccosine, arctangent, angle of elevation, angle of depression, simple harmonic motion, identities, sum and difference formulas, double angle formulas, half angle formulas, law of sine, law of cosine, area formula of a triangle, directed line segment, vector, magnitude, zero vector, initial point, terminal point, dot product, scalars, unit vector, velocity, speed, orthogonal, parametric curve, polar coordinate system, pole, polar coordinates, rose curves, limacon curves, lemniscate curves, complex plane, imaginary axis, modulus.

Related Concepts and skills (Identified by text book: Precalculus – Graphical, Numerical, Algebraic)