Englewood Public School District
Precalculus – Graphical, Numerical, Algebraic
Third Marking Period

Unit 3: Matrices, Conics, Sequences and Series

Overview: During this unit, students will review the concepts of matrices and expand their knowledge of matrix algebra, conic sections, the three-dimensional coordinate plane, sequences and series.

Time Frame: 43 to 47 Days

Enduring Understandings:

- *We can solve a system of equations using several different methods.*
- *Data can be organized in a matrix in exactly the same way that data can be organized in a rectangular table.*
- *We can use matrices to solve systems of equations.*
- *The identity matrix is comprised of 1's on the main diagonal and 0's everywhere else.*
- *The determinant of a 2x2 matrix can be defined by ad – bc.*
- *We can use the Gaussian Elimination method to solve three equations with three unknowns.*
- *We can graph linear inequalities on the coordinate plane.*
- *We can solve systems of linear inequalities using the coordinate plane.*
- *Matrix operations can transform points in a plane.*
- *There are different types of conic sections each defined by its own set of properties.*
- *Both the ellipse and hyperbola's shapes are determined by its distance from its foci.*
- *The intersection of a cone and a plane parallel to the side of the cone is a parabola.*
- *The intersection of a cone and a plane parallel to the base of the cone is a circle.*
- *The intersection of cone and a plane not parallel to the base of the cone is an ellipse.*
- *The intersection of a cone and a plane parallel to the axis (or not) is a hyperbola.*
- *An ellipse and a hyperbola each have two foci.*
- *The discriminant can determine the nature of the roots of a quadratic function.*
- *We can graph conic sections on the polar coordinate plane.*
- *We can graph points, lines and conics on a 3-dimensional Cartesian coordinate plane.*
- *A sequence can be defined by describing its n^{th} term of by stating its first term and a formula that related the n – 1 and n^{th}terms.*
- *A geometric sequence can be modeled explicitly or recursively.*
- *A combination is a collection, whereas a permutation is an ordered collection.*
- *Using Pascal's triangle, we can expand polynomials.*
- *A sequence is an ordered progression of numbers in a set. A sequence can be finite or infinite.*
- *An arithmetic sequence has a common difference between the terms.*

- *A geometric sequence has a common ratio between the terms.*
- *We can add all the terms in a sequence by using summation notation.*

Essential Questions:

- *How can we solve a system of equations?*
- *How can you use a matrix to organize data?*
- *How can you use a matrix equation to model a real-world situation?*
- *How can a matrix represent a transformation of a geometric figure?*
- *How can we add, subtract and multiply matrices?*
- *What is the identity matrix and what is it used for?*
- *How can we use elimination to solve for three variables when we're given three equations?*
- *How do we solve systems of inequalities?*
- *What is the 3-dimensional plane?*
- *How do we find the distance between points on the 3-dimensional plane?*
- *How do we graph conic sections?*
- *How are conic sections formed?*
- *What is the intersection of a cone and a plane parallel to a line along the side of a cone?*
- *What is the focus and the directrix of a parabola?*
- *What do the algebraic representation of conic sections look like?*
- *What is the difference between the algebraic representation of ellipses and hyperbolas?*
- *What is the difference between a geometric sequence and an arithmetic sequence?*
- *How can you represent the terms of a sequence explicitly and recursively?*
- *What are equivalent explicit and recursive definitions for an arithmetic sequence?*
- *How can you model a geometric sequence and sum?*
- *What is Pascal's triangle and when should we use it?*
- *What is summation notation?*
- *What is the difference between a permutation and a combination?*

Standards	Topics and Objectives	Activities	Resources	Assessments
MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8 **A-REI.C.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables **A-REI.C.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically **A-REI-D.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **N-VM.C.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or	**Topics** Solving systems of equations, adding, subtracting, and multiplying matrices, determinants and inverses, inverse matrices and systems, triangular form of a linear system, Gaussian Elimination, row operations and row Echelon form, solving systems using matrices, systems of inequalities, conic sections, circles, ellipses, hyperbolas, parabolas, focus, directrix, discriminant, 3-D coordinate plane, sequences, series, geometric sequence, arithmetic sequence, the counting principal, permutations, combinations, Pascal's triangle, summation notation. Twenty-First Century Themes and Skills include: • The Four C's • Global awareness • Financial, economic, business and entrepreneurial literacy	Cryptology and Matrices https://education.ti.com/en/timathnspired/us/detail?id=101D1839004849A4964D99CCE07E1E5F&t=237818E620F5429EA798FFB795F837B3 Parabolas and Matrices https://education.ti.com/en/timathnspired/us/detail?id=4BF86A2C03184CA09E501C9131FCA52F&t=237818E620F5429EA798FFB795F837B3 3x3 Linear systems of equations https://education.ti.com/en/timathnspired/us/detail?id=BA3E75EC24CE44348E0AFC9F4D6B1EF5&t=237818E620F5429EA798FFB795F837B3 Representing inequalities graphically http://map.mathshell.org/lessons.php?unit=9265&collection=8 Introduction to conic sections https://education.ti.com/en/timathnspired/us/detail?id=2E8793B33E1048C1BA	Pearson Chapters 7, 8, 9 Illustrative Mathematics https://www.illustrativemathematics.org/ Alabama Learning Exchange http://alex.state.al.us/search.php?fa_submit=ALLPLANS Texas Instruments http://education.ti.com Mathematics Assessment Project http://map.mathshell.org/ E-math instruction workbook https://emathinstruction.com/	Text page: 565 – 569, 630 – 632, 673 – 675

incidence relationships in a network **N-VM.C.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled **N-VM.C.8** Add, subtract, and multiply matrices of appropriate dimensions **N-VM.C.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties **N.VM.C.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse **N-VM.C.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work	**Objectives** Students will • Solve systems of equations algebraically • Solve systems of equations graphically • Find the sums of matrices • Find the differences of matrices • Find the product of matrices • Find the inverses of matrices • Solve systems of linear equations using Gaussian elimination • Solve systems of linear equations using row reduction echelon form of a matrix • Solve systems of linear equations using an inverse matrix • Solve linear programming problems using graphical methods • Graph systems of linear inequalities • Solve systems of linear inequalities graphically • Graph a parabola • Find the equation, focus and directrix of a parabola • Graph an ellipse	57F2C79E6F2991&t=629487495F2C47CA93E4DD7B103F29E2 Conic sections – discovering the degenerates https://alex.state.al.us/lesson_view.php?id=32225 Conic sections – playing with parabolas https://alex.state.al.us/lesson_view.php?id=32175 Conic sections – playing with hyperbolas https://alex.state.al.us/lesson_view.php?id=32207 Kepler’s Third Law of Motion https://www.illustrativemathematics.org/content-standards/tasks/1842 Tilt of earth’s axis and the four seasons https://www.illustrativemathematics.org/content-standards/tasks/1140 Explaining the equation for a circle https://www.illustrativemathematics.org/content-standards/tasks/1425

with matrices as transformations of vectors **N-VM.C.12** Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area **A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A-REI.C.8** Represent a system of linear equations as a single matrix equation in a vector variable. **A – REI. C.9** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). **A-CED.A.2** Create equations and inequalities in one variable and use them to solve problems. **A-CED.A.3** Represent constraints by equations or	• Find the equation, vertices and foci of an ellipse • Graph a hyperbola • Find the equation, vertices and foci of a hyperbola • Determine equations for translated and rotated axes for conic sections • Understand the general focus-directrix definition of a conic section • Write equations of conic sections in polar form • Draw 3-dimensional figures • Analyze vectors in 3-dimensional space • Use the multiplication principle of counting, permutations or combinations to count the number of ways that a task can be done • Expand the power of a binomial using the Binomial Theorem or Pascal's triangle • Find the coefficient of a given term of a binomial expansion • Express arithmetic and geometric sequences explicitly and recursively	Elliptic variations https://education.ti.com/en/timathnspired/us/detail?id=598D74949FDC4E55A74C8F99C5F490F6&t=629487495F2C47CA93E4DD7B103F29E2 Foci definition of ellipses and hyperbolas https://education.ti.com/en/timathnspired/us/detail?id=EA1D525B2BA54BC38ACF79E42FC75835&t=629487495F2C47CA93E4DD7B103F29E2 Focus / Directrix definition of conics https://education.ti.com/en/timathnspired/us/detail?id=2C39DF5CAF34454B9B090D2B2CB6AC74&t=629487495F2C47CA93E4DD7B103F29E2 Polar conics https://education.ti.com/en/timathnspired/us/detail?id=2310D54D170F4D0CB86526F9D82065E2&t=629487495F2C47CA93E4DD7B103F29E2 Sum of infinite geometric series	

inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. **A-REI-D.12** Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **C-GO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). **G.SRT.A.1** Verify experimentally the properties of dilations given by a center and a scale factor: **G.SRT.A.1a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	• Find the limits of convergent sequences • Use sigma notation and find finite sums of terms in arithmetic and geometric sequences • Find the sums of convergent geometric series • Use the principle of mathematical induction to prove mathematical generalizations	https://education.ti.com/en/timathnspired/us/detail?id=51568CB8EDD146A38E723142C6E8280E&t=237818E620F5429EA798FFB795F837B3 Building sequences and series with spreadsheet https://education.ti.com/en/timathnspired/us/detail?id=B255A2616C884E76A1D246A8F5E011A3&t=237818E620F5429EA798FFB795F837B3 Unit 13 – sequences and series https://emathinstruction.com/algebra-2-trigonometry/unit-13-sequences-and-series/ Binomial expansion – shortcut please https://alex.state.al.us/lesson_view.php?id=35699	

G.SRT.A.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor

A-REI-D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix

G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects

G-GPE.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-GMD.A.2 Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling **N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **A-APR.C.5** Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle **F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers **F-BF.A.2** Combine standard function types using arithmetic operations. **A-SSE.B.4** Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems	

Modifications:

- New Jersey Department of Education – Instructional Supports and Scaffolds
- Suggested Strategies for English Language Learners
- Secondary activities were created to allow for greater personalized learning to meet the needs of all learners including students with gifts and talents

Key Vocabulary: matrix, row, column, scalar multiple, zero matrix, additive identity, additive inverse, multiplicative identity, identity matrix, determinant, Gaussian elimination, augmented matrix, row Echelon form, vertex, focus, directrix, standard form, focal length, chord, focal width, ellipse, circle, major axis, minor axis, hyperbola, transverse axis, conjugate axis, semitransverse axis, semiconjugate axis, eccentricity, angle of rotation, discriminant, combination, permutation, Pascal's triangle, sequence, finite sequence, infinite sequence, recursively, converge, diverge, arithmetic sequence, geometric sequence, Fibonacci sequence, summation notation, infinite series, partial sums, induction, deduction

Related Concept and Skills (Identified by Text)