Englewood Public School District
College and Career Readiness Math
Second Marking Period

Unit 2: A review of Algebra 2

Overview: This unit goes through a review of Algebra 2. Topics include complex numbers, functions, quadratic, exponential and logarithmic functions, factoring, conic sections, and probability and statistics.

Time Frame: 43 to 47 Days

Enduring Understandings:

- *Imaginary numbers exist and* $\sqrt{-1} = i$.
- *A complex number can be written in the form a+bi, where 'a' is a real number and 'bi' is an imaginary number.*
- *Complex numbers can be added, subtracted, multiplied and divided.*
- *In order to divide complex numbers, we must use conjugates.*
- *Mathematical problems can be illustrated using different models, including numerical, algebraic and graphical models.*
- *A function is a rule where every element of one set (the domain) corresponds to one and only one element of another set (the range).*
- *You can combine functions using the same operations as real numbers.*
- *Different functions can be used to illustrate real world scenarios.*
- *We use the properties of exponents to simplify expressions and solve equations.*
- *Logarithmic and exponential equations are inverses of each other.*
- *We can simplify quadratic expressions by factoring.*
- *When a quadratic equation equals zero, we can solve for the roots of the equation by a variety of ways.*
- *The roots of a quadratic equation occur where the equation crosses the x-axis.*
- *We can graph quadratic inequalities on the coordinate plane.*
- *There are different types of conic sections each defined by its own set of properties.*
- *Both the ellipse and hyperbola's shape are determined by its distance from its foci.*
- *The intersection of a cone and a plane parallel to the side of the cone is a parabola.*
- *The intersection of a cone and a plane parallel to the base of the cone is a circle.*
- *The intersection of cone and a plane not parallel to the base of the cone is an ellipse.*
- *The intersection of a cone and a plane parallel to the axis (or not) is a hyperbola.*
- *An ellipse and a hyperbola each have two foci.*
- *We can graph conic sections on the polar coordinate plane.*
- *Experimental probability is based on the results of an experiment. Theoretical probability is the mathematical chance it will happen.*
- *The probability of an event is the value that its relative frequency of occurrence approaches in the long run.*
- *Probability calculates the likelihood that something will happen and assists us in making predictions and informed decisions.*

- *We can use decimals, fractions or percentages to describe the chance of an event happening.*
- *We can use diagrams, models or algebraic equations and expressions to illustrate different probabilities and outcomes.*
- *Statistics describes properties of data, such as counts, percentages or averages.*
- *We can use charts, graphs, plots, histograms and frequency tables to illustrate different statistics.*
- *Sometimes, one representation will be better than another to describe certain statistics.*
- *To effectively compare sets of data, we need to think about how the data is distributed by looking at the shape, the center and the spread of distribution.*
- *A box plot (box-and-whisker plot) includes five pieces of data: the three quartiles, the maximum and the minimum.*

Essential Questions:

- *What is a complex number?*
- *How can we perform operations with complex numbers?*
- *How do we find the conjugate of a complex number and when do we use it?*
- *What makes a relation a function?*
- *What is the domain and range of a function?*
- *How do we graph linear, quadratic, logarithmic and exponential functions?*
- *Why is illustrating a mathematical problem in different ways helpful?*
- *How do we solve exponential functions?*
- *How can you model data with a function?*
- *How are exponential functions and logarithmic functions related?*
- *How do we factor a quadratic expression?*
- *Why is knowing how to factor an important skill?*
- *How do we solve quadratic equations?*
- *What do the different forms of a quadratic equation tell us about the graph of that equation?*
- *How do we find the roots, or zeros, of a quadratic equation?*
- *How can we graph a quadratic inequality?*
- *How are conic sections formed?*
- *How do we graph different conic sections?*
- *What is the focus and the directrix of a parabola?*
- *What do the algebraic representation of conic sections look like?*
- *What is the difference between the algebraic representation of ellipses and hyperbolas?*
- *What effects the probability that a specific event will occur?*
- *How do we know whether an event is independent or dependent?*
- *How can we use modeling to form a prediction of an event occurring?*
- *How do we illustrate the probability of certain events?*
- *How can we illustrate different sets of statistics?*

- *What is the best illustration to use to describe different statistics?*
- *When comparing sets of data, what do we need to look at?*
- *What are the five points illustrated in a box-and-whisker plot?*

Standards	Topics and Objectives	Activities	Resources	Assessments
MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8 **N-CN.A.1** Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real **N-CN.A.2** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers **N-CN.A.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **F-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.C.7a** Graph linear and quadratic functions and show	**Topics** Complex numbers, conjugates, functions and relations, domain and range, quadratic, logarithmic and exponential functions, factoring, solving quadratics, completing the square, quadratic formula, quadratic inequalities, conics, probability and statistics Twenty-First Century Themes and Skills include: • The Four C's • Global awareness • Financial, economic, business and entrepreneurial literacy **Objectives** Students will • Expand their understanding of numbers to include imaginary numbers	Complex numbers battle ship: https://betterlesson.com/lesson/491418/complex-number-battleship Complex numbers hexagonal puzzle: https://www.stem.org.uk/resources/elibrary/resource/25623/complex-numbers-hexagonal-jigsaw-puzzle Complex numbers bingo: http://archive.jc-schools.net/dynamic/math/worksheets_secondary/CmplxNmbrsBingo_Activity_Alg2.pdf Complex numbers with calculator: https://education.ti.com/en/activity/detail?id=5F809E0C58F74C558667B9615C0D5565	https://www.illustrativemathematics.org/ https://betterlesson.com/ National Library of Virtual Manipulatives http://nlvm.usu.edu/ Alabama Learning Exchange http://alex.state.al.us/search.php?fa_submit=ALLPLANS Mathematics Assessment Project http://map.mathshell.org/ http://www.jmap.org/ https://www.khanacademy.org/ kutasoftware.com https://www.emathinstruction.com/ https://www.teacherspayteachers.com/	SAT and Accuplacer tests should be administered when appropriate

intercepts, maxima and minima **F-IF.A. 1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*) **F–IF.A.2** Use function notation evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context **F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes **F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **N-Q.A.1** Use units as a way to understand problems and to	• Add, subtract, multiply and divide complex numbers • Find the conjugate of a complex number • Identify the domain and range of a function • Use function notation • Compare and contrast functions given a table or graph • Be able to graph functions, including linear, quadratic, logarithmic and exponential functions • Create equations given a scenario and then graph the equations • Be able to identify key aspects of a graph and describe their meaning in the context of the problem • Use and apply the properties of exponents • Identify an exponential function given a table or graph • Write and solve exponential equations given a scenario • Factor a quadratic expression	Videos, worksheets, activities for functions and relations: https://www.onlinemathlearning.com/relations-functions.html Illustrates how to use a math notebook: https://mathequalslove.blogspot.com/2013/12/algebra-1-introduction-to-relations-and.html Last man standing – introduction to exponential functions: https://www.teacherspayteachers.com/Product/Last-Man-Standing-A-Hands-On-Activity-to-Introduce-Exponential-Functions-1195202 Zombies and exponential equations: http://theenlightenedelephant.blogspot.com/2015/03/zombies-and-exponential-functions.html Cut and glue factoring: https://www.teacherspayteachers.com/Product/Cut-and-Glue-Factoring-Polynomials-Activity-FREEBIE-545067	https://collegereadiness.collegeboard.org/sat/practice https://blog.prepscholar.com/complete-list-of-free-sat-math-practice https://accuplacer.collegeboard.org/student/practice

guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays **A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales **F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship **F-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases **F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)	• Solve quadratic equations by using square roots, the quadratic formula by completing the square • Identify the zeros of a quadratic equation • Solve a linear – quadratic system • Graph quadratic inequalities • Find the equation, focus and directrix of a parabola • Graph an ellipse • Find the equation, vertices and foci of an ellipse • Graph a hyperbola • Find the equation, vertices and foci of a hyperbola • Determine equations for translated and rotated axes for conic sections • Understand the general focus-directrix definition of a conic section • Identify a sample space • Calculate probabilities and conditional probabilities with	Quadratic quandary: http://www.cpalms.org/Public/PreviewResourceLesson/Preview/51208 Solving quadratics with the quadratic formula: https://betterlesson.com/lesson/588146/solving-quadratic-functions-using-the-quadratic-formula Investigating quadratic inequalities: http://www.projectmaths.ie/documents/LCActivities/SolvingQuadraticInequalities.pdf Paper folding and conics: https://nagt.org/nagt/profdev/twp/whatcom/activities/168596.html Probability and statistics using game shows: https://www.nctm.org/Publications/mathematics-teacher/2005/Vol98/Issue8/Activities-for-Students_-Teaching-Probability-and-Statistics-Using-Game-Shows/

F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function **A-SSE.B.3c** Use the properties of exponents to transform expressions for exponential functions. **F-IF.C.8b** Use the properties of exponents to interpret expressions for exponential functions **F-LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.	equally likely or unlikely outcomes • Distinguish between categorical and quantitative variables • Use various kinds of graphs to display data • Write outcomes of different events • Read tables and graphs illustrating different events and be able to answer questions based on those representations • Find and use measures of central tendency • Find and use the five-number summary of data • Use a box plot to describe data • Create a box plot given data • Create a histogram given data • Create a probability model for a random variable • Determine statistical significance • Understand important aspects of study design • Understand the role of randomness in	

A-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines **A-SSE.B.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines **A-APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **A-REI.B.4** Solve quadratic equations in one variable. **A-REI.B.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **F-IF.C.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context **G-GPE.A.1** Derive the equation of a circle of given	sampling, experimentation and simulation	

center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix.

G-GPE.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

S-CP. A. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

S-CP. A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP. A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-MD.B.5a Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.

S-CP. A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

S-CP. B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP. B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

S-CP. B.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

S-CP. B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

S-ID. A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID. B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-CP. A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

S-ID. A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **S-ID. A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **S-IC.A.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S-MD.A.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **S-MD.B.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values	

S-MD.B.5b Evaluate and compare strategies on the basis of expected values. **S-MD.B.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S-MD.B.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) **S-IC.A.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **S-IC.B.3**. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S-IC.B.5.** Use data from a randomized experiment to	

compare two treatments; use simulations to decide if differences between parameters are significant. **S-IC.B.6.** Evaluate reports based on data. **S-ID. C.8.** Compute (using technology) and interpret the correlation coefficient of a linear fit. **S-ID. C.9.** Distinguish between correlation and causation.	

Modifications:

- New Jersey Department of Education – Instructional Supports and Scaffolds
- Suggested Strategies for English Language Learners
- Secondary activities were created to allow for greater personalized learning to meet the needs of all learners including students with gifts and talents

Key Vocabulary: imaginary numbers, complex numbers, conjugates, function, relation, domain, range, exponential growth, exponential decay, quadratic, maximum, minimum, zeros, factoring, GCF, difference of two perfect squares, completing the square, quadratic formula, inequality, focus, directrix, circle, hyperbola, ellipse, parabola, sample space, event, outcome, probability function, probability distribution, Venn diagram, tree diagram, conditional probability, statistics, distribution, pie chart, bar graph, circle graph, stem-and-leaf plots, association, independent variables, frequency tables, histograms, frequency distribution, unimodal, bimodal, symmetric, mode, line graph, time plot, median, first quartile, second quartile, third quartile, range, interquartile range, box-and-whisker plot, mean