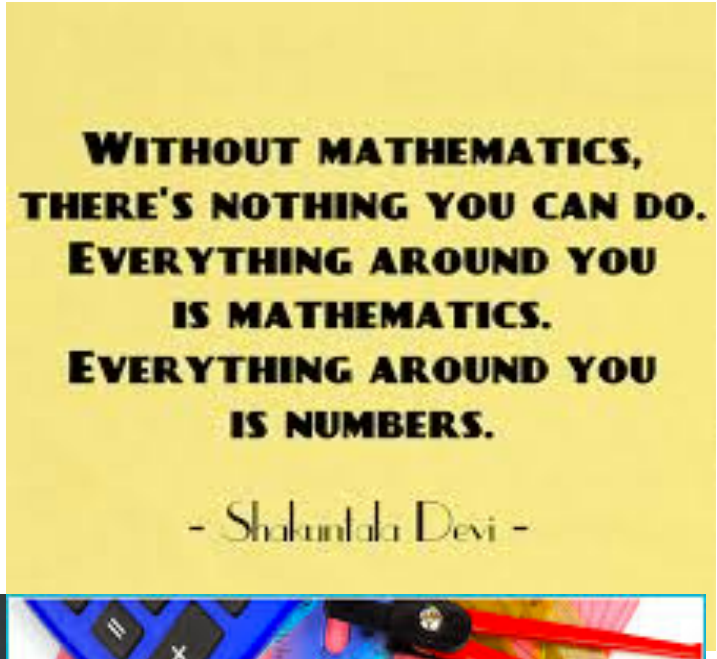
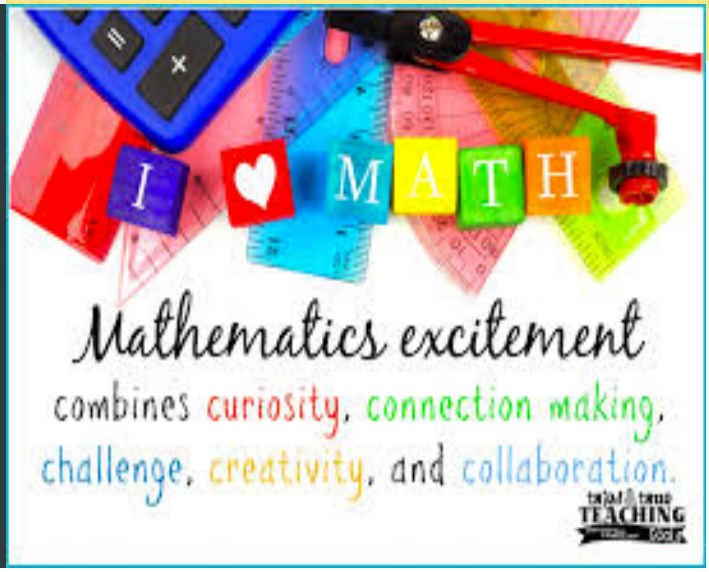
SWEETWATER COUNTY SCHOOL DISTRICT #1



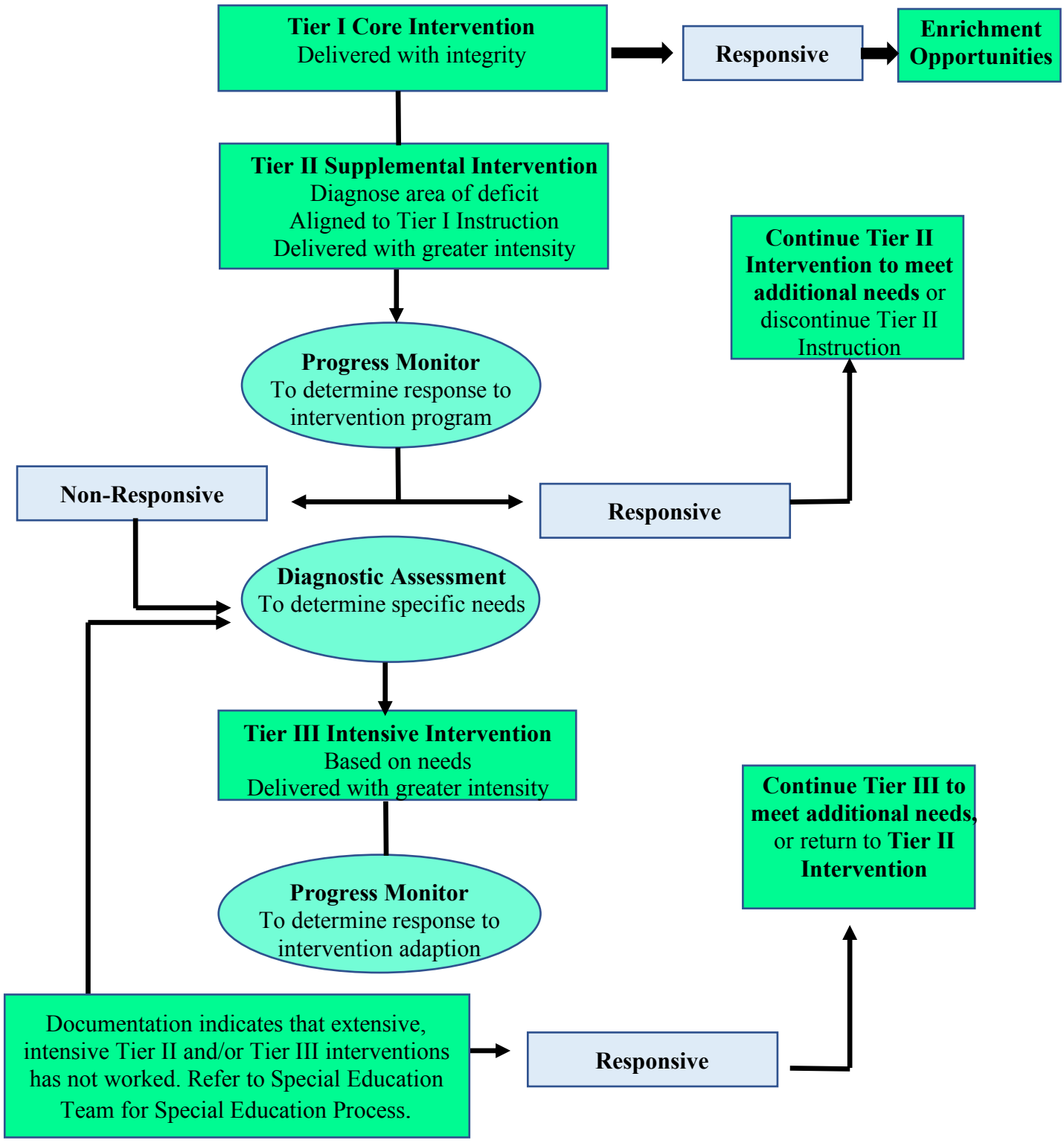


Intervention Handbook

SCSD #1 Intervention Plans, Menu of Options, Tier 2 and Tier 3 Support

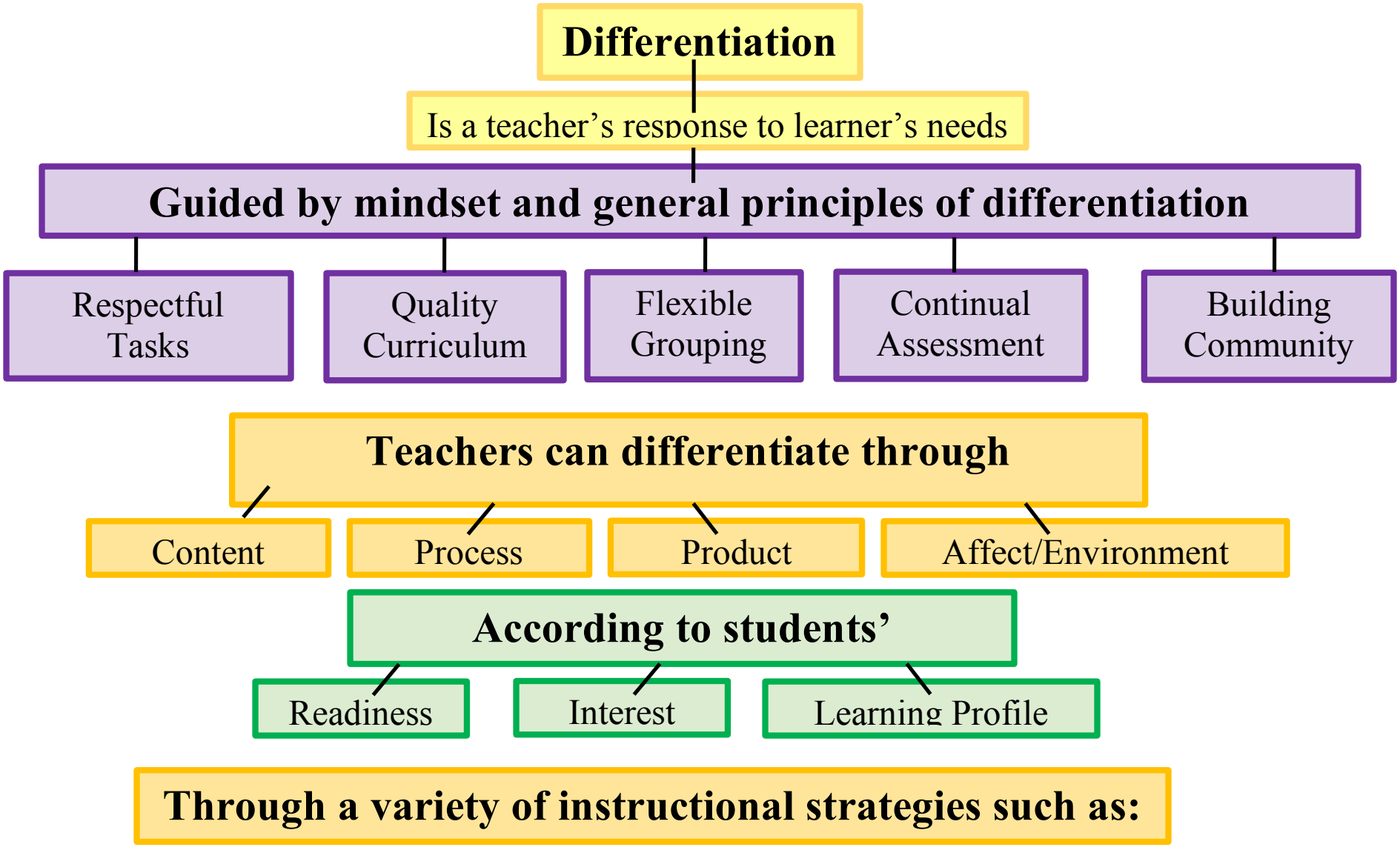
Table of Contents

K-12 Intervention Plan Framework



3/23/18

SCSD #1 Definition of Differentiation
(Adapted from Carol Ann Tomlinson)



Graphic Organizers, Scaffolding Reading, Cubing, Think-Tac-Toe, Learning Contracts, Independent Studies, Intelligence Preferences, Orbitals, Complex Instruction, Web Quests, etc.

Welcome to

Kindergarten!

Kindergarten Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Kindergarten Intervention Plan

Kindergarten Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Kindergarten Menu of Options

Benchmark MK.1	Benchmark MK.2	Benchmark MK.3	Benchmark MK.4	Benchmark MK.5	Benchmark MK.6
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/Activities	Games/Activities	Games/Activities	Games/Activities	Games/Activities	Games/Activities
Strategies			Fluency Ideas		


Welcome
to
1st Grade

First Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom, or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

First Grade Intervention Plan

First Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

First Grade Menu of Options

Benchmark M1.1	Benchmark M1.2	Benchmark M1.3	Benchmark M1.4	Benchmark M1.5	Benchmark M1.6	Benchmark M1.7
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities
Strategies				Fluency Templates		


Welcome to
2nd Grade!

Second Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom, or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Second Grade Intervention Plan

Second Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Second Grade Menu of Options

Benchmark M2.1	Benchmark M2.2	Benchmark M2.3	Benchmark M2.4	Benchmark M2.5	Benchmark M2.6	Benchmark M2.7	Benchmark M2.8	Benchmark M2.9	Benchmark M2.10	Benchmark M2.11
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities
Strategies										


hello
third
GRADE

Third Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom, or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Third Grade Intervention Plan

Third Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Third Grade Menu of Options

Benchmark M3.1	Benchmark M3.2	Benchmark M3.3	Benchmark M3.4	Benchmark M3.5	Benchmark M3.6	Benchmark M3.7
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities
Strategies						


hello fourth
GRADE

Fourth Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom, or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Fourth Grade Intervention Plan

Fourth Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Fourth Grade Menu of Options

Benchmark M4.1	Benchmark M4.2	Benchmark M4.3	Benchmark M4.4	Benchmark M4.5	Benchmark M4.6	Benchmark M4.7	Benchmark M4.8	Benchmark M4.9	Benchmark M4.10	Benchmark M4.11	Benchmark M4.12
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities
Strategies											


hello
fifth
GRADE

Fifth Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom, or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Fifth Grade Intervention Plan

Fifth Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Fifth Grade Menu of Options

Benchmark M5.1	Benchmark M5.2	Benchmark M5.3	Benchmark M5.4	Benchmark M5.5	Benchmark M5.6	Benchmark M5.7	Benchmark M5.8	Benchmark M5.9	Benchmark M5.10	Benchmark M5.11	Benchmark M5.12
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities	Games/ Activities
Strategies											


hello
sixth
GRADE

Sixth Grade Diagnosing Criteria

Screeners: In-Program Assessments, Classroom Assessments, District Assessments

I

No areas of deficits identified on assessments

Tier I (Core Instruction) on Intervention Plan

Identify deficit on in-program, classroom or other assessments

Determine area of deficit: (e.g.)

Tier II (Supplemental Instruction) on Intervention Plan

Tier II (Supplemental Instruction) has not worked, give Diagnostic Assessment

Tier III (Intensive Intervention) on Intervention Plan

Documentation indicates that extensive, intensive Tier II and/or Tier III interventions has not worked. Refer to Special Education Team for Special Education Process.

Sixth Grade Intervention Plan

Sixth Grade Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbers, factor strategies, etc.).

4. Incorporate math practices that support instruction (i.e. perserverance, reasoning, quantitatively and qualitatively).

5. Explicitly teach skills of accuracy, efficiency, and flexibility using Rapid Whiteboard Exchange.

6. Explicitly teach vocabulary related to content.

7. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align with core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options.

5. Explicitly re-teach vocabulary related to content.

6. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

7. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

Sixth Grade Menu of Options

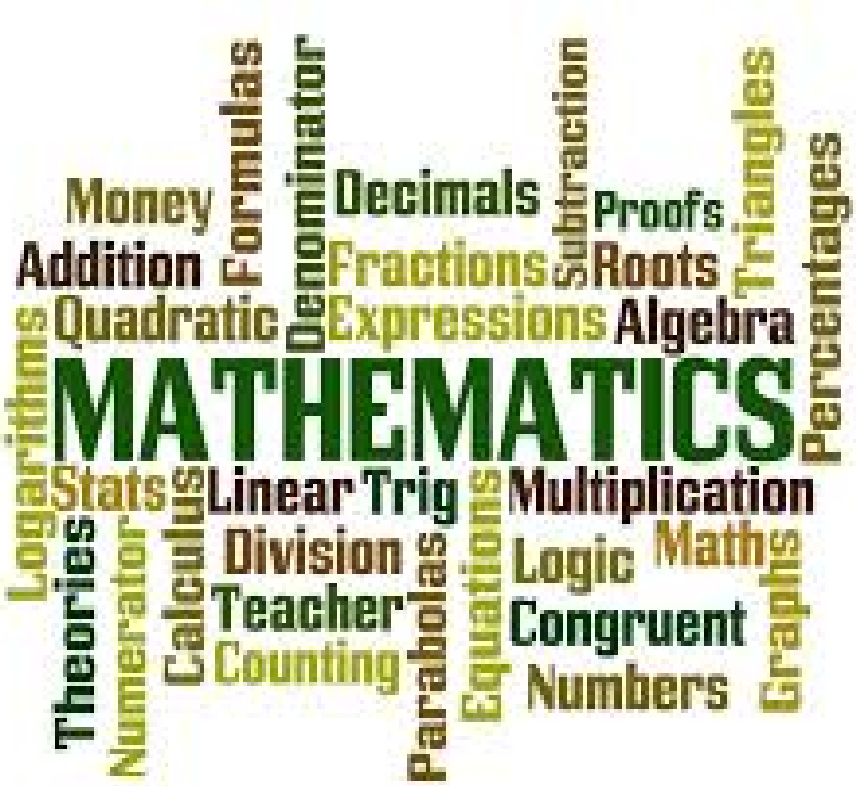
Benchmark M6.1	Benchmark M6.2	Benchmark M6.3	Benchmark M6.4	Benchmark M6.5	Benchmark M6.6
Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary	Vocabulary
Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments	Skill Assessments
Games/Activities	Games/Activities	Games/Activities	Games/Activities	Games/Activities	Games/Activities
Strategies					


hello
seventh
GRADE

Seventh Grade Intervention Plan


hello
eighth
GRADE

Eighth Grade Intervention Plan


MATHEMATICS
Money
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Fractions
Expressions
Subtraction
Proofs
Roots
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Triangles
Percentages
Logarithms
Stats
Linear Trig
Multiplication
Theories
Numerator
Calculus
Division
Teacher
Counting
Paraboloas
Equations
Logic
Math
Congruent
Numbers
Graphs

High School Intervention Plan

High School Math Intervention Plan

Click here for activities to support instruction (Menu of Options).

Tier I-Core Instruction

1. Implement a standards-aligned math program that supports fluency practice using the concrete-pictorial-abstract approach to instruction.

2. Manipulatives are readily accessible, used to teach new skill, and are available for students use alongside standard algorithm.

3. Incorporate daily opportunities for fluency routines such as, but not limited to sprints, number talks, fluency, math games, with explicit teacher support.

3. Explicitly model strategies for developing fluency (composing-decomposing numbes, factor strategies, etc.).

4. Incorporate math practices that support fluency (i.e. perserverance, reasoning, quantitately and qualitatively).

5. Explicitly teach fluency skills of accuracy, efficiency, and flexibility).

6. Periodically assess learning of all students to determine effectiveness of core instruction, and identify students in need of additional supports.

Tier II-Supplemental Instruction

1. Reteach deficit skill area properly using concrete manipulative to model concept.

2. Use companion evidence-based materials that aligns with the core program.

3. Provide explicit preteaching of skills underlying core content.

3. Provide small-group **(10-15 students)** instruction with multiple response formats and explicit corrective feedback.

4. Incorporated additional small-group or individual behavior strategies targeted to individual needs in engagement and motivation.

5. Collect progress monitoring data on fluency at least one or two times per month using core materials supports or a valid, reliable tool.

Tier III-Intensive Intervention

1. Reteach deficit skill area properly using concrete manipulative to model/teach concept.

2. Use companion evidence-based materials that align wit core program and work on specific skill deficit.

3. Provide alternative strategies to standard algorithm. Use Menu of Options.

4. Create an individualized set of support for student (i.e vocabulary book, alternative stratgies, support tools, manipulatives, charts, etc.). Use Menu of Options

5. Collect progress monitoring data on fluency weekly, at a level that is sensitive to change and adjust instruction as needed. Use core materials supports or a valid, reliable tool.

6. Determine next steps including the possibility of teaching the use of a tool such as a calculator as a permanent support tool.

High School Menu of Options

Algebra I	Algebra II	Geometry
Vocabulary	Vocabulary	Vocabulary
STEM Videos	STEM Videos	STEM Videos
Uplifting Videos	Uplifting Videos	Uplifting Videos
Additional Practice	Additional Practice	Additional Practice

Research

APPENDIX D. EIGHT EFFECTIVE MATHEMATICS TEACHING PRACTICES

Teaching Practice	Purpose	What the Teacher Does	What the Students Do
1. Establish mathematics goals to focus learning.	• Set the stage to guide instructional decisions. • Expect students to understand the purpose of a lesson beyond simply repeating the standard.	Considers broad goals as well as the goals of the unit and the actual lesson, including the following: • What is to be learned? • Why is the goal important? • Where do students need to go? • How can learning be extended?	• Make sense of new concepts and skills, including connections to concepts learned in previous grades. • Experience connections among the standards and across domains. • Deepen their understanding and expect mathematics to make sense.
2. Implement tasks that promote reasoning and problem solving.	• Provide opportunities for students to engage in exploration and make sense of important mathematics. • Encourage students to use procedures in ways that are connected to understanding.	Chooses tasks that • are built on current student understandings. • have various entry points with multiple ways for the problems to be solved. • are interesting to students.	• Work to make sense out of the task and persevere in solving problems. • Use a variety of models and materials to make sense of the mathematics in the task. • Convince themselves and others the answer is reasonable.
3. Use and connect mathematical representations.	• Provide concrete representations that lead students to develop conceptual understanding and later connect that understanding to procedural skills. • Provide a variety of representations that range from using physical models to using abstract notations.	• Uses tasks that allow students to use a variety of representations. • Encourages the use of different representations, including concrete models, pictures, words, and numbers, that support students in explaining their thinking and reasoning.	• Use materials to make sense out of problem situations. • Connect representations to mathematical ideas and the structure of big ideas, including operational sense with whole numbers, fractions, and decimals.
4. Facilitate meaningful mathematical discourse.	• Provide students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. • Advance the mathematical thinking of the whole class by talking and sharing aloud.	• Engages students in explaining their mathematical reasoning in small group and classroom situations. • Facilitates discussions among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Explain their ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of others. • Ask questions of others to make sense of their ideas.

Teaching Practice	Purpose	What the Teacher Does	What the Students Do
5. Pose purposeful questions.	• Reveal students' current understanding of a concept. • Encourage students to explain, elaborate, and clarify their thinking. • Make the learning of mathematics more visible and accessible for students.	• Asks questions that build on and extend student thinking. • Is intentional about the kinds of questions asked to make the mathematics more visible to students. • Uses wait time to provide students with time to think and examine their ideas.	• Think more deeply about the process of the mathematics rather than simply focusing on the answer. • Listen to and comment on the explanations of others in the class.
6. Build procedural fluency from conceptual understanding.	• Provide experiences with concrete materials that allow students to make sense of important mathematics and flexibly choose from a variety of methods to solve problems.	• Provides opportunities for students to reason about mathematical ideas. • Expects students to explain why their strategies work. • Connects student methods to efficient procedures as appropriate.	• Understand and explain the procedures they are using and why they work. • Use a variety of strategies to solve problems and make sense of mathematical ideas. • Do not rely on shortcuts or tricks to do mathematics.
7. Support productive struggle in learning mathematics.	• Provide opportunities for productive struggle, which is significant and essential to learning mathematics with understanding. • Allow students to grapple with ideas and relationships. • Give students ample time to work with and make sense of new ideas, which is critical to their learning with understanding.	• Supports student struggle without showing and telling a procedure but rather focusing on the important mathematical ideas. • Asks questions that scaffold and advance student thinking. • Builds questions and plans lessons based on important student misconceptions rather than focusing on the correct answer. • Recognizes the importance of effort as students work to make sense of new ideas.	• Stick to a task and recognize that struggle is part of making sense. • Ask questions that will help them to better understand the task. • Support each other with ideas rather than telling others the answer or how to solve a problem.
8. Elicit and use evidence of student thinking.	• Elicit and use evidence of student thinking, which helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. • Assess student thinking and understanding by using formative assessment through student written and oral ideas.	• Determines what to look for in gathering evidence of student learning. • Poses questions and answers student questions that provide information about student understanding, strategies, and reasoning. • Uses evidence to determine next steps of instruction.	• Accept that reasoning and understanding are as important as the answer to a problem. • Use mistakes and misconceptions to rethink their understanding. • Ask questions of the teacher and peers to clarify confusion or misunderstanding. • Assess progress toward developing mathematical understanding.

Source: Adapted from National Council of Teachers of Mathematics (2014).

13 RULES *That Expire*

Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in students' math careers.

By Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty

FUSE/THINKSTOCK

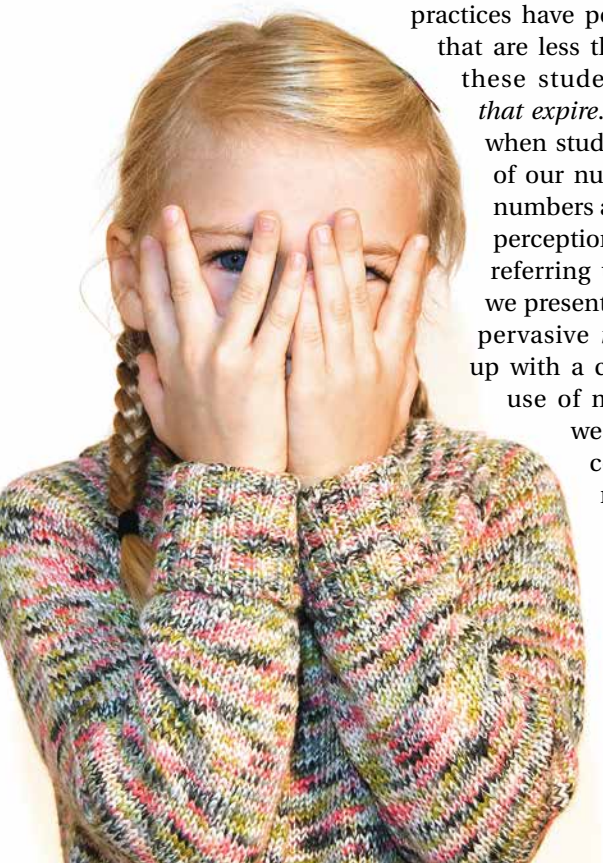
Imagine the following scenario: A primary teacher presents to her students the following set of number sentences:

$$3 + 5 = \square$$
$$\square + 2 = 7$$
$$8 = \square + 3$$
$$2 + 4 = \square + 5.$$

Stop for a moment to think about which of these number sentences a student in your class would solve first or find easiest. What might they say about the others? In our work with young children, we have found that students feel comfortable solving the first equation because it "looks right" and students can interpret the equal sign as *find the answer*. However, students tend to hesitate at the remaining number sentences because they have yet to interpret and understand the equal sign as a symbol indicating a relationship between two quantities (or amounts) (Mann 2004).

In another scenario, an intermediate student is presented with the problem 43.5 × 10. Immediately, he responds, "That's easy; it is 43.50 because my teacher said that when you multiply any number times ten, you just add a zero at the end."

In both these situations, hints or repeated practices have pointed students in directions that are less than helpful. We suggest that these students are experiencing *rules that expire*. Many of these rules "expire" when students expand their knowledge of our number systems beyond whole numbers and are forced to change their perception of what can be included in referring to *a number*. In this article, we present what we believe are thirteen pervasive *rules that expire*. We follow up with a conversation about incorrect use of mathematical language, and we present alternatives to help counteract common student misunderstandings.



ALINA555/THINKSTOCK

The Common Core State Standards (CCSS) for Mathematical Practice advocate for students to become problem solvers who can reason, apply, justify, and effectively use appropriate mathematics vocabulary to demonstrate their understanding of mathematics concepts (CCSSI 2010). This, in fact, is quite opposite of the classroom in which the teacher does most of the talking and students are encouraged to memorize facts, "tricks," and tips to make the mathematics "easy." The latter classroom can leave students with a collection of explicit, yet arbitrary, rules that do not link to reasoned judgment (Hersh 1997) but instead to learning without thought (Boaler 2008). The purpose of this article is to outline common rules and vocabulary that teachers share and elementary school students tend to overgeneralize—tips and tricks that do not promote conceptual understanding, rules that "expire" later in students' mathematics careers, or vocabulary that is not precise. As a whole, this article aligns to Standard of Mathematical Practice (SMP) 6: *Attend to precision*, which states that mathematically proficient students "…try to communicate precisely to others. …use clear definitions … and … carefully formulated explanations…" (CCSSI 2010, p. 7). Additionally, we emphasize two other mathematical practices: SMP 7: *Look for and make use of structure* when we take a look at properties of numbers; and SMP 2: *Reason abstractly and quantitatively* when we discuss rules about the meaning of the four operations.

"Always" rules that are not so "always"

In this section, we point out rules that seem to hold true at the moment, given the content the student is learning. However, students later find that these rules are not always true; in fact, these rules "expire." Such experiences can be frustrating and, in students' minds, can further the notion that mathematics is a mysterious series of tricks and tips to memorize rather than big concepts that relate to one another. For each rule that expires, we do the following:

1. State the rule that teachers share with students.
2. Explain the rule.
3. Discuss how students inappropriately overgeneralize it.
4. Provide counterexamples, noting when the rule is not true.

5. State the "expiration date" or the point when the rule begins to fall apart for many learners. We give the expiration date in terms of grade levels as well as CCSSM content standards in which the rule no longer "always" works.

Thirteen rules that expire

1. When you multiply a number by ten, just add a zero to the end of the number.

This "rule" is often taught when students are learning to multiply a whole number times ten. However, this directive is not true when multiplying decimals (e.g., $0.25 \times 10 = 2.5$, not 0.250). Although this statement may reflect a regular pattern that students identify with whole numbers, it is not generalizable to other types of numbers. Expiration date: Grade 5 (5.NBT.2).

2. Use keywords to solve word problems.

This approach is often taught throughout the elementary grades for a variety of word problems. Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside of the problem context (Clement and Bernhard 2005). Unfortunately, many keywords are common English words that can be used in many different ways. Yet, a list of keywords is often given so that word problems can be translated into a symbolic, computational form. Students are sometimes told that if they see the word *altogether* in the problem, they should always add the given numbers. If they see *left* in the problem, they should always subtract the numbers. But reducing the meaning of an entire problem to a simple scan for key words has inherent challenges. For example, consider this problem:

> John had 14 marbles in his left pocket. He had 37 marbles in his right pocket. How many marbles did John have?

If students use keywords as suggested above, they will subtract without realizing that the problem context requires addition to solve. Keywords become particularly troublesome when students begin to explore multistep word problems, because they must decide which keywords work with which component of the problem. Keywords can be informative but must be used in conjunction with all other words in the problem to grasp the full meaning. Expiration date: Grade 3 (3.OA.8).

3. You cannot take a bigger number from a smaller number.

Students might hear this phrase as they first learn to subtract whole numbers. When students are restricted to only the set of whole numbers, subtracting a larger number from a smaller one results in a negative number, an integer that is not in the set of whole numbers, so this rule is true. Later, when students encounter application or word problems involving contexts that include integers, students learn that this "rule" is not true for all problems. For example, a grocery store manager keeps the temperature of the produce section at 4 degrees Celsius, but this is 22 degrees too hot for the frozen food section. What must the temperature be in the frozen food section? In this case, the answer is a negative number, ($4^{\circ} - 22^{\circ} = -18^{\circ}$). Expiration date: Grade 7 (7.NS.1).

4. Addition and multiplication make numbers bigger.

When students begin learning about the operations of addition and multiplication, they are often given this rule as a means to develop a generalization relative to operation sense. However, the rule has multiple counterexamples. Addition with zero does not create a sum larger than either addend. It is also untrue when adding two negative numbers (e.g., $-3 + -2 = -5$), because –5 is less than both addends. In the case of the equation below, the product is smaller than either factor.

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

This is also the case when one of the factors is a negative number and the other factor is positive, such as $-3 \times 8 = -24$. Expiration date: Grade 5 (5.NF.4 and 5.NBT.7) and again at Grade 7 (7.NS.1 and 7.NS.2).

5. Subtraction and division make numbers smaller.

This rule is commonly heard in grade 3: both subtraction and division will result in an answer that is smaller than at least one of the

numbers in the computation. When numbers are positive whole numbers, decimals, or fractions, subtracting will result in a number that is smaller than at least one of the numbers involved in the computation. However, if the subtraction involves two negative numbers, students may notice a contradiction (e.g., –5 – (–8) = 3). In division, the rule is true if the numbers are positive whole numbers, for example:

$$8 \div 4 = 2 \text{ or } 4 \div 8 = \frac{1}{2}$$

However, if the numbers you are dividing are fractions, the quotient may be larger:

$$\frac{1}{4} \div \frac{2}{5} = \frac{5}{8}$$

This is also the case when dividing two negative factors: (e.g., –9 ÷ –3 = 3). Expiration dates: Grade 6 (6.NS.1) and again at Grade 7 (7.NS.1 and 7.NS.2c).

6. You always divide the larger number by the smaller number.

This rule may be true when students begin to learn their basic facts for whole-number division and the computations are not contextually based. But, for example, if the problem states that Kate has 2 cookies to divide among herself and two friends, then the portion for each person is 2 ÷ 3. Similarly, it is possible to have a problem in which one number might be a fraction:

> Jayne has $\frac{1}{2}$ of a pizza and wants to share it with her brother. What portion of the whole pizza will each get?

In this case, the computation is as follows:

$$\frac{1}{2} \div 2 = \frac{1}{4}$$

Expiration date: Grade 5 (5.NF.3 and 5.NF.7).

7. Two negatives make a positive.

Typically taught when students learn about multiplication and division of integers, rule 7 is to help them determine the sign of the product or quotient. However, this rule does not always hold true for addition and subtraction of integers, such as in –5 + (–3) = –8. Expiration date: Grade 7 (7.NS.1).

8. Multiply everything inside the parentheses by the number outside the parentheses.

As students are developing the foundational skills linked to order of operations, they are often told to first perform multiplication on the numbers (terms) within the parentheses. This holds true only when the numbers or variables inside the parentheses are being added or subtracted, because the distributive property is being used, for example, 3(5 + 4) = 3 × 5 + 3 × 4. The rule is untrue when multiplication or division occurs in the parentheses, for example, 2 (4 × 9) ≠ 2 × 4 × 2 × 9. The 4 and the 9 are not two separate terms, because they are not separated by a plus or minus sign. This error may not emerge in situations when students encounter terms that do not involve the distributive property or when students use the distributive property without the element of terms. The confusion seems to be an interaction between students' partial understanding of terms and their partial understanding of the distributive property—which may not be revealed unless both are present. Expiration date: Grade 5 (5.OA.1).

9. Improper fractions should always be written as a mixed number.

When students are first learning about fractions, they are often taught to always change improper fractions to mixed numbers, perhaps so they can better visualize how many *wholes* and *parts* the number represents. This rule can certainly help students understand that positive mixed numbers can represent a value greater than one whole, but it can be troublesome when students are working within a specific mathematical context or real-world situation that requires them to use improper fractions. This frequently first occurs when students begin using improper fractions to compute and again when students later learn about the slope of a line and must represent the slope as the rise/run, which is sometimes appropriately and usefully expressed as an improper fraction. Expiration dates: Grade 5 (5.NF.1) and again in Grade 7 (7.RP.2).

10. The number you say first in counting is always less than the number that comes next.

In the early development of number, students are regularly encouraged to think that number

TABLE 1

Some commonly used language "expires " and should be replaced with more appropriate alternatives.

Expired mathematical language and suggested alternatives

What is stated	What should be stated
Using the words *borrowing or carrying* when subtracting or adding, respectively	Use *trading* or *regrouping* to indicate the actual action of trading or exchanging one place value unit for another unit.
Using the phrase ___ *out of* __ to describe a fraction, for example, one out of seven to describe $\frac{1}{7}$	Use the fraction and the attribute. For example, say *one-seventh of the length of the string.* The *out of* language often causes students to think a part is being subtracted from the whole amount (Philipp, Cabral, and Schappelle 2005).
Using the phrase *reducing fractions*	Use *simplifying fractions*. The language of *reducing* gives students the incorrect impression that the fraction is getting smaller or being reduced in size.
Asking how shapes are *similar* when children are comparing a set of shapes	Ask, *How are these shapes the same? How are the shapes different?* Using the word *similar* in these situations can eventually confuse students about the mathematical meaning of similar, which will be introduced in middle school and relates to geometric figures.
Reading the equal sign as *makes*, for example, saying, *Two plus two makes four* for 2 + 2 = 4	Read the equation 2 + 2 = 4 as *Two plus two equals or is the same as four.* The language *makes* encourages the misconception that the equal sign is an action or an operation rather than representative of a relationship.
Indicating that a number *divides evenly* into another number	Say that a number *divides* another number a whole number of times or that it *divides without a remainder.*
Plugging a number into an expression or equation	Use *substitute values* for an unknown.
Using *top number* and *bottom number* to describe the numerator and denominator of a fraction, respectively	Students should see a fraction as one number, not two separate numbers. Use the words *numerator* and *denominator* when discussing the different parts of a fraction.

relationships are fixed. For example, the relationship between 3 and 8 is always the same. To determine the relationship between two numbers, the numbers must implicitly represent a count made by using the same unit. But when units are different, these relationships change. For example, three dozen eggs is more than eight eggs, and three feet is more than eight inches. Expiration date: Grade 2 (2.MD.2).

11. The longer the number, the larger the number.

The length of a number, when working with whole numbers that differ in the number of digits, does indicate this relationship or magnitude. However, it is particularly troublesome to apply this rule to decimals (e.g., thinking that 0.273 is larger than 0.6), a misconception noted by Desmet, Grégoire, and Mussolin (2010). Expiration date: Grade 4 (4.NF.7).

12. Please Excuse My Dear Aunt Sally.

This phrase is typically taught when students begin solving numerical expressions involving multiple operations, with this mnemonic serving as a way of remembering the order of operations. Three issues arise with the application of this rule. First, students incorrectly believe that they should always do multiplication before division, and addition before subtraction, because of the order in which they appear in the mnemonic PEMDAS (Linchevski and Livneh 1999). Second, the order is not as strict as students are led to believe. For example, in the expression $3^2 - 4(2 + 7) + 8 \div 4$, students have options as to where they might start. In this case, they may first simplify the 2 + 7 in the grouping symbol, simplify 3^2, or divide before doing any other computation—all without affecting the outcome. Third, the *P* in PEMDAS suggests that parentheses are first, rather than

Other rules that expire

We invite *Teaching Children Mathematics* (*TCM*) readers to submit additional instances of "rules that expire" or "expired language" that this article does not address. If you would like to share an example, please use the format of the article, stating the rule to avoid, a case of how it expires, and when it expires in the Common Core State Standards for Mathematics. If you submit an illustration of expired language, include "What is stated" and "What should be stated" (see **table 1**).

Join us as we continue this conversation on *TCM*'s blog at **www.nctm.org/TCMblog/MathTasks** or send your suggestions and thoughts to **tcm@nctm.org**. We look forward to your input.

grouping symbols more generally, which would include brackets, braces, square root symbols, and the horizontal fraction bar. Expiration date: Grade 6 (6.EE.2).

13. The equal sign means *Find the answer* or *Write the answer*.
An equal sign is a relational symbol. It indicates that the two quantities on either side of it represent the same amount. It is not a signal prompting the answer through an announcement to "do something" (Falkner, Levi, and Carpenter 1999; Kieran 1981). In an equation, students may see an equal sign that expresses the relationship but cannot be interpreted as *Find the answer*. For example, in the equations below, the equal sign provides no indication of an answer. Expiration date: Grade 1 (1.OA.7).

$$6 = \square + 4$$
$$3 + x = 5 + 2x$$

Expired language

In addition to helping students avoid the thirteen rules that expire, we must also pay close attention to the mathematical language we use as teachers and that we allow our students to use. The language we use to discuss mathematics (see **table 1**) may carry with it connotations that result in misconceptions or misuses by students, many of which relate to the Thirteen Rules That Expire listed above. Using accurate and precise vocabulary (which aligns closely with SMP 6) is an important part of developing student understanding that supports student learning and withstands the need for complexity as students progress through the grades.

No expiration date

One characteristic of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) is to have fewer, but deeper, more rigorous standards at each grade—and to have less overlap and greater coherence as students progress from K–grade 12. We feel that by using consistent, accurate rules and precise vocabulary in the elementary grades, teachers can play a key role in building coherence as students move from into the middle grades and beyond. No one wants students to realize in the upper elementary grades or in middle school that their teachers taught "rules" that do not hold true.

With the implementation of CCSSM, now is an ideal time to highlight common instructional practices that teachers can tweak to better prepare students and allow them to have smoother transitions moving from grade to grade. Additionally, with the implementation of CCSSM, many teachers—even those teaching the same grade as they had previously—are being required to teach mathematics content that differs from what they taught in the past. As teachers are planning how to teach according to new standards, now is a critical point to think about the rules that should or should not be taught and the vocabulary that should or should not be used in an effort to teach in ways that do not "expire."

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Progression of Number Concepts for Young Children

The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. They represent expectations for children, but each child will reach these indicators at their own pace and in their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

	~ 3 years old	~ 4 years old	~ 5 years old	~ End of kindergarten
Verbally count	Recites number words to 10 with occasional errors	Recites number words to 20, with occasional errors most likely in the teens	Recites number words to 40, with occasional errors most likely in the teens	Counts to 100 by ones and tens
Count objects	Uses one-to-one correspondence for small groups of objects (under 5)	Uses one-to-one correspondence when counting (up to 10 objects)	Uses one-to-one correspondence when counting (up to 15)	Uses one-to-one correspondence when counting (up to 25)
Cardinality	Begins to understand that the last number name said tells the total number of objects in a group	Understands that the last number said tells the number of objects counted; begins to count out objects up to 5; tells the number of objects counted for small numbers (<6)	Understands that the last number name said tells the number of objects counted; can count out objects up to 10	Counts to answer how many for up to 20 objects arranged in a line, array, or circle, or up to 10 in a scattered configuration; can count out objects up to 20
Subitize	Begins to recognize the number of objects in a group of two or three without counting	Quickly sees how many for 1, 2, and 3 objects; may begin to subitize visually or conceptually up to 5 objects (by seeing 2 and 3)	Quickly sees how many with 1–10 objects when they are in a familiar arrangement; uses chunking for numbers 6–10 with a 5 group (array, fingers, dice pattern)	Quickly sees how many with 1–10 objects when they are in a familiar arrangement; uses chunking for numbers 6–10 with a 5 group (array, fingers, dice pattern)
Read and write numerals	Identifies numerals as being different than letters and identifies some, such as 3	Reads numerals 1–5; may begin to write numerals	Reads numerals 1–10, begins to write numerals	Reads and writes numerals 0–20
Compare numbers	Uses language to compare the number of objects in two groups (*more, less, same*)	Begins using strategies to find which is more for two numbers ≤ 5	Uses counting to find which is more for two numbers ≤ 5; uses the words *less (fewer) than/ more than/same as*	Identifies whether the number of objects in one group is greater than, less than, or equal to another group of objects; compares two written numerals between 1 and 10
Composing and decomposing numbers	Knows the whole is bigger than the parts, but may not know by how many	Beginning to know number combinations up to 4 or 5 (4 has 3 and 1 in it)	Uses objects or fingers to decompose numbers <5 into its parts (5 has 4 and 1 inside it); names parts of numbers up to 5	Decomposes numbers to 10 into pairs using objects, drawings, and/or equations. Knows the pairs that make 10. Fluently adds and subtracts within 5.

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4. Facilitate meaningful mathematical discourse.	• Provide students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. • Advance the mathematical thinking of the whole class by talking and sharing aloud.	• Engages students in explaining their mathematical reasoning in small group and classroom situations. • Facilitates discussions among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Explain their ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of others. • Ask questions of others to make sense of their ideas.

Teaching Practice	Purpose	What the Teacher Does	What the Students Do
5. Pose purposeful questions.	• Reveal students' current understanding of a concept. • Encourage students to explain, elaborate, and clarify their thinking. • Make the learning of mathematics more visible and accessible for students.	• Asks questions that build on and extend student thinking. • Is intentional about the kinds of questions asked to make the mathematics more visible to students. • Uses wait time to provide students with time to think and examine their ideas.	• Think more deeply about the process of the mathematics rather than simply focusing on the answer. • Listen to and comment on the explanations of others in the class.
6. Build procedural fluency from conceptual understanding.	• Provide experiences with concrete materials that allow students to make sense of important mathematics and flexibly choose from a variety of methods to solve problems.	• Provides opportunities for students to reason about mathematical ideas. • Expects students to explain why their strategies work. • Connects student methods to efficient procedures as appropriate.	• Understand and explain the procedures they are using and why they work. • Use a variety of strategies to solve problems and make sense of mathematical ideas. • Do not rely on shortcuts or tricks to do mathematics.
7. Support productive struggle in learning mathematics.	• Provide opportunities for productive struggle, which is significant and essential to learning mathematics with understanding. • Allow students to grapple with ideas and relationships. • Give students ample time to work with and make sense of new ideas, which is critical to their learning with understanding.	• Supports student struggle without showing and telling a procedure but rather focusing on the important mathematical ideas. • Asks questions that scaffold and advance student thinking. • Builds questions and plans lessons based on important student misconceptions rather than focusing on the correct answer. • Recognizes the importance of effort as students work to make sense of new ideas.	• Stick to a task and recognize that struggle is part of making sense. • Ask questions that will help them to better understand the task. • Support each other with ideas rather than telling others the answer or how to solve a problem.
8. Elicit and use evidence of student thinking.	• Elicit and use evidence of student thinking, which helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. • Assess student thinking and understanding by using formative assessment through student written and oral ideas.	• Determines what to look for in gathering evidence of student learning. • Poses questions and answers student questions that provide information about student understanding, strategies, and reasoning. • Uses evidence to determine next steps of instruction.	• Accept that reasoning and understanding are as important as the answer to a problem. • Use mistakes and misconceptions to rethink their understanding. • Ask questions of the teacher and peers to clarify confusion or misunderstanding. • Assess progress toward developing mathematical understanding.

Source: Adapted from National Council of Teachers of Mathematics (2014).

13 RULES *That Expire*

Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in students' math careers.

By Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty

FUSE/THINKSTOCK

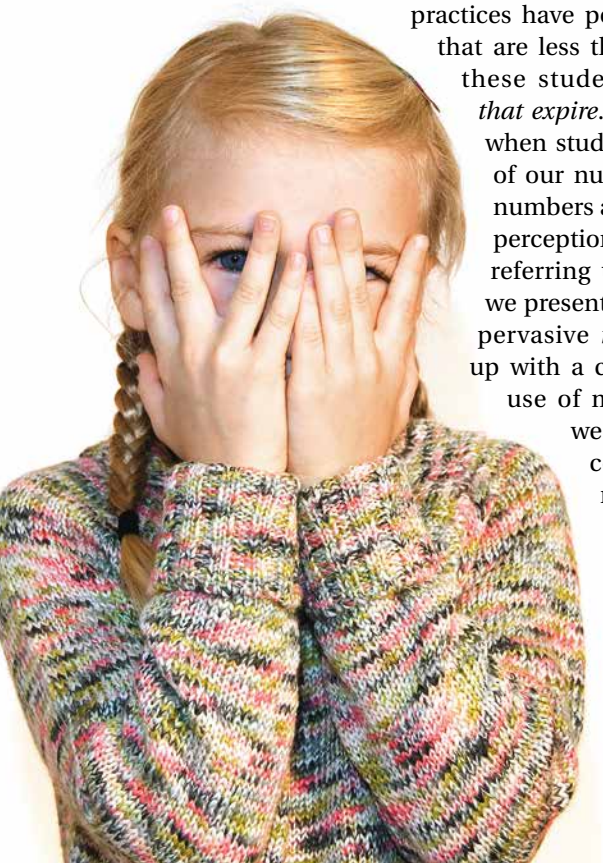
Imagine the following scenario: A primary teacher presents to her students the following set of number sentences:

$$3 + 5 = \square$$
$$\square + 2 = 7$$
$$8 = \square + 3$$
$$2 + 4 = \square + 5.$$

Stop for a moment to think about which of these number sentences a student in your class would solve first or find easiest. What might they say about the others? In our work with young children, we have found that students feel comfortable solving the first equation because it "looks right" and students can interpret the equal sign as *find the answer*. However, students tend to hesitate at the remaining number sentences because they have yet to interpret and understand the equal sign as a symbol indicating a relationship between two quantities (or amounts) (Mann 2004).

In another scenario, an intermediate student is presented with the problem 43.5 × 10. Immediately, he responds, "That's easy; it is 43.50 because my teacher said that when you multiply any number times ten, you just add a zero at the end."

In both these situations, hints or repeated practices have pointed students in directions that are less than helpful. We suggest that these students are experiencing *rules that expire*. Many of these rules "expire" when students expand their knowledge of our number systems beyond whole numbers and are forced to change their perception of what can be included in referring to *a number*. In this article, we present what we believe are thirteen pervasive *rules that expire*. We follow up with a conversation about incorrect use of mathematical language, and we present alternatives to help counteract common student misunderstandings.



ALINA555/THINKSTOCK

The Common Core State Standards (CCSS) for Mathematical Practice advocate for students to become problem solvers who can reason, apply, justify, and effectively use appropriate mathematics vocabulary to demonstrate their understanding of mathematics concepts (CCSSI 2010). This, in fact, is quite opposite of the classroom in which the teacher does most of the talking and students are encouraged to memorize facts, "tricks," and tips to make the mathematics "easy." The latter classroom can leave students with a collection of explicit, yet arbitrary, rules that do not link to reasoned judgment (Hersh 1997) but instead to learning without thought (Boaler 2008). The purpose of this article is to outline common rules and vocabulary that teachers share and elementary school students tend to overgeneralize—tips and tricks that do not promote conceptual understanding, rules that "expire" later in students' mathematics careers, or vocabulary that is not precise. As a whole, this article aligns to Standard of Mathematical Practice (SMP) 6: *Attend to precision*, which states that mathematically proficient students "…try to communicate precisely to others. …use clear definitions … and … carefully formulated explanations…" (CCSSI 2010, p. 7). Additionally, we emphasize two other mathematical practices: SMP 7: *Look for and make use of structure* when we take a look at properties of numbers; and SMP 2: *Reason abstractly and quantitatively* when we discuss rules about the meaning of the four operations.

"Always" rules that are not so "always"

In this section, we point out rules that seem to hold true at the moment, given the content the student is learning. However, students later find that these rules are not always true; in fact, these rules "expire." Such experiences can be frustrating and, in students' minds, can further the notion that mathematics is a mysterious series of tricks and tips to memorize rather than big concepts that relate to one another. For each rule that expires, we do the following:

1. State the rule that teachers share with students.
2. Explain the rule.
3. Discuss how students inappropriately overgeneralize it.
4. Provide counterexamples, noting when the rule is not true.

5. State the "expiration date" or the point when the rule begins to fall apart for many learners. We give the expiration date in terms of grade levels as well as CCSSM content standards in which the rule no longer "always" works.

Thirteen rules that expire

1. When you multiply a number by ten, just add a zero to the end of the number.

This "rule" is often taught when students are learning to multiply a whole number times ten. However, this directive is not true when multiplying decimals (e.g., $0.25 \times 10 = 2.5$, not 0.250). Although this statement may reflect a regular pattern that students identify with whole numbers, it is not generalizable to other types of numbers. Expiration date: Grade 5 (5.NBT.2).

2. Use keywords to solve word problems.

This approach is often taught throughout the elementary grades for a variety of word problems. Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside of the problem context (Clement and Bernhard 2005). Unfortunately, many keywords are common English words that can be used in many different ways. Yet, a list of keywords is often given so that word problems can be translated into a symbolic, computational form. Students are sometimes told that if they see the word *altogether* in the problem, they should always add the given numbers. If they see *left* in the problem, they should always subtract the numbers. But reducing the meaning of an entire problem to a simple scan for key words has inherent challenges. For example, consider this problem:

> John had 14 marbles in his left pocket. He had 37 marbles in his right pocket. How many marbles did John have?

If students use keywords as suggested above, they will subtract without realizing that the problem context requires addition to solve. Keywords become particularly troublesome when students begin to explore multistep word problems, because they must decide which keywords work with which component of the problem. Keywords can be informative but must be used in conjunction with all other words in the problem to grasp the full meaning. Expiration date: Grade 3 (3.OA.8).

3. You cannot take a bigger number from a smaller number.

Students might hear this phrase as they first learn to subtract whole numbers. When students are restricted to only the set of whole numbers, subtracting a larger number from a smaller one results in a negative number, an integer that is not in the set of whole numbers, so this rule is true. Later, when students encounter application or word problems involving contexts that include integers, students learn that this "rule" is not true for all problems. For example, a grocery store manager keeps the temperature of the produce section at 4 degrees Celsius, but this is 22 degrees too hot for the frozen food section. What must the temperature be in the frozen food section? In this case, the answer is a negative number, ($4^{\circ} - 22^{\circ} = -18^{\circ}$). Expiration date: Grade 7 (7.NS.1).

4. Addition and multiplication make numbers bigger.

When students begin learning about the operations of addition and multiplication, they are often given this rule as a means to develop a generalization relative to operation sense. However, the rule has multiple counterexamples. Addition with zero does not create a sum larger than either addend. It is also untrue when adding two negative numbers (e.g., $-3 + -2 = -5$), because –5 is less than both addends. In the case of the equation below, the product is smaller than either factor.

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

This is also the case when one of the factors is a negative number and the other factor is positive, such as $-3 \times 8 = -24$. Expiration date: Grade 5 (5.NF.4 and 5.NBT.7) and again at Grade 7 (7.NS.1 and 7.NS.2).

5. Subtraction and division make numbers smaller.

This rule is commonly heard in grade 3: both subtraction and division will result in an answer that is smaller than at least one of the

numbers in the computation. When numbers are positive whole numbers, decimals, or fractions, subtracting will result in a number that is smaller than at least one of the numbers involved in the computation. However, if the subtraction involves two negative numbers, students may notice a contradiction (e.g., –5 – (–8) = 3). In division, the rule is true if the numbers are positive whole numbers, for example:

$$8 \div 4 = 2 \text{ or } 4 \div 8 = \frac{1}{2}$$

However, if the numbers you are dividing are fractions, the quotient may be larger:

$$\frac{1}{4} \div \frac{2}{5} = \frac{5}{8}$$

This is also the case when dividing two negative factors: (e.g., –9 ÷ –3 = 3). Expiration dates: Grade 6 (6.NS.1) and again at Grade 7 (7.NS.1 and 7.NS.2c).

6. You always divide the larger number by the smaller number.

This rule may be true when students begin to learn their basic facts for whole-number division and the computations are not contextually based. But, for example, if the problem states that Kate has 2 cookies to divide among herself and two friends, then the portion for each person is 2 ÷ 3. Similarly, it is possible to have a problem in which one number might be a fraction:

> Jayne has $\frac{1}{2}$ of a pizza and wants to share it with her brother. What portion of the whole pizza will each get?

In this case, the computation is as follows:

$$\frac{1}{2} \div 2 = \frac{1}{4}$$

Expiration date: Grade 5 (5.NF.3 and 5.NF.7).

7. Two negatives make a positive.

Typically taught when students learn about multiplication and division of integers, rule 7 is to help them determine the sign of the product or quotient. However, this rule does not always hold true for addition and subtraction of integers, such as in –5 + (–3) = –8. Expiration date: Grade 7 (7.NS.1).

8. Multiply everything inside the parentheses by the number outside the parentheses.

As students are developing the foundational skills linked to order of operations, they are often told to first perform multiplication on the numbers (terms) within the parentheses. This holds true only when the numbers or variables inside the parentheses are being added or subtracted, because the distributive property is being used, for example, 3(5 + 4) = 3 × 5 + 3 × 4. The rule is untrue when multiplication or division occurs in the parentheses, for example, 2 (4 × 9) ≠ 2 × 4 × 2 × 9. The 4 and the 9 are not two separate terms, because they are not separated by a plus or minus sign. This error may not emerge in situations when students encounter terms that do not involve the distributive property or when students use the distributive property without the element of terms. The confusion seems to be an interaction between students' partial understanding of terms and their partial understanding of the distributive property—which may not be revealed unless both are present. Expiration date: Grade 5 (5.OA.1).

9. Improper fractions should always be written as a mixed number.

When students are first learning about fractions, they are often taught to always change improper fractions to mixed numbers, perhaps so they can better visualize how many *wholes* and *parts* the number represents. This rule can certainly help students understand that positive mixed numbers can represent a value greater than one whole, but it can be troublesome when students are working within a specific mathematical context or real-world situation that requires them to use improper fractions. This frequently first occurs when students begin using improper fractions to compute and again when students later learn about the slope of a line and must represent the slope as the rise/run, which is sometimes appropriately and usefully expressed as an improper fraction. Expiration dates: Grade 5 (5.NF.1) and again in Grade 7 (7.RP.2).

10. The number you say first in counting is always less than the number that comes next.

In the early development of number, students are regularly encouraged to think that number

TABLE 1

Some commonly used language "expires " and should be replaced with more appropriate alternatives.

Expired mathematical language and suggested alternatives

What is stated	What should be stated
Using the words *borrowing or carrying* when subtracting or adding, respectively	Use *trading* or *regrouping* to indicate the actual action of trading or exchanging one place value unit for another unit.
Using the phrase ___ *out of* __ to describe a fraction, for example, one out of seven to describe $\frac{1}{7}$	Use the fraction and the attribute. For example, say *one-seventh of the length of the string.* The *out of* language often causes students to think a part is being subtracted from the whole amount (Philipp, Cabral, and Schappelle 2005).
Using the phrase *reducing fractions*	Use *simplifying fractions*. The language of *reducing* gives students the incorrect impression that the fraction is getting smaller or being reduced in size.
Asking how shapes are *similar* when children are comparing a set of shapes	Ask, *How are these shapes the same? How are the shapes different?* Using the word *similar* in these situations can eventually confuse students about the mathematical meaning of similar, which will be introduced in middle school and relates to geometric figures.
Reading the equal sign as *makes*, for example, saying, *Two plus two makes four* for 2 + 2 = 4	Read the equation 2 + 2 = 4 as *Two plus two equals or is the same as four.* The language *makes* encourages the misconception that the equal sign is an action or an operation rather than representative of a relationship.
Indicating that a number *divides evenly* into another number	Say that a number *divides* another number a whole number of times or that it *divides without a remainder.*
Plugging a number into an expression or equation	Use *substitute values* for an unknown.
Using *top number* and *bottom number* to describe the numerator and denominator of a fraction, respectively	Students should see a fraction as one number, not two separate numbers. Use the words *numerator* and *denominator* when discussing the different parts of a fraction.

relationships are fixed. For example, the relationship between 3 and 8 is always the same. To determine the relationship between two numbers, the numbers must implicitly represent a count made by using the same unit. But when units are different, these relationships change. For example, three dozen eggs is more than eight eggs, and three feet is more than eight inches. Expiration date: Grade 2 (2.MD.2).

11. The longer the number, the larger the number.

The length of a number, when working with whole numbers that differ in the number of digits, does indicate this relationship or magnitude. However, it is particularly troublesome to apply this rule to decimals (e.g., thinking that 0.273 is larger than 0.6), a misconception noted by Desmet, Grégoire, and Mussolin (2010). Expiration date: Grade 4 (4.NF.7).

12. Please Excuse My Dear Aunt Sally.

This phrase is typically taught when students begin solving numerical expressions involving multiple operations, with this mnemonic serving as a way of remembering the order of operations. Three issues arise with the application of this rule. First, students incorrectly believe that they should always do multiplication before division, and addition before subtraction, because of the order in which they appear in the mnemonic PEMDAS (Linchevski and Livneh 1999). Second, the order is not as strict as students are led to believe. For example, in the expression $3^2 - 4(2 + 7) + 8 \div 4$, students have options as to where they might start. In this case, they may first simplify the 2 + 7 in the grouping symbol, simplify 3^2, or divide before doing any other computation—all without affecting the outcome. Third, the *P* in PEMDAS suggests that parentheses are first, rather than

Other rules that expire

We invite *Teaching Children Mathematics* (*TCM*) readers to submit additional instances of "rules that expire" or "expired language" that this article does not address. If you would like to share an example, please use the format of the article, stating the rule to avoid, a case of how it expires, and when it expires in the Common Core State Standards for Mathematics. If you submit an illustration of expired language, include "What is stated" and "What should be stated" (see **table 1**).

Join us as we continue this conversation on *TCM*'s blog at **www.nctm.org/TCMblog/MathTasks** or send your suggestions and thoughts to **tcm@nctm.org**. We look forward to your input.

grouping symbols more generally, which would include brackets, braces, square root symbols, and the horizontal fraction bar. Expiration date: Grade 6 (6.EE.2).

13. The equal sign means *Find the answer* or *Write the answer*.
An equal sign is a relational symbol. It indicates that the two quantities on either side of it represent the same amount. It is not a signal prompting the answer through an announcement to "do something" (Falkner, Levi, and Carpenter 1999; Kieran 1981). In an equation, students may see an equal sign that expresses the relationship but cannot be interpreted as *Find the answer*. For example, in the equations below, the equal sign provides no indication of an answer. Expiration date: Grade 1 (1.OA.7).

$$6 = \square + 4$$
$$3 + x = 5 + 2x$$

Expired language

In addition to helping students avoid the thirteen rules that expire, we must also pay close attention to the mathematical language we use as teachers and that we allow our students to use. The language we use to discuss mathematics (see **table 1**) may carry with it connotations that result in misconceptions or misuses by students, many of which relate to the Thirteen Rules That Expire listed above. Using accurate and precise vocabulary (which aligns closely with SMP 6) is an important part of developing student understanding that supports student learning and withstands the need for complexity as students progress through the grades.

No expiration date

One characteristic of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) is to have fewer, but deeper, more rigorous standards at each grade—and to have less overlap and greater coherence as students progress from K–grade 12. We feel that by using consistent, accurate rules and precise vocabulary in the elementary grades, teachers can play a key role in building coherence as students move from into the middle grades and beyond. No one wants students to realize in the upper elementary grades or in middle school that their teachers taught "rules" that do not hold true.

With the implementation of CCSSM, now is an ideal time to highlight common instructional practices that teachers can tweak to better prepare students and allow them to have smoother transitions moving from grade to grade. Additionally, with the implementation of CCSSM, many teachers—even those teaching the same grade as they had previously—are being required to teach mathematics content that differs from what they taught in the past. As teachers are planning how to teach according to new standards, now is a critical point to think about the rules that should or should not be taught and the vocabulary that should or should not be used in an effort to teach in ways that do not "expire."

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Progression of Number Concepts for Young Children

The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. They represent expectations for children, but each child will reach these indicators at their own pace and in their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

	~ 3 years old	~ 4 years old	~ 5 years old	~ End of kindergarten
Verbally count	Recites number words to 10 with occasional errors	Recites number words to 20, with occasional errors most likely in the teens	Recites number words to 40, with occasional errors most likely in the teens	Counts to 100 by ones and tens
Count objects	Uses one-to-one correspondence for small groups of objects (under 5)	Uses one-to-one correspondence when counting (up to 10 objects)	Uses one-to-one correspondence when counting (up to 15)	Uses one-to-one correspondence when counting (up to 25)
Cardinality	Begins to understand that the last number name said tells the total number of objects in a group	Understands that the last number said tells the number of objects counted; begins to count out objects up to 5; tells the number of objects counted for small numbers (<6)	Understands that the last number name said tells the number of objects counted; can count out objects up to 10	Counts to answer how many for up to 20 objects arranged in a line, array, or circle, or up to 10 in a scattered configuration; can count out objects up to 20
Subitize	Begins to recognize the number of objects in a group of two or three without counting	Quickly sees how many for 1, 2, and 3 objects; may begin to subitize visually or conceptually up to 5 objects (by seeing 2 and 3)	Quickly sees how many with 1–10 objects when they are in a familiar arrangement; uses chunking for numbers 6–10 with a 5 group (array, fingers, dice pattern)	Quickly sees how many with 1–10 objects when they are in a familiar arrangement; uses chunking for numbers 6–10 with a 5 group (array, fingers, dice pattern)
Read and write numerals	Identifies numerals as being different than letters and identifies some, such as 3	Reads numerals 1–5; may begin to write numerals	Reads numerals 1–10, begins to write numerals	Reads and writes numerals 0–20
Compare numbers	Uses language to compare the number of objects in two groups (*more, less, same*)	Begins using strategies to find which is more for two numbers ≤ 5	Uses counting to find which is more for two numbers ≤ 5; uses the words *less (fewer) than/ more than/same as*	Identifies whether the number of objects in one group is greater than, less than, or equal to another group of objects; compares two written numerals between 1 and 10
Composing and decomposing numbers	Knows the whole is bigger than the parts, but may not know by how many	Beginning to know number combinations up to 4 or 5 (4 has 3 and 1 in it)	Uses objects or fingers to decompose numbers <5 into its parts (5 has 4 and 1 inside it); names parts of numbers up to 5	Decomposes numbers to 10 into pairs using objects, drawings, and/or equations. Knows the pairs that make 10. Fluently adds and subtracts within 5.

Intervention Planning Forms

Date: **Focus Skill:**

Focus Math Practice:

Small Group Members:

Warm Up: **(ongoing review)**	
Skill Lesson: **C—P—A**	
Math Fact Practice: **(strategy focus)**	
Assessment	

Observations and Next Steps:

Tier 2 Intervention Planning Sheet

Student Name _____

Formal Assessment Data:

Math Observations and Conferencing

Student Entry Points : (Interests, Strengths, Learning Styles, etc.)	Math Goal(s):

Date	Intervention/Instruction/Extension	Observation	Level (1-2-3-4)	Next Steps

Date: **Focus Skill:**

Focus Math Practice:

Small Group Members:

Warm Up: **(ongoing review)**	
Skill Lesson: **C—P—A**	
Math Fact Practice: **(strategy focus)**	
Assessment	

Observations and Next Steps:

Tier 2 Intervention Planning Sheet

Student Name _____

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Math Observations and Conferencing

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