

# Englewood Public School District

## Algebra II

### Fourth Marking Period

#### Unit 4: Matrices and Trigonometric Functions

**Overview:** During this unit, students will learn about matrices, graphing trigonometric functions, and exploring trigonometric identities and equations.

**Time Frame:** 43 to 47 Days

#### Enduring Understandings:

- *Data can be organized in a matrix in exactly the same way that data can be organized in a rectangular table.*
- *If you can model a real world situation with a system of equations, you can represent the system with a matrix equation.*
- *Matrix operations can transform points in a plane.*
- *Combinations of circular functions model natural periodic behavior.*
- *The interrelationships among the six basic trigonometric functions make it possible to write trigonometric expressions in various equivalent forms.*
- *To solve some trigonometric equations you can use an inverse function to find one solution. Then you can use periodicity to find all solutions.*
- *If you know two angles and a side of a triangle, you can use trigonometry to solve the triangle. If you know two sides and the included angle you can find the area of the triangle.*
- *Several trigonometric identities involve a single angle. Other identities involve two angles.*
- *The double angle identities are special cases of angle sum identities.*

#### Essential Questions:

- *How can you use a matrix to organize data?*
- *How can you use a matrix equation to model a real world situation?*
- *How can a matrix represent a transformation of a geometric figure?*
- *How can you model periodic behavior?*
- *How can you write a formula to represent a trigonometric function?*
- *How can you find the value of  $\cos$ ,  $\tan$ , cosecant, secant, and cotangent from the sine value?*
- *How do you verify that an equation involving the variable  $x$  is an identity?*
- *How is the inverse of a trigonometric function also a function?*
- *How do trigonometric functions relate to trigonometric ratios for a right angle?*

<p>N-VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes</p> <p>N-VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point</p> <p>N-VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p>N-VM.B.4a, b, c Add and subtract vectors</p> <p>N-VM.B.5.a, b Multiply a vector by a scalar</p> <p>N-VM.C.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network</p> <p>N-VM.C.7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled</p> <p>N-VM.C.8 Add, subtract, and multiply matrices of appropriate dimensions</p> <p>N-VM.C.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties</p> <p>N-VM.C.10 Understand that</p>	<p><b>Topics</b></p> <p>Adding, subtracting and multiplying matrices, determinants and inverses, inverse matrices and systems, vectors, angles and the unit circle, radian measure, sine, cosine, and tangent functions, reciprocal trigonometric functions, trigonometric identities, Law of Sines, Law of Cosines, angle identities, double angle and half angle identities</p> <p>Twenty-First Century Themes and Skills include:</p> <ul style="list-style-type: none"> <li>• The Four C's</li> <li>• <a href="#">Global awareness</a></li> <li>• <a href="#">Financial, economic, business and entrepreneurial literacy</a></li> </ul> <p><b>Objectives</b></p> <p>Students will</p> <ul style="list-style-type: none"> <li>• Add and subtract matrices to solve matrix equations</li> <li>• Multiply matrices using scalar and matrix multiplication</li> <li>• Find the inverse of a matrix</li> <li>• Solve systems of matrix inverses and multiplication</li> <li>• Transform geometric figures using matrix operations</li> <li>• Use basic vector operations and the dot product</li> <li>• Identify cycles, periods, and</li> </ul>	<p>Standards Solution Common Core Function Lessons:</p> <ul style="list-style-type: none"> <li>• Proving and Applying Trigonometric Identities</li> <li>• Trigonometric Functions to Model Periodic Phenomena</li> <li>• Trigonometric Functions and the Unit Circle</li> </ul> <p>Exploring Sinusoidal Functions <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/tasks/1647">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/tasks/1647</a></p> <p>What exactly is a radian? <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1874">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1874</a></p> <p>Bicycle Wheel <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1873">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1873</a></p> <p>Trigonometric Identities and Rigid Motions <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1698">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1698</a></p> <p>Properties of Trigonometric Functions <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1704">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1704</a></p> <p>Trig Functions and the Unit Circle <a href="https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1704">https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1704</a></p>	<p>Pearson Realize Chapters 12, 13, and 14</p> <p>Standards Solution Common Core Lessons</p> <p>Illustrative Mathematics <a href="https://www.illustrativemathematics.org/">https://www.illustrativemathematics.org/</a></p> <p>Alabama Learning Exchange <a href="http://alex.state.al.us/search.php?fa_submit=ALLPLANS">http://alex.state.al.us/search.php?fa_submit=ALLPLANS</a></p> <p>Arizona Math Flipbook <a href="http://www.azed.gov/azcommoncore/files/2012/11/high-school-ccss-flip-book-usd-259-2012.pdf">http://www.azed.gov/azcommoncore/files/2012/11/high-school-ccss-flip-book-usd-259-2012.pdf</a></p> <p>NYC Department of Education <a href="http://schools.nyc.gov/default.htm">http://schools.nyc.gov/default.htm</a></p> <p>Mathematics Assessment Project <a href="http://map.mathshell.org/">http://map.mathshell.org/</a></p> <p>Texas Instruments <a href="https://education.ti.com/en/us/home">https://education.ti.com/en/us/home</a></p> <p>Desmos <a href="https://teacher.desmos.com/">https://teacher.desmos.com/</a></p> <p>Worksheets for every topic: <a href="http://kutasoftware.com/free.html">http://kutasoftware.com/free.html</a> (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)</p> <p>Algebra assessments, interactive, videos, games,</p>	<p><b>Formative Assessments:</b> Textbook Pages 791, 817, 821, 859, 897, 898, 927, 963, 964</p> <p>Math journal (NJSLSA.R1, NJSLSA.W2)</p> <p><b>Summative Assessments:</b> Multiple choice / short answer assessments (CRP2, CRP4, CRP8)</p> <p>Chapter quizzes/tests</p> <ul style="list-style-type: none"> <li>• Pearson Realize</li> <li>• MathXL</li> </ul> <p>Grade 11 Algebra II Common Core Assessment 4, Standards Solution</p> <p><b>Benchmark Assessment:</b> End of Year Assessment</p> <p><b>Alternative Assessments:</b> Learning centers: each learning center focuses on a different type of problem (9.3.ST.2, 9.3.ST-ET.5)</p> <p>Create posters illustrating the main objectives of the unit (CRP6)</p>
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the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse

**N-VM.C.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors

**N-VM.C.12** Work with  $2 \times 2$  matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area

**F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship

**F-IF.C.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude

**F-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle

**F-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as

- amplitudes of periodic functions
- Work with angles in standard position
- Find coordinates of points on the unit circle
- Use radian measure for angles
- Find the length of an arc of a circle
- Identify properties of the sine function and graph sine curves
- Graph and write cosine functions
- Solve trigonometric equations
- Graph the tangent function
- Graph and write equations of translations of trigonometric functions
- Evaluate reciprocal trigonometric functions
- Graph reciprocal trigonometric functions
- Verify trigonometric functions
- Evaluate trigonometric functions
- Solve trigonometric equations
- Find lengths of sides in a right triangle
- Find measures of angles in a right triangle
- Find the area of any triangle
- Use the law of sines
- Use the law of cosines in finding the measures of sides and angles of a triangle

[atics.org/content-standards/HSF/TF/A/2/tasks/1820](https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1820)

Trigonometric Functions for Arbitrary Angles  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1692>

Equilateral Triangles and Trigonometric Functions  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/A/3/tasks/1847>

Special Triangles 1  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/A/3/tasks/1819>

Special Triangles 2  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/A/3/tasks/1898>

Properties of Trigonometric Functions  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/A/4/tasks/1704>

As the Wheel Turns  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/595>

lessons, homework:  
[https://www.opened.com/search?area=mathematics&grade=9&offset=0&resource\\_type=interactive-assessment](https://www.opened.com/search?area=mathematics&grade=9&offset=0&resource_type=interactive-assessment)  
 (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5, 8.1.12.A.3)

Algebra common core worksheets:  
<https://www.ixl.com/math/algebra-1>  
 (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)

Khan Academy – videos, lessons, assessments  
[www.khanacademy.org](http://www.khanacademy.org)  
 (CRP2, CRP4, CRP8, CRP11, 9.3.ST.2, 9.3.ST-ET.5, 8.1.12.A.3)

Worksheets / assessment items for all topics based on standards:  
[http://jmap.org/JMAP\\_RESOURCES\\_BY\\_TOPIC.htm#AI](http://jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI)  
 (CRP2, CRP4, CRP8, 9.3.ST.2, 9.3.ST-ET.5)

radian measures of angles traversed counterclockwise around the unit circle

**F-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline

F-TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed

F-TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-SRT.D.9 Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side

G-SRT.D.10 Prove the Laws of Sines and Cosines and use them to solve problems

G-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare

- Verify and use angle identities
- Verify and use sum and difference identities
- Verify and use double angle and half angle identities

Foxes and Rabbits 2  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/816>

Foxes and Rabbits 3  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/817>

Hours of Daylight 1  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/1832>

Coordinates of Points on a Circle  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/tasks/1894>

Finding Trig Values  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835>

Trigonometric Ratios and The Pythagorean Theorem  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1693>

Calculations with Sine and Cosine  
<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1868>

transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch)  
G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure

Finding Trig Values

<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835>

Sum and Difference Angle Formulas

<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/9/tasks/1116>

Coordinates of Equilateral Triangles

<https://www.illustrativemathematics.org/content-standards/HSF/TF/C/9/tasks/1687>

Matrices, We Don't Die, We Multiply!

[http://alex.state.al.us/lesson\\_view.php?id=33241](http://alex.state.al.us/lesson_view.php?id=33241)

Does Size Matter?

[http://alex.state.al.us/lesson\\_view.php?id=33217](http://alex.state.al.us/lesson_view.php?id=33217)

Trigonometric Art

[http://alex.state.al.us/lesson\\_view.php?id=32800](http://alex.state.al.us/lesson_view.php?id=32800)

Traveling Around the Unit Circle

[http://alex.state.al.us/lesson\\_view.php?id=27478](http://alex.state.al.us/lesson_view.php?id=27478)

Representing Trigonometric Functions

<http://map.mathshell.org/lessons.php?unit=9255&collection=8>

Operating on Matrices

<https://education.ti.com/en/us/activity/detail?id=E6D135D7780449FAA56B2D5728B51C79&ref=/en/us/activity/search/advanced>

Application of Matrix  
Multiplication

<https://education.ti.com/en/us/activity/detail?id=02556DBF5D5F42E8873DBD4CF0C3E6B4&ref=/en/us/activity/search/advanced>

The Determinant of a Matrix

<https://education.ti.com/en/us/activity/detail?id=2993B76D66AC4954970DF9238461D0AE&ref=/en/us/activity/search/advanced>

Coded Messages

<https://education.ti.com/en/us/activity/detail?id=CA722DBD12A040229EB5071701E5CFFC&ref=/en/us/activity/search/advanced>

The Unit Circle

<https://education.ti.com/en/us/activity/detail?id=873ACBF93E584712A9D24804C9E92F83&ref=/en/us/activity/search/advanced>

Trig Ratios

<https://education.ti.com/en/us/activity/detail?id=3F04FAE238FB456293A441879B751538&ref=/en/us/activity/search/advanced>

Radian Measure

<https://education.ti.com/en/us/activity/detail?id=8D7D391056E44037894A9CC2CF45C078&ref=/en/us/activity/search/advanced>

Vertical and Phase Shifts

<https://education.ti.com/en/us/activity/detail?id=4D4C4711421B4E0FAE74966514035E4F&ref=/en/us/activity/search/advanced>

Sinusoids with Vertical Transformations

<https://teacher.desmos.com/polygraph/custom/561fe4a7e104f0f5312bb6b0>

Match My Trig Function

<https://teacher.desmos.com/activitybuilder/custom/56036c6c6f26469f23250fb5>

Connect the Dots (Trig Style)

<https://teacher.desmos.com/activitybuilder/custom/56097557686358ae0730006c>

Everything you need to know about math journals:

<https://thecornerstoneforteachers.com/math-journals/>

(NJSLSA.R1, NJSLSA.W2)

Additional texts:

[www.newsela.com](http://www.newsela.com)

[www.readworks.org](http://www.readworks.org)

[www.commonlit.org](http://www.commonlit.org)



**Key Vocabulary:**

Determinant, dilation, equal matrices, image, matrix equation, preimage, scalar multiplication, variable matrix, zero matrix, amplitude, central angle, cosine, cycle, midline, period, periodic function, phase shift, radian, sine, tangent, unit circle, Law of Sines, Law of Cosines, trigonometric identity, trigonometric ratios

**Accommodations and Modifications:**

**Students with special needs:** Support staff will be available to aid students related to IEP specifications. 504 accommodations will also be attended to by all instructional leaders. Modifications, alternative assessments, and scaffolding strategies will be used to support this learning. The use of Universal Design for Learning (UDL) will be considered for all students as teaching strategies are considered. Additional staff should be included so all students can fully participate in the standards associated with this curriculum.

**ELL/ESL students:** Students will be supported according to the recommendations for “can do’s” as outlined by WIDA - [https://www.wida.us/standards/CAN\\_DOs/](https://www.wida.us/standards/CAN_DOs/)

**Students at risk of school failure:** Formative and summative data will be used to monitor student success at first signs of failure. Student work will be reviewed to determine support. This may include parent consultation, basic skills review and differentiation strategies. With considerations to UDL, time may be a factor in overcoming developmental considerations. More time will be made available with a certified instructor to aid students in reaching the standards.

**Gifted and Talented Students:** Students excelling in mastery of standards will be challenged with complex, high level challenges.

**English Language Learners:**

- Teaching modeling
- Peer modeling
- Word walls
- Give directions in small steps and in as few words as possible
- Provide visual aids
- Group similar problems together
- Repeat directions when necessary
- Provide a vocabulary list with definitions
- Use of alge-tiles when

**Special Education:**

- Utilize modifications & accommodations delineated in the students’ IEP
- Work with paraprofessional
- Work with a partner
- Shorten assignments to focus on mastery or key concepts
- Maintain adequate space between desks
- Keep workspaces clear of unrelated materials
- Provide fewer problems to attain passing grades

**At-Risk:**

- Use visual demonstrations, illustrations and models
- Give directions / instructions verbally and in simple written format
- Peer support
- Increased one – on – one time
- Teachers may modify instructions by modeling what the student is expected to do
- Instructions may be printed out in large print and hung

**Gifted and Talented:**

- Inquiry based instruction
- Independent study
- Higher order thinking skills
- Adjusting the pace of the lessons
- Real world scenarios
- Student driven instruction
- Allow students to complete an independent project as an alternative test



<p>needed</p> <ul style="list-style-type: none"> <li>• Use of number line when needed</li> </ul>	<ul style="list-style-type: none"> <li>• Tape a number line to the student's desk</li> <li>• Create a math journal that they can use during class, on assignments and (if teacher allows) on assessments</li> <li>• Provide extra time to complete a task when needed</li> <li>• Provide definitions of different graphs / charts with illustrations</li> <li>• Allow tests to be taken in a separate room</li> <li>• Allow students to use a calculator when appropriate</li> <li>• Divide test into small sections of similar questions or problems</li> <li>• Use of alge-tiles when needed</li> <li>• Use of number line when needed</li> </ul>	<p>up for the students to see during the time of the lesson</p> <ul style="list-style-type: none"> <li>• Review behavior expectations and make adjustments</li> <li>• Create a math journal that they can use during class, on assignments and (if teacher allows) on assessments</li> <li>• Allow students to complete an independent project as an alternative test</li> <li>• Use of alge-tiles when needed</li> <li>• Use of number line when needed</li> </ul>	
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#### **Interdisciplinary Connections: ELA**

**NJSLSA.R1.** Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

**NJSLSA.W2.** Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content

#### **21<sup>st</sup> Century Standards**

**9.2.12.C.1:** Review career goals and determine steps necessary for attainment.

**9.2.12.C.2:** Modify Personalized Student Learning Plans to support declared career goals.

**9.3.ST.2:** Use technology to acquire, manipulate, analyze and report data.

**9.3.ST-ET.5:** Apply the knowledge learned in STEM to solve problems.

**Career Ready Practices:**

**CRP2:** Apply appropriate academic and technical skills

**CRP4:** Communicate clearly and effectively and with reason

**CRP6:** Demonstrate creativity and innovation

**CRP8:** Utilize critical thinking to make sense of problems and persevere in solving them

**CRP11:** Use technology to enhance productivity

**Technology Standards:**

**8.1.12.A.3:** Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.

**Major** **Supporting** **Additional** (Identified by PARCC Model Content Frameworks)