**Englewood Public School District**
**Precalculus – Graphical, Numerical, Algebraic**
**First Marking Period**

## Unit 1: Polynomial, Power, Rational, Exponential and Logarithmic Functions

**Overview:** During this unit, students will review the concepts of linear equations and inequalities, complex numbers, functions and their properties, as well as begin to study many different functions, including polynomial, exponential and logarithmic functions.

**Time Frame:** 43 to 47 Days

**Enduring Understandings:**

- *You can use variables to represent variable quantities in real-world situations and in patterns.*
- *Linear equations can be represented using either slope-intercept, point-slope, or standard form.*
- *The point of intersection of the graphs of equations is the solution to the system y = f(x), y = g(x).*
- *A complex number consists of both a real number and an imaginary number.*
- *The properties of the real number system also apply to complex numbers.*
- *Mathematical problems can be illustrated using different models, including numerical, algebraic and graphical models.*
- *If f and $f^{-1}$ are inverse functions and if one maps a to b, then the other maps b to a.*
- *A function is a rule where every element of one set (the domain) corresponds to one and only one element of another set (the range).*
- *You can combine functions using the same operations as real numbers.*
- *You can use the values of a, h, and k in the standard form of a function to determine how the parent function has been transformed.*
- *Any function is possibly a stretch or compression, a reflection, and/or a translation of the parent function.*
- *A power function can be written in the form f(x) = $k \cdot a^x$ where k and a are nonzero constants.*
- *Power functions can be seen throughout chemistry and physics, as well as geometry.*
- *The end behavior of a function describes how the function behaves as x approaches infinity.*
- *The Intermediate Value Theorem tells us that a sign change implies a real zero.*
- *You can divide polynomials to find the zeros of a function.*
- *The Fundamental Theorem of Algebra states that a polynomial function of degree n has n complex zeros (real and nonreal) and some of these zeros may be repeated.*
- *If f and g are polynomial functions with g(x) ≠ 0, then the function given by r(x) = $\frac{f(x)}{g(x)}$ is a rational function.*
- *The function y = $ab^x$ illustrates an exponential function.*
- *Exponential functions and logarithmic functions are inverse operations.*
- *Logarithms and exponents have consistent properties.*
- *You can solve logarithmic problems by applying the properties of logarithms.*

**Essential Questions:**

- *How do variables help you model real world situations?*
- *How do you solve an equation or inequality?*
- *Does it matter which form of a linear equation you use?*
- *What is a complex number?*
- *Why is illustrating a mathematical problem in different ways helpful?*
- *How are the function and its inverse related?*
- *How do you use transformations to help graph functions?*
- *How can you model data with a function?*
- *Why is combining functions useful?*
- *What does the end behavior of a function tell us, and why is it useful?*
- *How do you determine the number of zeros of a function?*
- *How are exponential functions and logarithmic functions related?*
- *How do we use properties of exponents and logarithms to solve problems?*

| Standards | Topics and Objectives | Activities | Resources | Assessments |
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| **MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**<br><br>**N-RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents<br><br>**N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number | **Topics**<br><br>Real numbers, Equations, Inequalities, Complex numbers, Functions, Graphs, Linear Systems, Transformations, Quadratic functions, Polynomial functions, Fundamental Theorem of Algebra, Rational functions, Exponential functions, Logarithmic functions<br><br>Twenty-First Century Themes and Skills include:<br>• The Four C's<br>• Global awareness<br>• Financial, economic, business and entrepreneurial literacy | Powers of complex numbers https://www.illustrativemathematics.org/content-standards/tasks/1689<br><br>Computations with complex numbers https://www.illustrativemathematics.org/content-standards/tasks/617<br><br>Complex number battleship https://betterlesson.com/lesson/491418/complex-number-battleship<br><br>Complex number bingo | Pearson Chapters P, 1, 2, 3<br><br>Illustrative Mathematics https://www.illustrativemathematics.org/<br><br>Betterlesson.com<br><br>jc-schools.net<br><br>National Library of Virtual Manipulatives http://nlvm.usu.edu/<br><br>Alabama Learning Exchange http://alex.state.al.us/search.php?fa_submit=ALLPLANS<br><br>Mathematics Assessment Project http://map.mathshell.org/ | Textbook Pages 60, 61, 62, 152, 153, 154, 155, 245, 246, 247, 248, 249, 313, 314, 315, 316, 317 |

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| and an irrational number is irrational<br><br>**F-IF.C.8b** Use the properties of exponents to interpret expressions for exponential functions.<br><br>**G-GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.<br><br>**G-GPE.B.4** Use coordinates to prove simple geometric theorems algebraically.<br><br>**A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.<br><br>**A-REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | **Objectives**<br><br>Students will<br>• Understand the rational number system and be able to identify numbers specifically<br>• Use interval notation when describing a set of numbers<br>• Write inequalities<br>• Apply the properties of algebra<br>• Work with exponents<br>• Graph points on the coordinate plane<br>• Use the distance and midpoint formulas<br>• Write the standard form of the equation of a circle on the coordinate plane<br>• Solve linear equations in one variable<br>• Solve inequalities in one variable<br>• Determine the slope of a line<br>• Write the equation of a line in two variables<br>• Solve quadratic equations<br>• Solve absolute value equations<br>• Solve equations with fractional expressions | http://archive.jc-schools.net/dynamic/math/worksheets_secondary/CmplxNmbrsBingo_Activity_Alg2.pdf<br><br>Representing functions of everyday situations http://map.mathshell.org/download.php?fileid=1740<br><br>Identifying Quadratic Functions https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/2107<br><br>Zeros and Factorization of a General Polynomial https://www.illustrativemathematics.org/content-standards/HSA/APR/B/2/tasks/788<br><br>Transforming the Graph of a Function https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742<br><br>Function Transformations http://nlvm.usu.edu/en/nav/frames_asid_329_g_4_t_2.html?open=activities&from=category_g_4_t_2.html | jmap.org<br>khanacademy.org<br><br>kutasoftware.com |

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| **A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br><br>**G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).<br><br>**A-SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines<br><br>**A-SSE.B.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.<br><br>**A-REI.B.4** Solve quadratic equations in one variable.<br><br>**F-IF.C.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and | • Utilize technology when appropriate in order to solve equations involving quadratics, absolute value or fractional expressions.<br>• Extend their knowledge of the real number system into the imaginary number system<br>• Perform algebraic operations with imaginary numbers<br>• Find complex zeros of quadratic functions<br>• Solve inequalities involving absolute value, quadratic polynomials and expressions involving fractions<br>• Use different models (algebraic, numerical or graphical) to solve problems<br>• Translate from one data model to another<br>• Represent functions in a variety of ways<br>• Determine the domain and range of a function<br>• Analyze a function's characteristics, such as extreme values, asymptotes, symmetry and end behavior<br>• Recognize graphs of the 12 basic functions | Discover the Roots of a Polynomial Function<br>http://alex.state.al.us/lesson_view.php?id=27664<br><br>Parent Functions and Their Children<br>http://alex.state.al.us/lesson_view.php?id=30033<br><br>Composite function relay<br>http://images.pcmac.org/Uploads/JeffersonCountySchools/JeffersonCountySchools/Departments/DocumentsSubCategories/Documents/Math%20-%20Composition%20of%20Functions%20Relay%20-%20Chain%20Reaction%20Activity%20with%20Printable%20Cards.pdf<br><br>Exponential functions – Boom Town<br>https://www.illustrativemathematics.org/content-standards/tasks/2126<br><br>Exponential functions – Allergy medication<br>https://www.illustrativemathematics.org/content-standards/tasks/2125 |

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| interpret these in terms of a context.<br><br>**N-CN.A.1** Know there is a complex number $i$ such that $i^2 = –1$, and every complex number has the form $a + bi$ with $a$ and $b$ real<br><br>**N-CN.A.2** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers<br><br>**N-CN.A.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.<br><br>**N-CN.C.7** Solve quadratic equations with real coefficients that have complex solutions<br><br>**N-CN.C.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$<br><br>**N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units | • Combine the 12 basic functions in a variety of ways in order to create new functions<br>• Build new functions by adding, subtracting, multiplying or dividing<br>• Build new functions by composing functions<br>• Define functions and relations parametrically<br>• Find inverse functions<br>• Perform translations, reflections, stretches and shrinks of functions and parametric relations<br>• Model real world scenarios with functions<br>• Produce functions to model data, formulas, graphs and verbal descriptions<br>• Recognize linear and quadratic functions<br>• Graph linear and quadratic functions<br>• Solve problems involving linear and quadratic functions<br>• Sketch power functions<br>• Graph polynomial functions<br>• Use technology to find the zeros of a function<br>• Find the zeros of a function algebraically | Exponential functions – identifying graphs<br>https://www.illustrativemathematics.org/content-standards/tasks/2115<br><br>Exponents and logs 1<br>https://www.illustrativemathematics.org/content-standards/tasks/600<br><br>Exponents and logs 2<br>https://www.illustrativemathematics.org/content-standards/tasks/615 |

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| consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.<br><br>**A-CED.A.1a** Create equations and inequalities in one variable and use them to solve problems.<br><br>**A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br><br>**F-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.<br><br>**F-IF.C.7a** Graph linear and quadratic functions and show intercepts, maxima and minima<br><br>**A-CED.A.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context | • Predict the end behavior of a function<br>• Divide polynomials using either long division or synthetic division<br>• Apply the Remainder Theorem<br>• Apply the Factor Theorem<br>• Apply the Rational Zeros Theorem<br>• Determine the upper and lower bounds of zeros of polynomials<br>• Factor polynomials<br>• Identify asymptotes and predict the end behavior of rational functions<br>• Solve equations involving fractions<br>• Understand and identify extraneous solutions<br>• Solve inequalities involving polynomial or rational functions<br>• Evaluate exponential expressions<br>• Identify and graph exponential and logistic functions<br>• Model real life problems with exponential growth or decay<br>• Convert equations between logarithmic | |

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| **F-IF.A. 1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*)<br><br>**F–IF.A.2** Use function notation evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context<br><br>**F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes<br><br>**F-IF.C.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions<br><br>**F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in | form and exponential form<br>• Evaluate common and natural logarithms<br>• Graph common and natural logarithmic functions<br>• Apply the properties of logarithms to evaluate expressions<br>• Apply the properties of logarithms to solve exponential and logarithmic equations algebraically<br>• Solve real world application problems involving exponential and logarithmic equations | |

tables, or by verbal descriptions).

**F-BF.B.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

**F-BF. A. 1b** Combine standard function types using arithmetic operations

**F-BF.A.1c** Compose functions

**F-BF.B.4** Find inverse functions

**A-CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**G-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

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| **G-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems<br><br>**G-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects<br><br>**G-MG.A.3** Apply geometric methods to solve design problems<br><br>**S-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related<br><br>**N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling<br><br>**N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities<br><br>**F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship | |

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| **F-IF.C.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior<br><br>**F-IF.C.7d** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior<br><br>**S-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related<br><br>**S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.<br><br>**S-ID.C.8.** Compute (using technology) and interpret the correlation coefficient of a linear fit.<br><br>**S-ID.C.9** Distinguish between correlation and causation.<br><br>**A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to | |

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| construct a rough graph of the function defined by the polynomial<br><br>**A-REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions<br><br>**A-APR.B.2a** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x)<br><br>**N-CN.C.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.<br><br>**A-SSE.B.3c** Use the properties of exponents to | |

transform expressions for exponential functions

**F-LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions

**F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

**F-LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**F-LE.A.4** Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

**F-LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context.

**F-BF.B.5** Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents

**Modifications:**

- New Jersey Department of Education – Instructional Supports and Scaffolds
- Suggested Strategies for English Language Learners
- Secondary activities were created to allow for greater personalized learning to meet the needs of all learners including students with gifts and talents

**Key Vocabulary:** real numbers, natural (counting) numbers, whole numbers, integers, terminates, rational, irrational, imaginary, complex, intervals, exponent, base, bounded interval, unbounded interval, algebraic expression, variable, constant, additive inverse, multiplicative inverse, reciprocal, scatterplot, magnitude, absolute value, Cartesian plane, equation of a circle, midpoint, distance formula, Pythagorean Theorem, equivalent, equation, system of equations, inequality, system of inequalities, point-slope form, slope, slope-intercept form, general form, quadratic equation, completing the square, factor, quadratic formula, discriminant, complex conjugate, zero factor property, root, solution, zero, deductive reasoning, domain, range, function notation, discontinuity, removable discontinuity, jump discontinuity, infinite discontinuity, continuous, discontinuous, local maxima, local minima, absolute maxima, absolute minima, relative extrema, even function, odd function, asymptote, identity function, squaring function, cubing function, reciprocal function, square root function, exponential function, natural logarithmic function, sine function, cosine function, absolute value function, greatest integer function, logistic function, piecewise function, composition of functions, one-to-one, reflection, transformations, translation, regression line, leading coefficient, linear function, rate of change, correlation coefficient, axis of symmetry, vertex, power function, quotient, remainder, synthetic division, extraneous solutions, logistic growth function, logistic decay function, Newton's law of cooling, simple interest, compound interest, compounded continuously